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Network endurance against cascading overload failure

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ABSTRACT

Network endurance can be regarded as the upper limit of survival time before the system's complete breakdown, which is highly related to system resilience. Although network endurance against overload failure is critical for network design and operational management, the definition and corresponding evaluation method still remain challenging. In this paper, based on the load-dependent overload model, we define network endurance as the cascade duration at criticality before the complete network breakdown and develop an approach for endurance evaluation. We find that network endurance highly depends on initial disturbance intensity and cascade intensity. The network endurance with a uniform initial load distribution usually monotonically increases with decreasing initial disturbance intensity, while for other initial load distributions endurance behaviors are more complicated. We also provide theoretical analysis for the network endurance. Our findings may help to understand the network reliability mechanism against cascading overload failures and design a highly reliable network.

1. Introduction

Many complex systems such as power grid, transportation, and telecommunication system can be modeled as a complex network, where nodes represent elements of the system and edges stand for the interaction between nodes [1]. With the growing complexity of the network, the probability of cascading failure shift to an unprecedented level. Many accidents reveal that a small local perturbation can cause large-scale damage to the system [2,3]. For example, the major blackout of 2003 Northern America, originated from outage of one transmission line, has caused a large swath of districts paralyzed with more than 4 billion dollars financial loss [4]. For the city traffic system, jams have also become a major threat to system operational reliability [5]. Under certain conditions, congestion may be up to several kilometers in the highway. In Germany alone, the direct and indirect economic costs caused by traffic congestion are estimated at around 37.34 billion dollars in 2020 reported by Cebr [6]. With various increasing internal or external disturbances, large-scale collapse of system draws much research attention and is found as a phase transition of cascading failures between system function and failure [7]. Cascading failures take place under certain initial disturbance, leading to an avalanche of overloads on other nodes. As cascading failures continue, the failure size gradually or abruptly increases. The system will quickly lose its functionality and collapse when the network approaches the critical point. From the viewpoint of system reliability, the time from the start of the cascading failure to the end, indicates the maximal time opportunity allowing for repair, which we define as network endurance here. For example, D Zhou [8] et al have discussed the length of the cascade in a model of interdependent percolation. In this way, system resilience [5] is highly dependent on system endurance, and improving network endurance can be one of the possible ways to avoid complete system collapse against cascading failure.

For complex systems, system endurance depends on the cascading failure mechanism. Different system failure models are presented to explain the complicated cascading failures mechanism [9–12]. These models can be separated into two categories, i.e., static and dynamical. For the first type, studies quantify the reliability of a network by the network performance under removal of a fraction of nodes or edges without considering flow dynamics and relevant redistribution of load. Monte Carlo simulation is a generic method to study the static tolerance

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to both random and targeted attacks [13,14]. Besides numerical simulations, the problem can also be theoretically solved by percolation theory [15], where site and bond percolation are two types of standard models [16].

For the dynamical model, research mainly focuses on the effect of overloads with no visible or direct causality between component failures [17–19]. Instead, the coupling relation between failures is reflected by the redistribution of network flow as a global effect. For example in the power grid, when a line trips, its load in the form of power flow, will transfer to other functional parts through invisible alternative paths. A great number of models, including sand-pile model [20], CASCADE model [21], and ORNL-PSerc-Alaska (OPA) model [22], are proposed to account for the dynamical aspects of cascading overloads. For transportations, Li et al. [1] proposed a new method to study the congestion cascade with dynamical percolation. Based on geometry analysis, the propagation of cascading overloads is studied and predicted with theoretical framework [19].

To meet the challenge of cascading failures, system resilience enginnering [23] is proposed to build the ability of adaption and recover from perturbations. Meanwhile, resilience is found an intrinstic property of complex systems [5], many qualitative and quantitative evaluation procedures recently have been presented to describe the conceptual framework and assessment approaches [24–28]. To realize the recovery ability, different functional-based and structural-based self-healing models are proposed based on cascading failure models to investigate how recovery strategies can enhance system resilience. For example, Liu et al. develop two models of self-healing strategies for a single network based on global or local information respectively [18,29]. The restoration characteristics of the interdependent network are also investigated considering repair resource, timing and load tolerance for different coupling strength [30]. These models or strategies facilitate recovery in case of system collapse under the threat of cascading overload failure.

System endurance, measuring the survival time before system collapse, is critical for designing self-healing strategy. However, a valid definition for network endurance has not yet been developed. Furthermore, the relation between system endurance and cascading overloads is essentially unclear. Considering the relevant cascading failure model, here we wish to develop the definition of network endurance and understanding the relation between system endurance and cascading overloads mechanism.

In this study, we develop a method to analyze the system network endurance based on analysis of critical threshold. In Section 2, the CASCADE model is described in detail, and three different initial load distributions are considered. The proposed method for endurance analysis is described in Section 3. Simulation experiments for the endurance analysis framework are conducted in Section 4. In Section 5, we develop a theoretical method to understand the relation between cascading failure process and network endurance. Section 6 concludes the work.

2. Preliminaries

A cascading overload model is considered to characterize the feature of network flow redistribution during cascading failures.

2.1. Cascading failure model

Overload model is commonly used to capture the basic cascade dynamics of component failures in complex systems. Model of CASCADE is originally developed by Dobson et al. to study the cascading failure dynamics of power grid [21]. They use Galton-Watson branching process to analytically solve the size distribution of blackout. Based on this model, we study the network endurance against cascading failures due to redistribution of component load. Our theoretical analysis is for general network structure. Here, we take the square lattice as an example for analysis. Each node i has a random initial load L_i distributed in $[L_{\min}, L_{\max}]$, where $L_{\min}$ and $L_{\max}$ are respectively the lower and upper limits of the load distribution. Firstly, an initial disturbance D is exogenously imposed on each functional node (the load of a node is below load tolerance) to initiate the cascade process. A node fails if its load exceeds the limit of operation . When a node fails, it affects other nodes in the network through transmitting a fixed amount of load I' to all remaining functional nodes. A failure node can cause redistribution of additional loads to other functional nodes, which may in turn cause further overloads of other nodes. This cascade process continues in such a domino effect until no further nodes are overloaded. The disintegration of networks depends on the failure mechanism and network structures.

Network endurance is the cascade duration when system is at the critical threshold, which has a deep relation with cascading overload dynamics. To find this relation, we need two important parameters including cascade intensity coefficient λ and initial disturbance coefficient θ, which are defined as

$$\begin{cases} \lambda = N \cdot I' \\ \theta = N \cdot D \end{cases} \tag{1}$$

where N represents the network size and I' is the normalized load increment of each remaining functional node. Since the system size N can also increase the total overloads for a given cascading intensity, we need to scale the system parameters with N.

The simulation algorithm of the cascade process can be summarized as follows:

(1) For all nodes at stage 0, they are initially assigned with a load following certain distribution, with $L_i \in [L_{\min}, L_{\max}]$, $0 \le L_{\min} \le L_{\max} = L^{fail} = 1$. Here, the load distribution can be drawn from real systems and several possibilities, such as uniform, exponential and Gaussian distributions, will be considered in the later Section 2.2;
(2) Set $T = 1$, and add an initial disturbance $D_0 = D$ of additional load to each functional node;
(3) Test each functional node: for $i = 1, 2, \ldots, n$, if $L_i > L^{fail} = 1$, then the node i fails. Suppose m_T nodes fail at step T;
(4) A load increment $D_T = m_{T-1} \cdot I' = m_{T-1} \cdot \frac{\lambda}{N}$ is added to each remaining functional node, where I' is a normalized parameter representing the load transfer strength of a failed node;
(5) $T = T + 1$ and return to step (3), the cascade process proceeds until no functional nodes fail.

2.2. Initial load distributions

According to the recent observation, the initial load distribution of realistic systems, such as communication network, power grid, and traffic network may present different forms and evolve with time [31,32]. An example can be found in the power grid, where the load of a component is related to the number of transmission lines connected to it [33]. For other networks, such as Internet or air transportation network, the load of a node is positively related to the route choice [34,35]. We assume three initial load distributions including uniform, exponential and Gaussian distributions to study the network endurance under different scenarios. To compare the effect of different initial load distributions, we use the truncated distribution which restricts the domain of a distribution within a specific range for analysis. Under the same average load, we show the truncated distributions as follows.

For a truncated uniform initial load distribution, $L \sim U(L_{\min}, L_{\max})$,

$$f(x) = \begin{cases} \frac{1}{L_{\max} - L_{\min}}, & x \in (L_{\min}, L_{\max}) \\ 0 & elsewhere \end{cases} \tag{2}$$

where the average load $\bar{L} = \int_{L_{\min}}^{L_{\max}} x f(x) dx = (L_{\min} + L_{\max})/2$.

For a truncated Gaussian initial load distribution $f(x) \sim N(\mu, \sigma^2)$, $x \in (L_{min}, L_{max})$

$$f(x) = \begin{cases} \frac{e^{-(x-\mu)^2/2\sigma^2} / \sigma\sqrt{2\pi}}{\Phi(\frac{L_{max}-\mu}{\sigma}) - \Phi(\frac{L_{min}-\mu}{\sigma})}, x \in (L_{min}, L_{max}) \\ 0 \quad elsewhere \end{cases} \tag{3}$$

where $\bar{L} = \int_{L_{min}}^{L_{max}} xf(x)dx = \mu + \sigma^2[f(L_{min}) - f(L_{max})]$ is the average load of the truncated Gaussian distribution. μ is the average value of the Gaussian distribution and σ is the variance.

For a truncated exponential initial load distribution $f(x) \sim E(\xi)$, $x \in (L_{min}, L_{max})$

$$f(x) = \begin{cases} \frac{\xi e^{-\xi x}}{e^{-\xi L_{max}} - e^{-\xi L_{min}}}, x \in (L_{min}, L_{max}) \\ 0 \quad elsewhere \end{cases} \tag{4}$$

where $\bar{L} = \int_{L_{min}}^{L_{max}} xf(x)dx = \frac{1}{\xi} + \frac{1}{\xi}[L_{min}f(L_{min}) - L_{max}f(L_{max})]$ is the average load of the truncated Exponential distribution. ξ is the rate parameter of distribution.

3. Network endurance analysis

To quantify the effect of cascading overloads, we use the order parameter of percolation, i.e., the relative size of the giant component $G(t)$, as an indicator of system integrity. Percolation theory is usually applied to study the network robustness and vulnerability for complex system. Here, the propagation of cascading overloads can be regarded as a dynamical percolation process. The giant component here refers to the cluster that can span the entire network, which is usually regarded as the indicator of network connectivity. For demonstration, we consider a general network for analysis (see Fig. 1 left-hand). As the cascade process proceeds, the giant component gradually fragments into many small clusters. When the network reaches the critical point of breakdown, the network becomes totally disintegrated. We define the system endurance against cascading overload failures as the critical cascade duration, i.e. cascade time steps before the network collapse (see Fig. 1).

Definition 3.1. The network endurance, denoted as T_c, is defined as follows

$$T_c = \max[T(\lambda, D, \bar{L}, N)] \tag{5}$$

where $T(\lambda, D, \bar{L}, N)$ is the cascade duration under certain perturbations, and endurance T_c is the cascade duration under the critical state of system complete breakdown. Here, we obtain network endurance in Eq. (5) by changing the value of only one variable and fixing the other variables. The critical state of the network is determined by the percolation threshold $p = p_c$. This could be marked when SG reaches the maximal value according to the percolation theory [1]. For some cases, other indicators are more effective to mark the critical point [36]. This depends on whether the transition is continuous or discontinuous. We use both of them to analyze the network critical point. Endurance is the upper limit of survival time, where larger endurance provides more opportunities to recover the system, and finally, to improve system resilience.

For the uniform initial load distribution, the cascade intensity coefficient λ represents the mean size of cascading failure upon a single failure, while θ is the mean size of initial failure [21]. λ and θ are respectively two main parameters that determine the relation between cascade dynamics and network endurance, which reveal the failure dependency relation between nodes. Larger λ indicates more cascading overload failures during the failure propagation, while larger θ will result in larger initial failure.

4. Simulation analysis

In this section, we present numerical results of endurance analysis based on cascading overload model mentioned in Section 2.1.

4.1. Analysis of the critical threshold

Each node in the network is perturbed by an additional external load disturbance D. As shown in Fig. 2a, in the beginning, a small perturbation cannot result in system collapse. With the increasing of disturbance added to the network, the system undergoes a phase transition from functional state to collapsed state once it reaches the critical point. As shown in Fig 2a, the size of failure nodes continues to increase with increasing initial disturbance D and network becomes completely collapsed at D_c (D_c corresponds to the dashed line in Fig. 2b). Moreover, with more and more nodes disconnected from the

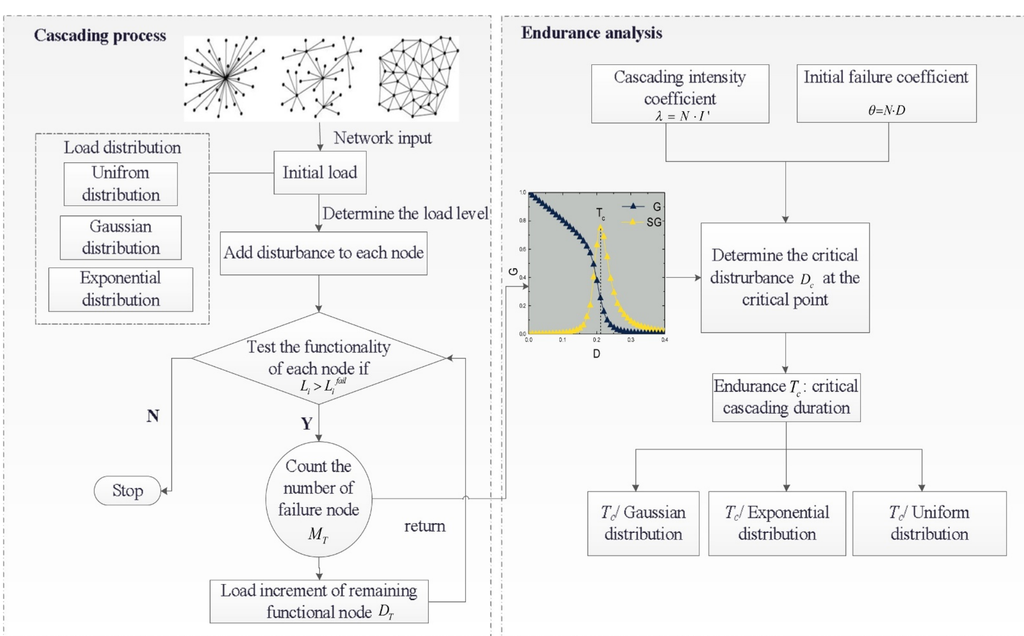

Fig. 1. . A framework for endurance analysis considering cascading failure. G represents the giant component and SG stands for the second largest component. Here, we take the maximal SG for example to determine the critical state.

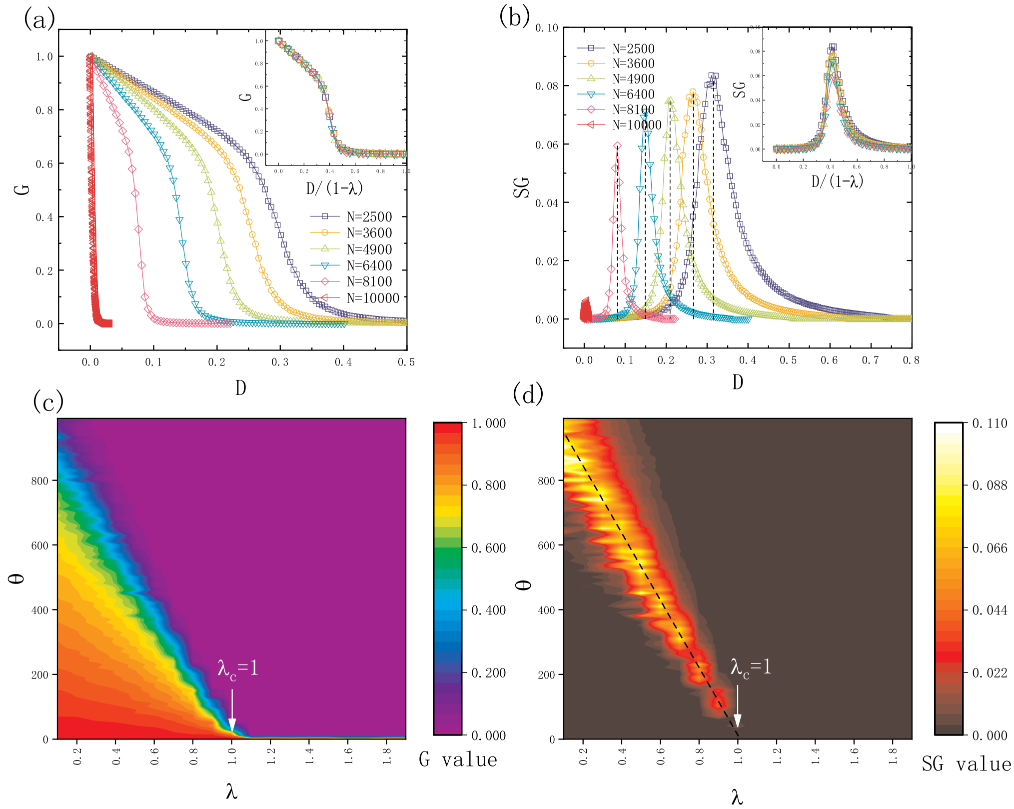

Fig. 2. . The critical threshold of the network with a uniform initial load distribution. (a) The relative size of the giant component (*G*) and (b) the second largest component (*SG*) as a function of disturbance *D*. The insets show the curves of *G* and *SG* when *D* is scaled by $1 - \lambda$. Note that all the rescaled curves collapse into a single curve for varying network size ($0 < \lambda < 1$). Here, cascade intensity is fixed with $I' = 10^{-4}$ with different system sizes. The dashed line represents the critical threshold of the system with the maximal value of *SG*; (c) and (d) are respectively the heat map of *G* and *SG* versus θ and λ when $N = 2500$. The results are averaged over 500 realizations on a square lattice.

giant component, the size of the second largest component *SG* gradually increases to its maximum value at D_c in Fig. 2b, indicating that the network reaches the critical state. In particular, we find that the curves of *G* and *SG* collapse into a single curve for varying network size when *D* is scaled by $1 - \lambda$, suggesting that network performance (*G* and *SG*) is independent of network size under the new indicator $D/(1 - \lambda)$. This transformation is intuitive and will be explained in Section 5 by theoretical analysis.

Moreover, we show the phase diagram of *G* and *SG* under different cascade intensity coefficient λ and the initial disturbance coefficient θ in Fig. 2c and d. When λ increases for $\theta = 0$, there is a critical threshold close to $\lambda_c = 1$, above which the system usually has no giant component (see Fig. 2c). Small λ indicates fewer cascading failure nodes induced by a single failure node in the next step of cascading overloads. In other words, the system with $\lambda < 1$ can withstand cascading overload and stay in a stable state, for the cascading failures will die out finally. Meanwhile, contour line in Fig. 2d shows the complementary effect of θ for λ. When the pair of variables (λ, θ) change along the dashed line in Fig. 2d, the system remains in a critical state when the size of *SG* reaches its maximal value at this dashed line.

4.2. Endurance under different initial disturbance levels

To study how the network endurance is influenced by initial disturbance, we impose different initial disturbance intensities on the network. As shown in Fig. 3a–c, we assume three types of initial disturbance intensity: (1) small disturbance ($D = 0.01$), (2) medium disturbance ($D = 0.1$), and (3) large disturbance ($D = 0.7$). For small disturbance in Fig. 3a, endurance is marked with a dashed line and is found to increase with increasing cascade intensity coefficient λ. One can also notice that for a given *D* and λ the duration of failure spreading first increases with the average load, and becomes decreasing after reaching the maximum near the critical point (see Fig. 3a). For the case of $\lambda = 1$, there is only decrease of duration with the increase of average load. The main reason for the maximum duration is due to the termination condition of the cascade process, which will be further discussed in Section 5 by theoretical analysis.

For medium disturbance in Fig. 3b, the maximum of the duration *T* has similar tendency. For a given λ, maximal duration is usually smaller than that in Fig. 3a, suggesting larger initial disturbance intensity leads to smaller system endurance. Moreover, when the initial disturbance intensity continues to increase, as shown in Fig. 3c, the system collapses even within a few steps, due to the large amount of initial overloads breaking the network. From the above results, initial disturbance *D* is found to have a significant effect on network endurance. To further observe the effect of *D*, we show the endurance as a function of *D* for a given cascade intensity in Fig. 3d. For each *D* value, when we change the average load, we can find the maximal *T* value that can be regarded as the endurance T_c under this *D* value. Therefore, T_c is a function of *D* accordingly. The increase of *D* could make more nodes overloaded initially. This will in turn generate smaller giant component, which permits short duration for cascading overloads after the initial disturbance.

In fact, these effects can be understood by the interaction of static percolation effect and dynamical overload effect. For small component dependence (low λ), network will only break when the failure from initial disturbance is enough, due to percolation effect. To the contrary, for large λ, it is suggested that system has more failure interactions and spreading easily among them due to overload effect. Therefore, network endurance is determined by the combination of these two effects.

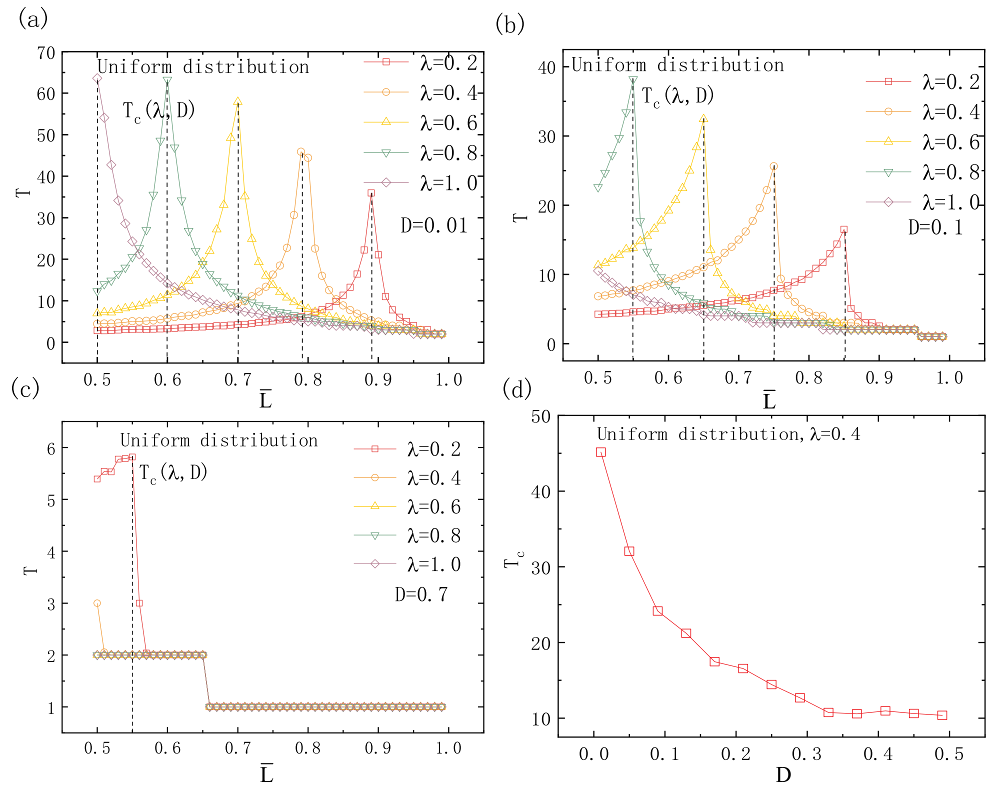

Fig. 3. The endurance T_c under different initial disturbances. (a), (b), (c) When disturbance D increases from small to large, the duration varies with average load levels. The corresponding endurance for each curve is drawn with the dashed line. (d) T_c as a function of disturbance D. Here, the network size is fixed with $N = 2500$ and $0 \le L_{\min} \le L_{\max} \le L^{fail}$. In order to find the critical point, we change the average load $\bar{L}$ by changing $L_{\min}$ and fixing $L_{\max}$. The results are averaged over 500 realizations.

4.3. Endurance with different initial load distributions

In general, real networks operate at different initial load distributions. For the comparison between different initial load distributions, we set the same value of average load $\overline{L}$ for different initial load distributions, as well as the load tolerance L_{fail}. It can be found in Fig. 4 that the network has different failure behaviors with distinct initial load distributions. For different distributions, we show how the duration T of the network depends on the cascade intensity coefficient λ. It is found that T usually first increases with λ and gradually decreases after reaching maximal T, where we consider network endurance as the maximal T.

As shown in Fig. 4a, for the uniform initial load distribution, the endurance increases with the decrease of D. In this case shown in Fig. 4b, network usually experiences a continuous collapse with the failure propagation. Unlike uniform initial load distribution, the network endurance under Gaussian initial load distribution does not have monotonic increase with the change of D. Instead, there is a single peak for certain D in Fig. 4c, which means that the largest network endurance is under a combination of initial disturbance intensity and cascade intensity. For large D, large part of network is initially fragmented, and failure only needs short cascade duration to destroy the remaining network. For smaller D, network is almost intact at first, high cascade intensity is required to disintegrate the network. However, large cascade intensity can lead to short cascade duration with large number of overloads, which spreads over the network. For exponential initial load distribution in Fig. 4e, behaviors of network endurance are similar to the case of Gaussian initial load distribution.

As shown in Fig. 5, single curves without average of result in Fig. 4(d, f) shows that the network with an uniform initial load distribution seems a continuous transition. However, the network with a Gaussian or an exponential initial load distribution seems to collapse in a discontinuous manner. It is suggested that abrupt type of collapses for certain load distributions occur because the load is more concentrated within the characteristic interval under exponential and Gaussian initial load distribution. Therefore, there could be combination of model parameters leading to abrupt transition, including Gaussian and exponential initial load distribution. These different results will be explained theoretically in Section 5.

5. Theoretical analysis

There are two types of conditions in which the cascading overload process can be ended. The first case is that the failure size at a single step decreases to be 1, i.e. $m_T = 1$ (see Fig. 6a, the red lines). The other case is that all the nodes fail at the end of the process, i.e. $M_T = N$, also indicating that there are no further failures and the process ends (see Fig. 6a, the green lines). These two driving conditions are determined by initial disturbance intensity and cascade intensity. For example, for uniform initial load distribution (see Fig. 4a), two driving conditions cause the cascade duration curve to increase first and then decrease, where the peak of the curve happens at the boundary of the two conditions.

We first start with a simple case of uniform initial load distribution. Given the initial condition of $D_0 = D$ and $M_0 = m_0 = ND$, the total failure size M_T at step T can be calculated in Fig. 6b,

$$M_T = NS_T = N\left(\sum_{i=1}^{T} D_i\right) \tag{6}$$

where S_T is the cumulative overload and the disturbance D_T is the incremental overload at step T.

For the cascading failure size of a single step, based on Eq. (6), we have $m_T = M_T - M_{T-1} = ND_T$. Furthermore, according to the overload

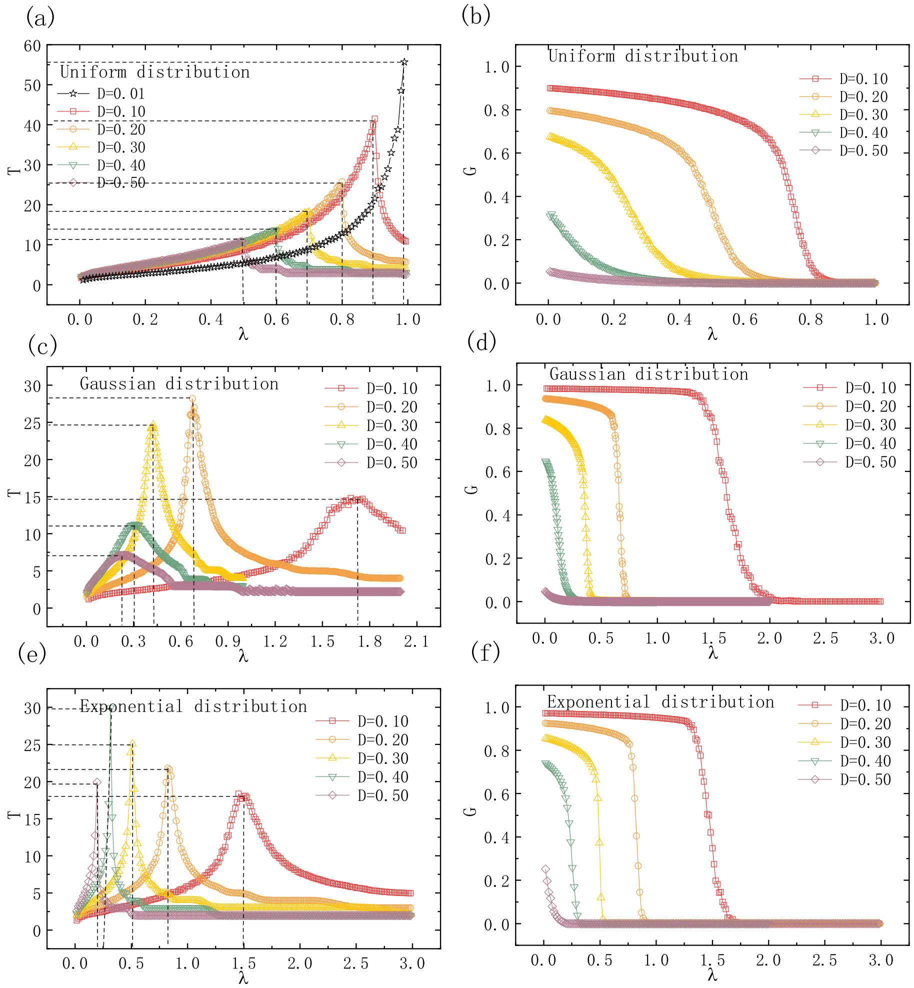

Fig. 4. Impact of different initial load distributions on endurance. (a, b) is the case of uniform initial load distribution, and (c, d) is the case of Gaussian initial load distribution with $\mu = 0.5$, $\sigma^2 = 0.2$ and (e, f) is the case of exponential initial load distribution with rate parameter $\xi = 4.2$. All nodes are initially loaded by the independent load L_1, L_2, ……, L_i in $[L_{\min}, L_{\max}]$, where for uniform and Gaussian load distribution $L_{\min} = 0$, $L_{\max} = 1$, while for exponential distribution $L_{\min} = 0.3$, $L_{\max} = 1$. The mean load of all above cases is $\bar{L} = 0.5$. T, G, and λ respectively represent the cascade duration, the size of the giant component, and cascade intensity coefficient. The network is also with $N = 2500$ nodes. Here, some curves in (d, f) seem not discontinuous because the averaging procedure of simulation removes the appearance of discontinuity.

model, the disturbance $D_T = m_{T-1}I' = m_{T-1}\cdot\frac{\lambda}{N}$. Hence, we can easily obtain the relation

$$m_T = \lambda m_{T-1} \tag{7}$$

This also indicates that λ represents the branching size of cascading failures. From Eqs. (6) and (7), the cumulative and incremental failure size can be shown as follows

$$\begin{cases} m_T = m_0\lambda^T = ND\lambda^T \\ M_T = NS_T = ND\left(\sum_{i=0}^{T}\lambda^i\right) = ND\cdot\frac{1-\lambda^{T+1}}{1-\lambda} \end{cases} \tag{8}$$

As shown in Eq. (8), the total failure size $M_T = ND\cdot\frac{1-\lambda^{T+1}}{1-\lambda}$. When cascading overload process ends, the cascade steps can be regarded as $T \to \infty$. Thus, we have $\frac{M_{T\to\infty}}{N} \approx \frac{D}{1-\lambda}$ when $0 < \lambda < 1$. The value of G and SG depend on the failure proportion, which is not related to network size. It explains why rescaled curves collapse into a single curve in

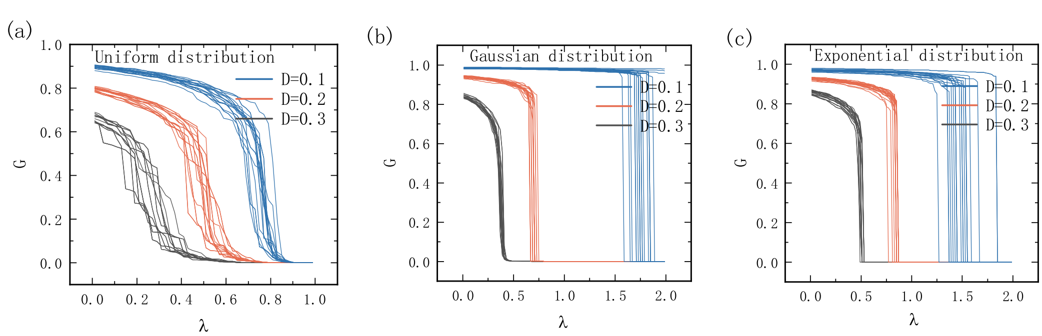

Fig. 5. G vs. λ under different initial load distributions for single realizations. Other parameters are the same as those in Fig 4. It is suggested that uniform distribution has a continuous transition while Gaussian and exponential distribution seem to have an abrupt breakdown. Here, we only show 20 realizations for clarity.

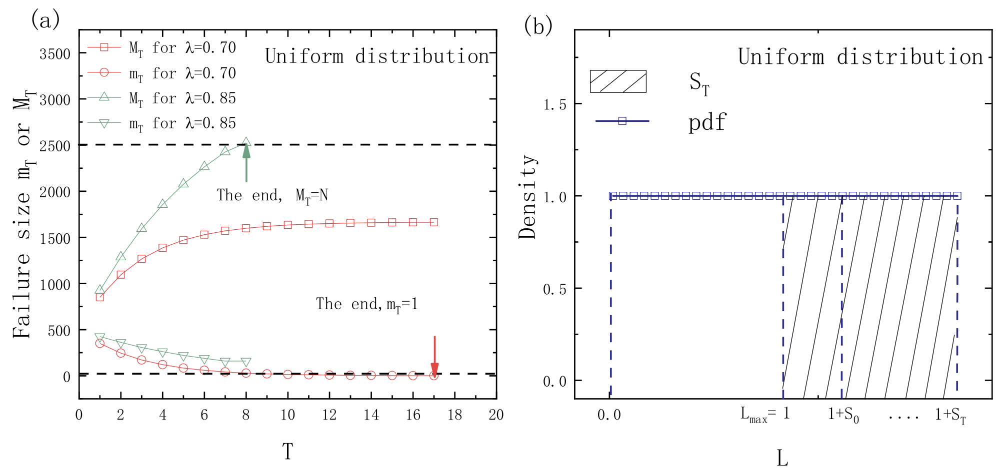

Fig. 6. Cascading failure size of different conditions. (a) Failure size vs. cascade duration T for two types of driving conditions. The dashed line represents two conditions: the total failure size $M_T = N$ (green arrow) or the failure size for a single step $m_T = 1$ (red arrow). Here $N = 2500$. (b) A schematic diagram for the calculation of cascading failure size under uniform initial load distribution. $S_T = \sum_{T=0}^{T} D_i$ stands for the cumulative incremental load of a node from the beginning to step T. With the initial disturbance D added to the network, the shaded area represents the cumulative overload S_T.

the inset of Fig. 2a and b.

The cascade duration T is determined by two driving conditions according to the above analysis. Hence, let $m_T = 1$ or $M_T = N$ in Eq. (8), then we can calculate the duration as follows

$$\begin{cases} T_1 = \frac{-\ln(ND)}{\ln\lambda}, & m_T = 1,\ M_T \le N,\ 0 < \lambda < 1 \\ T_2 = \frac{\ln(1-\frac{1-\lambda}{D})}{\ln\lambda} - 1, & M_T = N,\ m_T \ge 1,\ 0 < \lambda < 1 \end{cases} \tag{9}$$

from which T_1 and T_2 are theoretical cascade duration with the terminal condition $m_T = 1$ and $M_T = N$ respectively.

When the condition reaches $T_1 = T_2 = T$, we have $\lambda = N(D-1)/(1-N)$ according to Eq. (9). Then the cumulative and incremental failure size can be calculated by Eq. (8) as follows:

$$\begin{cases} m_T = ND\lambda^T = ND\lambda^{\frac{-\ln(ND)}{\ln\lambda}} = NDe^{\frac{-\ln(ND)}{\ln\lambda}\cdot\ln\lambda} = 1 \\ M_T = ND\cdot\frac{1-\lambda^{T+1}}{1-\lambda} = ND\cdot\frac{1-\lambda^{\frac{-\ln(ND)}{\ln\lambda}+1}}{1-\lambda} = ND\cdot\frac{1-\frac{\lambda}{ND}}{1-\lambda} = \frac{ND-\lambda}{1-\lambda} = N \end{cases} \tag{10}$$

We find that the total cascading failure size M_T exactly equals to the network size N at $T_1 = T_2$. The cascade duration T increases to its maximal value and thus we can determine the endurance T_c at $T_1 = T_2$.

Cascade process is a site percolation process on a network. The value of endurance T_c is determined by the critical threshold which can be calculated from network percolation, where the critical probability of square lattice is $p_c = 0.5927$ [15]. For the process ends with network endurance, we have the equation

$$\begin{cases} m_{T_c} = 1 \\ M_{T_c} = N(1-p_c) \end{cases} \tag{11}$$

By solving the Eq. (11), we get the network endurance T_c and λ_c of the network for a given initial disturbance D

$$\begin{cases} T_c = \frac{-\ln(ND)}{\ln\lambda_c} \\ \lambda_c = \frac{D-(1-p_c)}{\frac{1}{N}-(1-p_c)} \end{cases} \tag{12}$$

It can be found from Eq. (12) that $\lambda_c \approx 1$ when $D = 0$ and $N \to \infty$.

As for a general form of initial load distribution, we can also obtain the following self-consistent recursion equations based on the above analysis

$$\begin{cases} M_T = N\int_1^{1+S_T} f(x - \sum_{i=0}^{T} D_i)dx \\ m_T = M_T - M_{T-1} \\ D_T = m_{T-1}\cdot\frac{\lambda}{N} \end{cases} \tag{13}$$

subject to the initial condition $D_0 = D,\ m_0 = M_0 = N\int_1^{1+D_0} f(x - D_0)dx$, $T = 1, 2, 3, \ldots$. Here, the value of T is determined by the terminal condition of cascade process $m_T = 1$ and $M_T = N$. $f(x)$ is the probability density function of the initial load distribution. For truncated exponential initial load distribution in this paper, the pdf is $f(x) = \frac{\xi e^{-\xi x}}{e^{-\xi L_{\min}} - e^{-\xi L_{\max}}}$, whereas the pdf of truncated Gaussian initial load distribution is $f(x) = \frac{e^{-(x-\mu)^2/\sigma\sqrt{2\pi}}}{\Phi(\frac{L_{\max}-\mu}{\sigma}) - \Phi(\frac{L_{\min}-\mu}{\sigma})}$. We can calculate the endurance T_c with the same Eq. (11).

In general, it is usually hard to get a closed form of analytical expression except for the case under uniform initial load distribution. However, we can still calculate the cascade duration T and giant component G by iteratively solving the self-consistent recursion Eq. (14). The equations can always have solution from one of the two termination rules. Here, we take the case of initial disturbance $D = 0.2$ for an example. As can be seen in Fig. 7, the numerical results have good agreement with the experimental results for the cascade duration.

From the above theoretical analysis, we can better understand the continuous and abrupt transition for different initial load distributions. The slope of curves in Fig. 4b, d and f at criticality are calculated as following the differential equation

$$\frac{dG}{d\lambda} = \frac{dG}{dq}\cdot\frac{dq}{d\lambda} \tag{14}$$

where q is the fraction of failure nodes with $q = M_T/N$. As dG/dq has the same value for different initial load distributions, the transition is determined by $dq/d\lambda$, which can be calculated by iteratively solving the Eq. (12). As shown in Fig. 8, the value of $dq/d\lambda$ for Gaussian and exponential initial load distribution is much greater than that for uniform initial load distribution at the critical point, indicating that a small increase of λ at criticality will result in greater increase of node failures for Gaussian and exponential initial load distribution. Therefore, from the aspect of initial load distributions, a continuous phase transition can be observed for uniform initial load distribution, rather than an abrupt transition for Gaussian or exponential initial load distribution

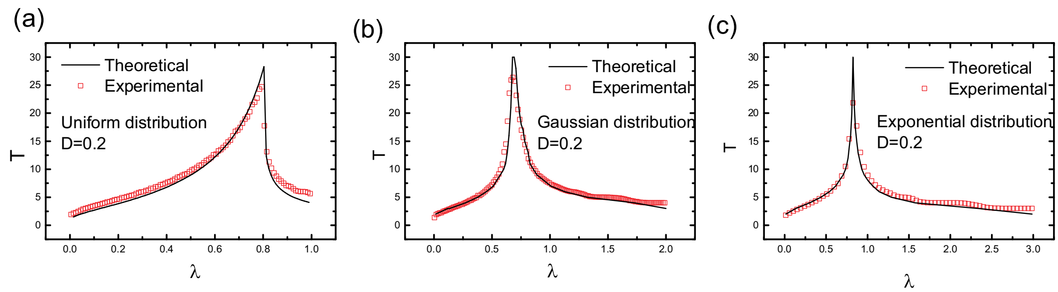

Fig. 7. Comparing the theoretical and simulation results. Symbols are simulation results on a network with $N = 2500$, whereas the lines are theoretical results for (a) uniform distribution, (b) Gaussian distribution, (c) exponential distribution.

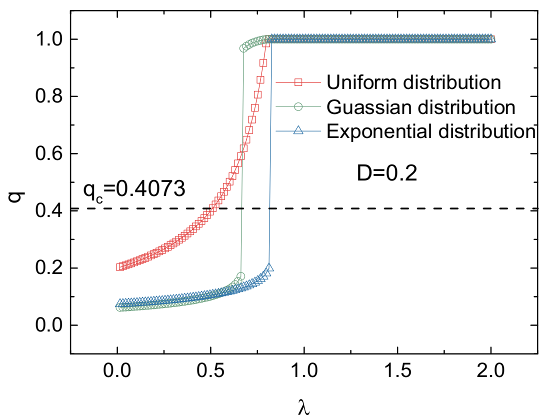

Fig. 8. The failure ratio q as a function of cascade intensity coefficient λ for a given disturbance $D = 0.2$. The dashed line corresponds to the percolation critical threshold of lattice network.

6. Conclusions

Network robustness refers to the ability of a system to resist perturbation and maintain its function. Endurance refers to the temporal scale for a network to withstand the cascading failures, which allows to perform certain healing activities to recover. Therefore, endurance is a basic and important concept for many complex systems, which is highly related to system resilience. Larger endurance provides more opportunities for system healing. Recent studies have proposed different frameworks including deterministic and probabilistic measures to describe system resilience, neglecting the study of system endurance. Many self-healing strategies also put forward to investigate system resilience after perturbation. However, the definition and corresponding endurance evaluation method remain challenging.

In this paper, taking into account cascading overload failure, we develop a general method to investigate network endurance by both numerical and theoretical approach. The endurance here is defined as the cascading failure duration when the network is close to its critical point. We find that the endurance highly depends on the initial disturbance intensity and cascade intensity. Initial load distribution also has a great impact on network endurance. It is found that the network endurance with a uniform initial load distribution monotonically increases with the decreasing initial disturbance, while for other initial load distributions endurance has a maximum with the certain combination of both initial disturbance intensity and cascade intensity. Moreover, two different transitions are also found, where a continuous transition usually occurs on uniformly distributed network, while for Gaussian and exponential initial load distributions network has an abrupt transition.

For effectiveness of self-healing strategy, the endurance analysis method presented in this paper can be used to help to design and assess recovery strategies of network resilience after perturbation. While the relation between endurance and network topology is unclear, future work is needed to extrapolate. Moreover, networks usually interact with each other. For example, the smart power grid is highly spatially and temporally interdependent with communication networks. Therefore, interdependency of the system will also affect the analysis of network endurance. We may extend our work to temporal network and interdependent network in the future.

Author's statement

S. Guo is the corresponding author of this work. J. Zhong and H. Sanhedrai contribute equally to this work.

Declaration of Competing Interest

No conflict of interest.

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Supplementary materials

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