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Precise Approach of Earth Gravity Field and Geoid

PAGravf4.5 User Reference

Classroom Teaching, Self-Exercise, Science Research and Engineer Computing

Chinese Academy of Surveying & Mapping
March 2023, Beijing, China

Precise Approach of Earth Gravity Field and Geoid (PAGravf4.5) is a large Windows package for geodetic scientific computation according to stationary gravity field theory, which includes five subsystems, namely, data analysis and preprocessing calculation of Earth gravity field, computation of various terrain effects on various field elements outside geoid, precision approach and full element modelling on Earth gravity field, optimization, unification, and application for regional height datum as well as editing, calculation, and visualization tools for geodetic data files.

Strictly according to the approach theory of gravity field, PAGravf4.5 efficiently deals with various terrain effects on various gravity field elements outside geoid. Scientifically constructs the gravity field approach system with the spatial domain integration algorithm based on boundary value theory and spectral domain radial basis function approach algorithm to realize the full element modelling on gravity field in full space outside geoid and the fine gravity prospecting from various heterogeneous observations. And develops some ingenious algorithms based on physical geodesy to improve and unify the regional height datum, so as to consolidate and expand the applications of Earth gravity field.

The basic principles, main methods and all the formulas in physical geodesy and Earth gravity field have been realized completely in PAGravf4.5 to popularize high education. Many long-term puzzles such as various terrain effects on various observations, full-element analytical modelling on gravity field, fine gravity prospecting from heterogeneous observations, external accuracy index measurement and computational performance control have been effectively solved to strengthen the application capacity of Earth gravity field.

PAGravf4.5 is suitable for senior undergraduates, graduate students, scientific researchers, and engineering technicians in in geodesy and geophysics, geology and geoscience, geomatics and geographic information, seismic and geodynamics.

Key words: terrain effect, gravity field approach, geoid, spherical radial basis function, height datum, geodetic computation.

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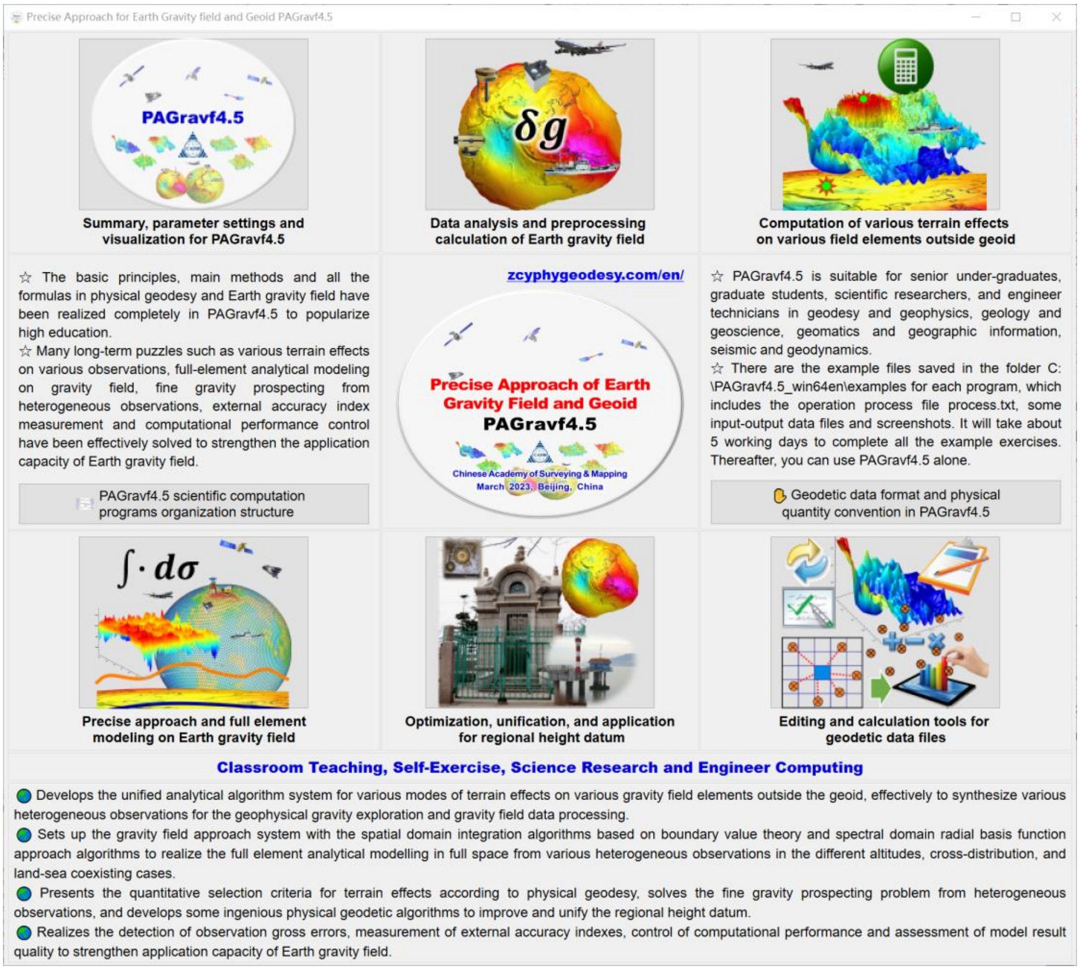
1 PAGravf4.5's features, strengths, concepts, and usage

Precise Approach of Earth Gravity Field and Geoid (PAGravf4.5) is a large Windows package for scientific computation rigorously based on stationary gravity field theory. Strictly according to physical geodesy, PAGravf4.5 constructs the unified analytical algorithm system of various terrain effects on various gravity field elements on the geoid or in the outer space outside the geoid to improve the geophysical gravity exploration and gravity field data processing. Scientifically constructs the gravity field approach system with the spatial domain integration algorithm based on boundary value theory and spectral domain radial basis function approach algorithm to realize the full element modelling on gravity field in full space outside geoid and fine gravity prospecting from various heterogeneous observations. And develops some ingenious physical geodetic algorithms to improve and unify the regional height datum, so as to consolidate and expand the applications of Earth gravity field.

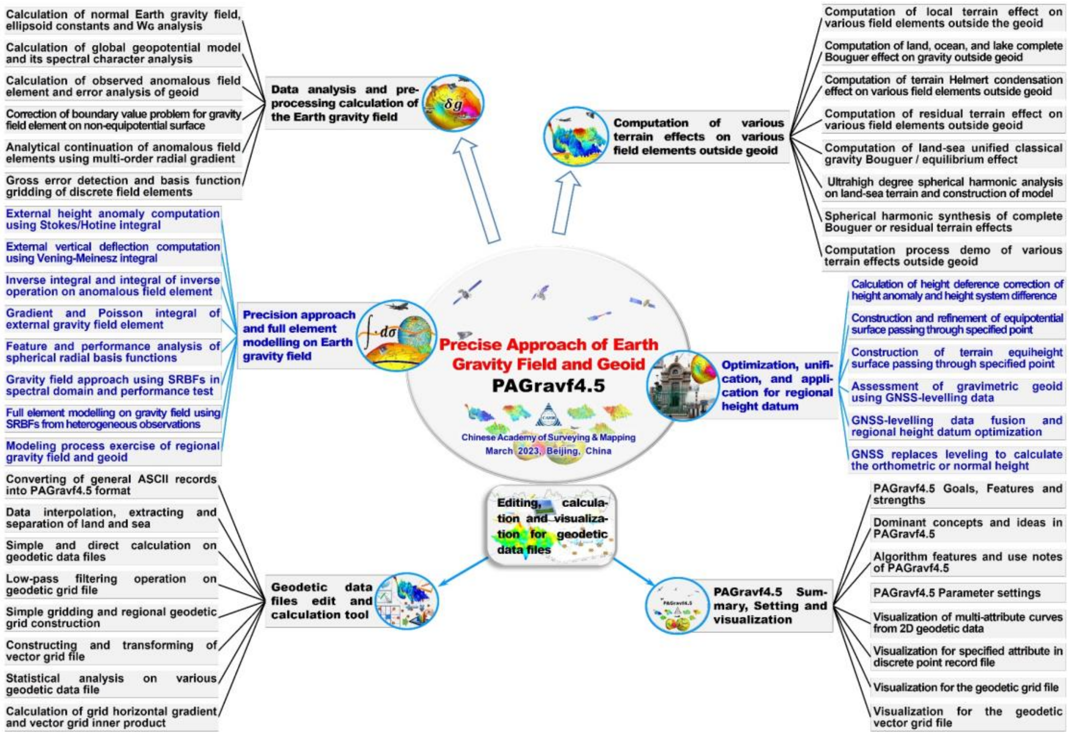


1.1 PAGravf4.5 structure of computation functions

The basic principles, main methods and all the formulas in physical geodesy and

Earth gravity field have been realized completely in PAGravf4.5 to popularize high education. Many long-term puzzles such as various terrain effects on various observations, full-element analytical modelling on gravity field, fine gravity prospecting from heterogeneous observations, external accuracy index measurement and computational performance control have been effectively solved to strengthen application capacity of Earth gravity field.

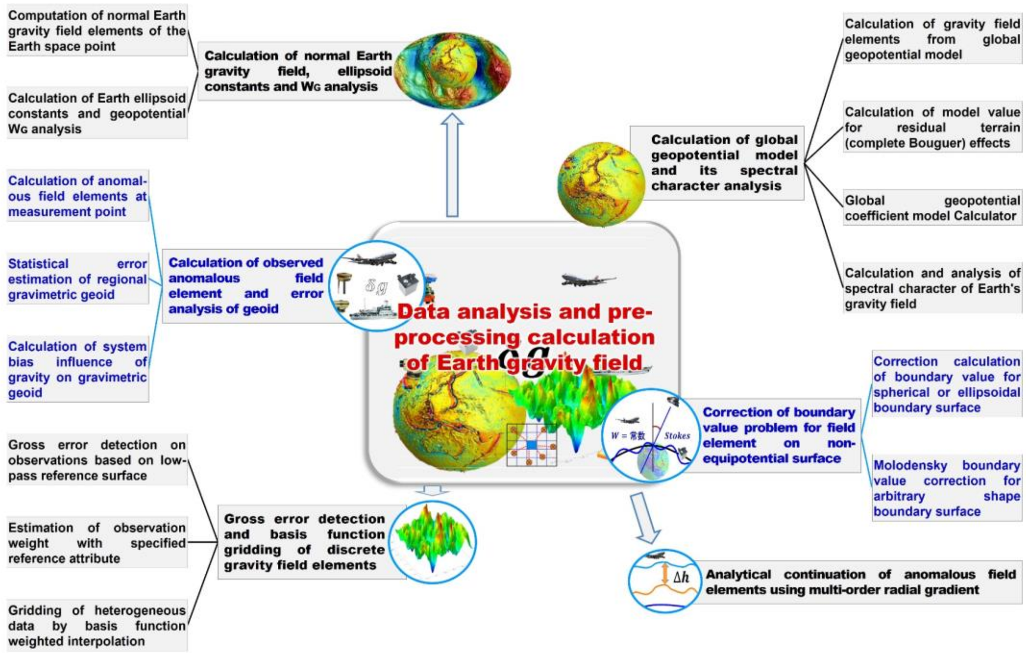
PAGravf4.5 has five subsystems, which includes data analysis and preprocessing calculation of Earth gravity field, computation of various terrain effects on various field elements outside geoid, precision approach and full element modelling on Earth gravity field, optimization, unification, and application for regional height datum as well as editing, calculation, and visualization tools for geodetic data files.



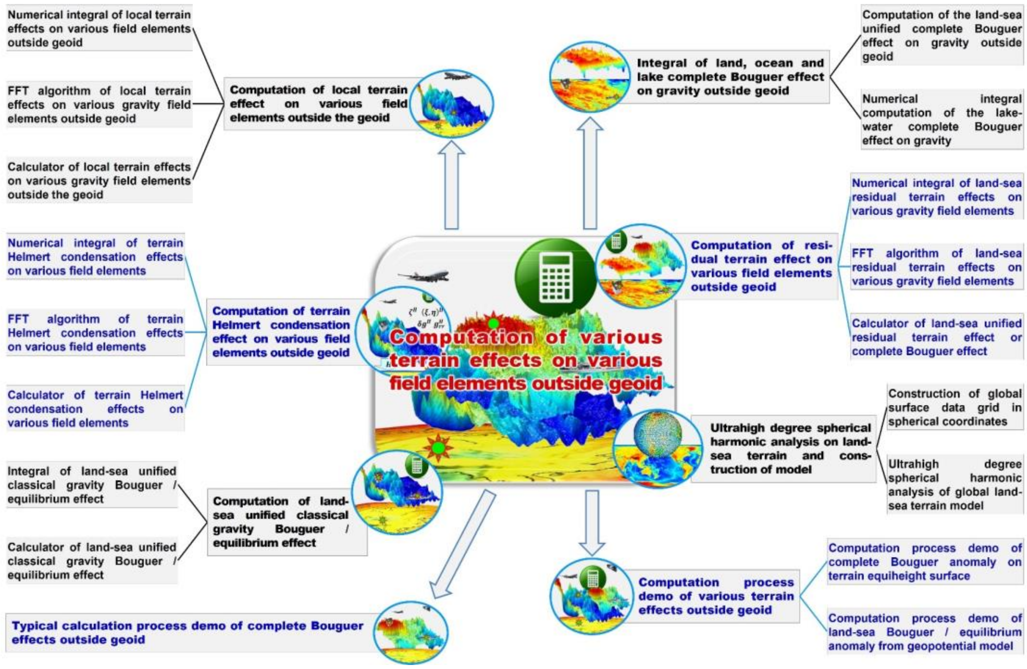
You can design your own schemes and processes, then organize flexibly the related programs and functions in PAGravf4.5, perform some scientific computations for various terrain effects outside geoid, full element modelling on gravity field, refinement of 1cm stationary geoid, fine gravity prospecting from heterogeneous observations, improvement of regional height datum and application of Earth gravity field.

1.1.1 Data analysis and preprocessing calculation of Earth gravity field

The subsystem is mainly used for the analysis and calculation of the normal Earth gravity field and Earth ellipsoid constants, calculation of the geopotential coefficient model and its spectral feature, correction of gravity field elements for the geodetic boundary value problem, and analytical continuation, gross error detection and griding operation for gravity field observations.



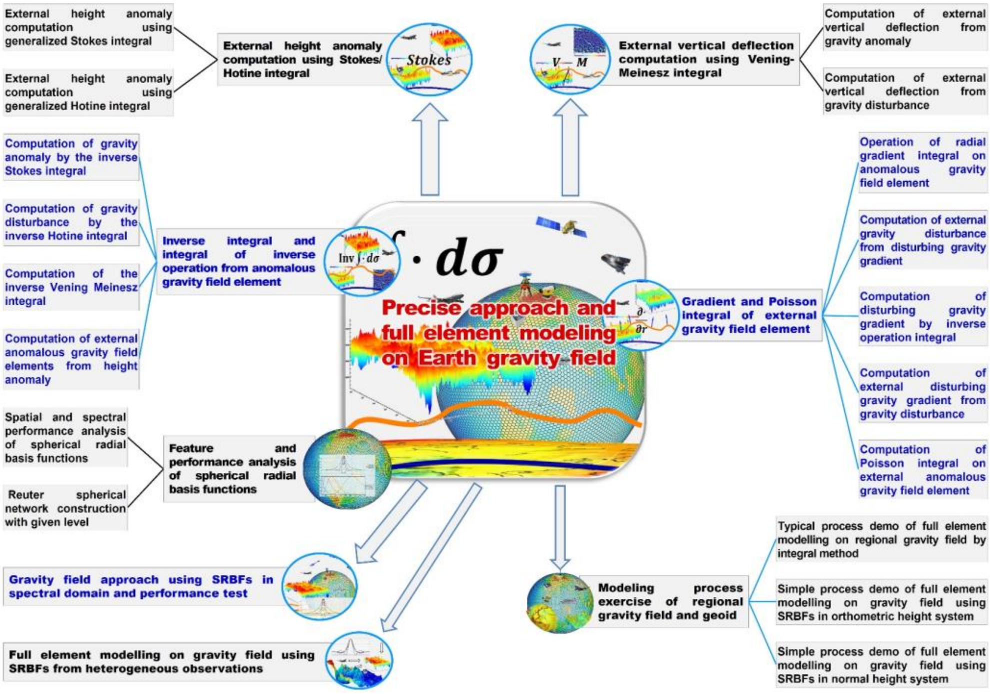
1.1.2 Computation of various terrain effects on various field elements outside geoid



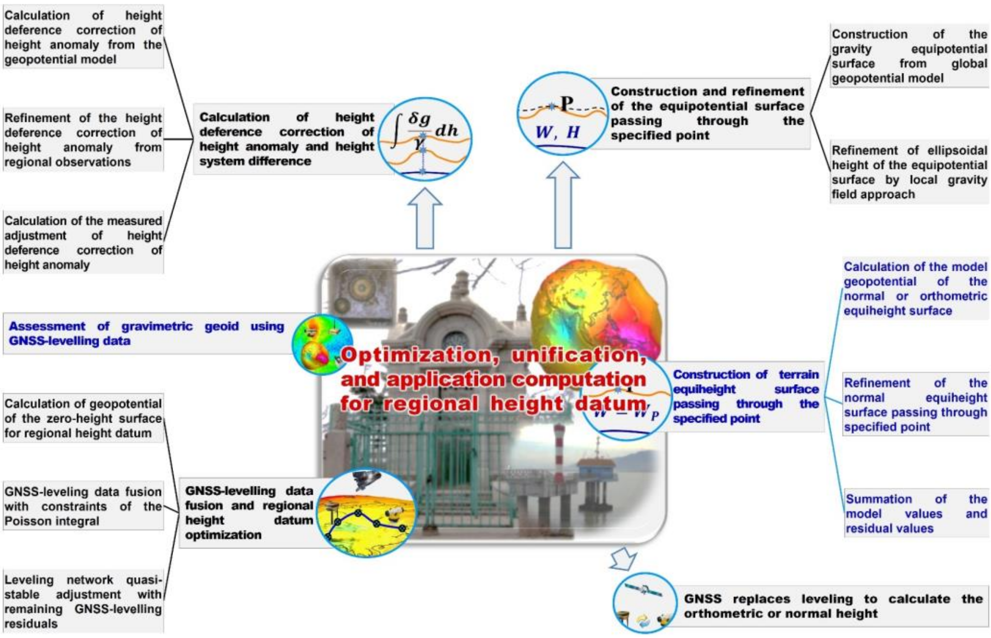
Constructs the harmonic terrain influence field, and then develops a rigorous and unified analytical algorithm system for various modes of terrain effects on various types of gravity field elements outside the geoid (on the geoid or in its outer space), to synthesize effectively various complex geodetic observations for geophysical gravity exploration and gravity field data processing.

1.1.3 Precise approach and full element modelling of Earth gravity field

PAGravf4.5 sets up the scientific gravity field approach system with the spatial domain integration algorithms based on boundary value theory and the spectral domain radial basis function approach algorithms to realize the full element analytical modelling on gravity field in full space outside the geoid from various heterogeneous observations in the different altitudes, cross-distribution and land-sea coexisting cases.

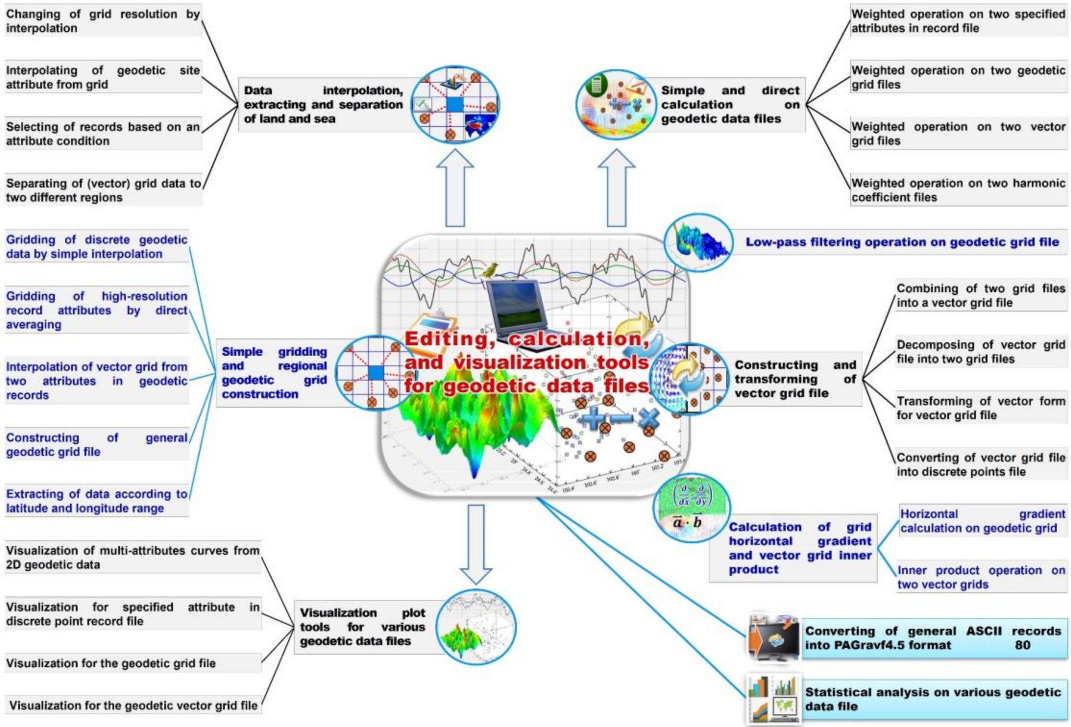


1.1.4 Optimization, unification, and application for regional height datum



Develops some ingenious physical geodetic algorithms to improve and unify the regional height datum, and then enhance the application capacity of Earth gravity field.

1.1.5 Editing, calculation, and visualization tools for geodetic data files



PAGravf4.5 were developed by QT C++ (Visual C++) for the user interface, Intel Fortran (Fortran90, 132 Columns fixed format) for the core function modules and mathGL C++ for the geodetic data file visualization in the Visual Studio 2017 x64 integrated environment, which is composed of more than 50 win64 executable programs with nearly 600 function modules.

1.2 Scientific goals and strengths of PAGravf4.5

1.2.1 Scientific goals of PAGravf4.5

(1) Solves the analytical compatibility and rigorous unified computation problems of various modes of terrain effects on various types of field elements, to fulfill the requirements of gravity field data processing in various cases and comprehensively improve the geophysical gravity exploration.

(2) Sets up the scientific and complete gravity field approach system with the positive-inverse integral in spatial domain and SRBF approach in spectral domain, to realize the full element analytical modelling in full space outside geoid from heterogeneous observations.

(3) Develops some ingenious physical geodetic algorithms based on the analytical relationship between Earth gravity field and height datum, to improve and unify the regional height datum, and consolidate and expand the applications of Earth gravity field.

1.2.2 Geodetic features and strengths

(1) Develops the unified analytical algorithm system for various modes of terrain effects on various gravity field elements outside the geoid, effectively to synthesize various heterogeneous observations for the geophysical gravity exploration and gravity field data processing.

(2) Sets up the gravity field approach system with the spatial domain integration algorithms based on boundary value theory and spectral domain radial basis function approach algorithms to realize the full element analytical modelling in full space from various heterogeneous observations in the different altitudes, cross-distribution, and land-sea coexisting cases.

(3) Presents the quantitative selection criteria for terrain effects according to physical geodesy, solves the fine gravity prospecting problem from heterogeneous observations, and develops some ingenious physical geodetic algorithms to improve and unify the regional height datum.

(4) Realizes the detection of observation gross errors, measurement of external accuracy indexes, control of computational performance and assessment of model result quality to strengthen application capacity of Earth gravity field.

1.3 Dominant concepts and ideas integrated into PAGravf4.5

1.3.1 Concepts and quantitative criterions for terrain effect

The terrain effect has always been a very tricky problem in physical geodesy and geophysical gravity exploration. According to the basic requirements of physical geodesy, PAGravf4.5 puts forward the convenient and practical quantitative criterion for terrain effects, which can give a reliable basis for effectively improving the performance and role of terrain effect in physical geodesy and geophysical gravity exploration.

The terrain effect on gravity field element has three key elements: The adjustment mode of the terrain or crustal masses, type of gravity field element effected by the terrain, and location of gravity field element (positional relationship with the terrain masses).

According to the different adjustment modes of terrain masses, the terrain effects usually include the local terrain effect, terrain Bouguer effect, ocean Bouguer effect, crustal equilibrium effect, terrain Helmert condensation and residual terrain effect, etc.

In physical geodesy, computation of the terrain effects on gravity field elements serves two basic purposes. One is to improve the griding performance of discrete field elements, and the other is to separate terrain ultrashort wave components for the gravity field approach. Accordingly, PAGravf4.5 defines the quantitative criteria for the selection of the terrain masses adjustment modes and algorithm parameters (such as integral radius, minimum degree of residual terrain model and spatial resolution, etc.).

(1) In order to improve the griding performance of discrete field elements, it is expected to improve the smoothness of discrete field elements after the terrain effect removed. In this case, the optimal criterion for terrain effect is that the standard deviation

of discrete field elements would decrease after the terrain effect removed. This quantitative criterion is also applicable for geophysical gravity exploration purposes.

(2) The terrain effect is expected to consist of only ultrashort wave components for gravity field approach purpose, so the optimal criterion is that the standard deviation of field elements would decrease, and the statistical mean of terrain effects in the range of tens of kilometers is small after the terrain effect removed.

(3) The ratio D/ε of difference D between the maximum and minimum of terrain effects on a certain field element and its standard deviation ε, reflects the outlier of ultrashort wave signal in this mode of terrain effect. D/ε is large, which means that the proportion of ultrashort wave signals is small, but the signal is large. It is beneficial to improve the data processing performance to process this type of field element using this mode of terrain effect.

(4) When the sizes of several modes of terrain effects are roughly same, the greater the ratio of the standard deviation of terrain effect on gravity disturbance to the standard deviation of terrain effect on height anomaly, the richer the ultrashort wave components of terrain effect, and the more favorable it is for geoid refinement.

Among the above four guideline criterions defined by PAGravf4.5, the first two are the binding regulations, which are globally applicable and need to be followed. The latter two can be the technical references and should be employed appropriately based on further analysis. These criterions will no longer be appropriate when gravity field observations are so scarce that their space statistical representation is severely underrepresented.

The statistical properties of terrain effects vary significantly with the terrain and gravity field nature in the target area. It is recommended to calculate, compare, and analyze different modes of terrain effects on various field elements and their differences each other in advance, and then summarize the spectral domain character of the terrain effects and properties of the effect on different types of target field elements to design a calculation scheme with high adaptability based on these analysis results.

The performance of terrain effect is closely related to the complexity of the local terrain, short-wave figure of the gravity field and space distribution of the observations. When the terrain complexity is low, local gravity field structure is not complex and space distribution of the observations is dense, there is a possibility that the performance of the griding or gravity field approach cannot be further improved using any mode of terrain effect.

1.3.2 The uniqueness of the geoid and PAGravf4.5's realization

The Stokes boundary value problem requires that there are no masses outside the geoid, and the terrain masses should be compressed into the geoid under the condition of keeping the disturbing geopotentials unchanging outside the Earth surface. PAGravf4.5 believes that there is some a way to compress the terrain masses that the disturbing geopotential between ground and geoid are equal to the analytical continuation value of the disturbing geopotential outside the Earth surface, thereby the corresponding geoidal height is the analytical continuation solution of the geoid.

Various terrain effect on gravity disturbances and that on height anomalies computed by PAGravf4.5 satisfy the Hotine integral formula. For example, the terrain Helmert condensation effect on gravity disturbances (direct effects) and that on height anomalies (indirect effects) satisfy the Hotine integral formula. Therefore, no matter whether you choose the local terrain effect, terrain Helmert condensation, or residual terrain effect, the regional geoid refined by PAGravf4.5 programs with the terrain effect remove-restore scheme is the analytical continuation solution of the geoid.

Obviously, the geoidal height determined from satellite gravity field or directly calculated from a global geopotential coefficients model are all the analytical continuation solution of the geoid. Maintaining the uniqueness of the geoid solution, PAGravf4.5 can deeply integrate satellite gravity, geopotential coefficient model and regional gravity field data, and then theoretically strictly approach the gravity field and geoid.

1.3.3 Classification and solution of gravity field boundary value problem

Most of the direct geodetic observations are based on the plumb line and level surface namely the natural coordinate system. For the sake of convenience, PAGravf4.5 divides the external boundary value problem into Stokes problem and Molodensky problem according to whether the inner normal of the boundary surface coincides with the plumb line, that is, whether the boundary surface is an equipotential surface.

(1) Stokes boundary value problem. The boundary value problem whose boundary surface is an equipotential surface, whose boundary value is a linear combination with the disturbing potential and its partial derivatives with respect to coordinates is called the Stokes boundary value problem. The inner normal lines of the boundary surface in the Stokes problem coincide with the plumb lines.

(2) Molodensky boundary value problem. The boundary value problem whose boundary surface is not an equipotential surface, whose boundary value is a linear combination with the disturbing potential and its partial derivatives with respect to coordinates is called the Molodensky boundary value problem. The inner normal lines of the boundary surface in the Molodensky boundary value problem do not coincide with the plumb lines.

When the boundary surface is not an equipotential surface, any one of the following three methods can be employed to solve the Molodensky boundary value problem.

(1) Analytically continuate the gravity field elements on the boundary surface to the equipotential surface close to the boundary surface. In this case, the new boundary surface becomes the equipotential surface, and the boundary value problem into the Stokes problem, and then solve the Stokes problem.

(2) Correct the gravity field elements on the boundary surface from the direction of

the inner normal line of boundary surface to the direction of plumb line, so that the boundary value problem becomes a Stokes problem, and then solve the Stokes problem.

(3) Directly solve the Molodensky boundary value problem with the non-equipotential surface as the boundary surface. Which is not recommended by PAGravf4.5 due to the generally low accuracy of the solution.

PAGravf4.5 suggests that the geodetic boundary value problem is mainly solved according to the Stokes boundary value theory, and the Molodensky boundary value theory is mainly employed for the reduction of gravity field data into the equipotential surface, and for error analysis for processing of gravity field data on the non-equipotential surface.

1.3.4 Geopotential, geoid, and height datum

(1) The gravimetric geoid is the solution of the boundary value problem, whose geopotential is a constant and equal to the normal potential of the ellipsoidal surface, and the ellipsoidal surface is also the starting surface of the geoidal height.

(2) Global geopotential W_0 is an appoint geopotential for the global height datum in IERS numerical standards, which can be calculated from a latest geopotential model and sea surface height observations according to the Gaussian geoid appoint.

(3) According to the define of height system, the zero normal height surface always coincides with the zero orthometric height surface everywhere, which is namely the geoid with the constant geopotential W_G or W_0.

(4) Essentially, the gravimetric geoid determined according to the geodetic boundary value theory is the realization of the constant geopotential W_G in the Earth coordinate system, namely determining of the ellipsoidal height of the geoid.

(5) PAGravf4.5 recommends that the geoidal geopotential W_G or U_0 should replace the empirical appoint W_0 in the IERS numerical standard. The latter has no direct geodetic relationship with the geopotential of the gravimetric geoid.

Whether for realizing of global height datum or for refining of regional height datum, employing the geoidal geopotential W_G as the global geopotential can not only effectively reflect the unique invariance of geodetic datum, but also make full use of physical geodesy (space geodesy) technology and method to approach the global geopotential with infinite precision. Which can effectively ensure the analytic rigor of gravity field approach in the realization and unification of height datum.

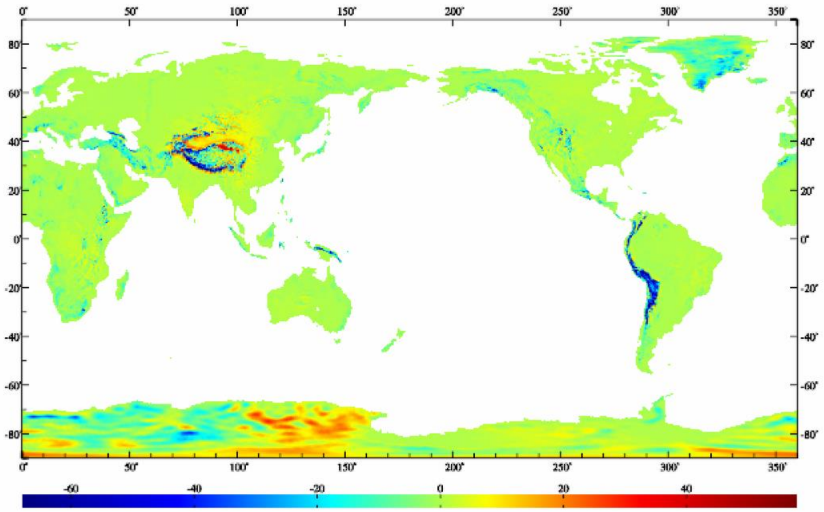
1.3.5 Analytical orthometric system is more suitable

(1) Let the gravity value of the move point between ground and geoid be equal to the gravity value analytically continued to the move point from the outer gravity field, and the resulting orthometric height is called the analytical orthometric height.

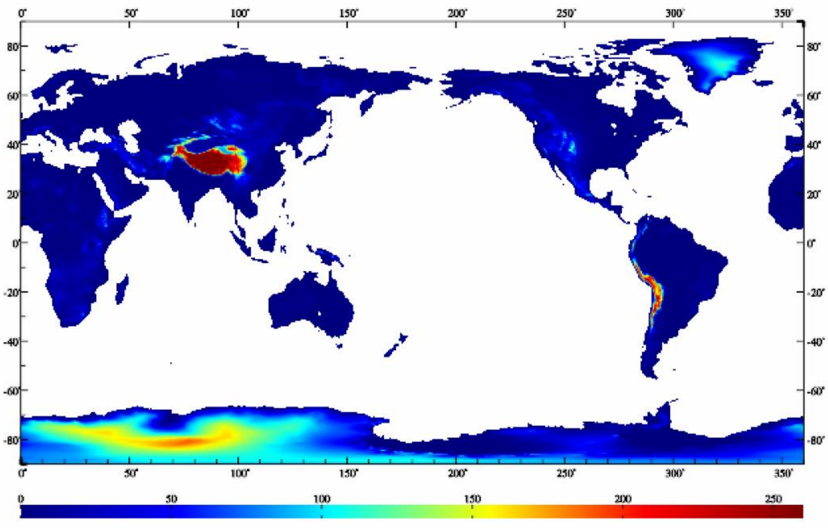
The analytic orthometric height of the global ground points is closer to the normal height, and the difference is about 60 cm from the Helmert orthometric height at 3000 m altitude.

(2) The geoidal height is the ellipsoidal height of the geoid, which is the solution of

the Stokes boundary value problem in the Earth coordinate system. The measurement scale of the geoidal height is the geometric scale of the Earth coordinate system, while the measurement scale of the analytical orthometric height difference in the vertical direction is also strictly expressed by the geometric scale. Thus, the analytical orthometric height is consistent with the geometric scales of the Earth coordinate system and the geoidal height.



The differences between the analytical orthometric and normal height of the global land ground (cm, 95%<±16cm)



The differences between the analytical orthometric and Helmert orthometric height of the global land ground (cm, 95%<±108cm)

(3) The analytical orthometric height is not directly related to the terrain density, which can be continuously refined with the latest gravity field data. On the view of uniqueness, repeatability, and measurability of geodetic datum, the analytical orthometric height is more suitable for height datum purpose than other types of orthometric height.

Different from the Helmert orthometric system, the analytic orthometric system and normal height system are compatible and consistent with each other and supported by the rigorous gravity field theory, they can also be directly employed for the moon and Earth-like planets.

1.4 Data format, convention, and exercise in PAGravf4.5

1.4.1 Format convention for geodetic data file

PAGravf4.5 adopts 5 types of geodetic data files in own format. The program [Converting of general ASCII records into PAGravf4.5 format] is the important interface for PAGravf4.5 to accept external text data. Using the program [Simple gridding and regional geodetic grid construction], you can construct a numerical grid with the given grid specifications. The other programs or functions only accept the format data generated by PAGravf4.5 own.

(1) The discrete geodetic record file

A discrete geodetic record data is represented by a one-dimensional array.

▪ Multiple rows of the file headers are allowed, whose content and format are not restricted.

▪ One record represents the geodetic data of one site. The attributes of each record include site number (name), longitude (degree decimal), latitude (degree decimal), attribute 4, …, attribute n.

▪ The attribute convention is a numeric format, the number of the attributes (n) is not more than 80, and the attributes are separated by spaces.

(2) The geodetic network observation file

A geodetic network observation file can store the baseline component data for the CORS network, height differences for the levelling network, or gravity differences for the gravity network.

▪ The file header occupies a row and includes the number of characters of the baseline or route name, number of characters of the site name, ……

▪ The file record includes the baseline or route name, starting site (longitude, latitude, height), ending site (longitude, latitude, height), ……, observations (default value is 9999).

▪ The relations between the baselines (or routes) and the sites in the geodetic network are reflected with the composition of the characters of their name. A baseline or route name is agreed to be composed of site names A and B at both ends (B***A), where the number of characters of all the sites names is required to be equal.

Therefore, the number of characters of the baseline or route name shall not be less than twice the number of characters of the site name.

(3) The geodetic numerical grid file

The geodetic numerical grid data is represented by a two-dimensional array.

▪ There is a row of file header at the beginning of the file. The file header contains minimum longitude, maximum longitude, minimum latitude, maximum latitude, longitude interval of a cell grid, latitude interval of a cell grid. The units of all the attributes are decimal degrees.

▪ The grid elements are sequentially stored in an increasing manner of row latitude and column longitude until all data is stored. The elements are separated by spaces.

The grid value of cell grid represents the mean value of the cell grid elements. In the numerical integral operation, the location of the center point of the cell grid is employed to calculate the integral distance from the cell grid to the calculation point.

(4) The geodetic vector grid file

A vector grid file is composed of the first component grid and the second component grid of the vector. The header file and the first component grid in the vector grid file are same as that in the geodetic grid file, and the second component grid follow the first component grid closely with the same way.

Vector grid such as vertical deflection and horizontal gradient vector grid in PAGravf4.5 are stored in the form of vector grid file.

(5) The spherical harmonic coefficient file

▪ The file header occupies a row and consists of two attributes for the scale parameters of the spherical harmonic coefficients model, namely the geocentric gravitational constant GM ($\times 10^{14}$m^2/s^2) and equatorial radius of the Earth a (m).

▪ The geopotential coefficient model, and terrain masses spherical harmonic coefficient model and terrain geopotential coefficient model in PAGravf4.5 are stored in the form of spherical harmonic coefficient file.

▪ The spherical harmonic coefficients correspond to the scale parameters of GM and a. Here, the surface harmonic functions in the spherical harmonic coefficient model are defined on the spherical surface whose radius is equal to the semi-major axis a of the Earth.

▪ The degree n order m spherical harmonic coefficient is expressed by a record with the format ‘degree n, order m, C_{nm}, S_{nm} (, C_{nm} error, S_{nm} error)’.

PAGravf4.5 does not require the degrees and orders of harmonic coefficients to be arranged and allows to exist insufficient orders.

1.4.2 The main physical quantity unit and direction

(1) Unit convention of anomalous gravity field elements: Height anomaly or geoidal height in the unit of m, gravity anomaly or gravity disturbance in the unit of mGal, vertical deflection in the unit of as namely ", and gravity gradient in the unit of E.

(2) The unit of terrain effect on some a gravity field element is the same as that of the gravity field element.

(3) Longitude (decimal degrees), latitude (decimal degrees), ellipsoidal height (m), normal (orthometric) height / depth (m).

(4) Vertical deflection vector (SW). The first component points to the south direction,

and the second component points to the west direction, which forms a right-handed rectangular coordinate system with the gravity disturbance direction. This coordinate system is a natural coordinate system.

(5) Tangential gravity gradient vector (NW). The first component points to the north direction, and the second component points to the west direction, which forms a right-handed rectangular coordinate system with the disturbing gravity gradient direction (radial or zenith direction).

In PAGravf4.5, the disturbing gravity gradient T_{rr} and tangential gravity gradient are the second-order partial derivatives of the disturbing potential T to the coordinate, which are the main diagonal components of the gravity gradient tensor.

Please distinguish the concepts of the tangential gravity gradient and gravity horizontal gradient. The latter is the horizontal gradient of the vertical derivative of the disturbing potential, which is only the non-diagonal component of the gravity gradient tensor (cross item, such as T_{zx} or T_{zy}).

1.4.3 For classroom teaching and self-study exercise

To ease the classroom teaching and self-study, there are the example files saved in the folder C:\PAGravf4.5_win64en\examples for each Win64 program. Each example includes the operation process file process.txt, some input-output data files and screenshots. The folder name of the example files is the same as the name of the executable program.

Before using the PAGravf4.5 programs, it is recommended to perform completely the program example using the input-output example data files with comparing the screenshots and the process information in process.txt. It will take about 5 working days to complete all the example exercises. Thereafter, you can use PAGravf4.5 alone.

PAGravf4.5 is suitable for senior undergraduates, graduate students, scientific researchers, and engineering technicians in geodesy and geophysics, geology and geoscience, geomatics and geographic information, seismic and geodynamics, aerospace and satellite dynamics, which can be employed in the classroom teaching, self-exercise, science research, and engineer computing.

You can design your own schemes and processes, then organize flexibly the related programs and functions in PAGravf4.5, perform some scientific computations for various terrain effects outside geoid, full element modelling on gravity field, refinement of 1cm stationary geoid, fine gravity prospecting from heterogeneous observations, improvement of regional height datum and application of Earth gravity field.

1.4.4 Ignoring and expanding several classic concepts

(1) Ignoring the terrain correction and direct effect concepts

In the classic terrain correction, the correction object is only the terrestrial gravity. PAGravf4.5 need deal with various modes of terrain effects on various types of gravity field elements on the geoid or in its outer space. The classic direct effect is the effect of terrain mass on gravity (gravity disturbance or gravity anomaly), while the indirect effect is the effect of terrain mass on geopotential (disturbing potential, height anomaly or geoid). PAGravf4.5 need deal with various terrain effect on full elements gravity field elements. The concept of terrain correction, direct and indirect effect can no longer meet the needs of PAGravf4.5.

PAGravf4.5 adopts the concept of terrain effect uniformly, and strictly distinguishes the adjustment mode of terrain masses, the type of field elements effected and the location of field elements. For example, the local terrain, terrain Helmert condensation and residual terrain effects on the ground gravity disturbance, external gravity disturbance and geoidal gravity disturbance include 3 × 3 = 9 different terrain effect quantities.

The terrain effect on various types of gravity field elements is equal to the negative value of its terrain correction. For example, the local terrain effect is equal to the negative value of classic local terrain correction.

(2) Recommending $W_0=W_G$ as the global geopotential

PAGravf4.5 proposes that the scale parameters (GM, a) of global geopotential coefficient model, the second-degree zonal harmonic coefficient $\bar{C}_{20}$ and the rotation mean angular velocity ω should be employed as the four basic parameters of the normal ellipsoid. In this case, the second-degree zonal harmonic term of anomalous gravity field is always zero, which is beneficial to improve the performance of the gravity field approach.

The geoid determined according to the geodetic boundary value theory is essentially the realization of the constant geopotential W_G in the Earth coordinate system, namely determining of the ellipsoidal height of the geoid. The geoidal geopotential W_G is always equal to the normal potential U_0 of the normal ellipsoid. PAGravf4.5 suggests that the geoidal geopotential W_G replaces the appoint W_0 in the IERS numerical standard. The latter is usually calculated from the global geopotential model and satellite altimetry data according to the Gaussian geoid appoint.

Whether for realizing of global height datum or for refining of regional height datum, when the geoidal geopotential W_G is the global geopotential W_0, it can not only effectively reflect the unique invariance of geodetic datum, but also make full use of physical geodesy (space geodesy) technology and method to approach the global geopotential with infinite precision, which can ensure the analytic rigor of gravity field approach method in the realization and unification of height datum.

(3) Recommending the analytical orthometric system

The analytic orthometric height of the global ground points is closer to the normal height, and the difference is about 60 cm from the Helmert orthometric height at 3000 m altitude. The analytical orthometric height is not directly related to the terrain density, which can be continuously refined with the latest gravity field data. The geometric scale of the analytical orthometric height is consistent with that of the Earth coordinate system and the geoidal height.

On the view of uniqueness, repeatability, and measurability of geodetic datum, analytical orthometric height is more suitable for height datum purpose than other types of orthometric height. Different from the Helmert orthometric system, the analytic orthometric system and normal height system are analytically consistent with each other and supported by the rigorous gravity field theory, they can also be directly employed for the moon and Earth-like planets.

1.5 Algorithm features and use notes of PAGravf4.5

1.5.1 Complete and analytical terrain effect algorithm system

PAGravf4.5 independently developed the complete set of terrain effect algorithm system to realize various modes of terrain effects on different types of gravity field elements on the geoid or in its outer space.

(1) The set of algorithms are rigorous in theory, the numerical integral has no calculation error, and the accuracy of the fast FFT algorithms is controllable.

(2) There are various modes of terrain masses adjustment, the type of gravity field element affected by terrain can be arbitrary and the field element can be located on the geoid or in its outer space.

(3) Strictly follows the analytical relationship of gravity field between the terrain effects on different types of field elements.

(4) Makes full use of the analytical compatibility between different modes of terrain effects, so that the algorithm codes can be short and concise.

From the terrain effect formulas in sections 7.5 to 7.8, it is easy to see that many algorithm formulas are very similar. Some terrain effect algorithms only adjust some parameters and call the same codes.

1.5.2 Technical features of gravity field Integral algorithm

(1) The fixed integral radius of gravity field

Limiting the definition domain of kernel function, PAGravf4.5 executes the gravity field integral operation with the given radius, including numerical integral and FFT integral algorithm (kernel function windowing), to coordinate and unify various gravity field approach algorithms. Two-dimensional FFT adopts the modified planar two-dimensional kernel function, and its calculation accuracy is not significantly different from that of one-dimensional FFT in the range of latitude 10°.

(2) The calculation point and the move point (integral running area element)

The coordinates of geodetic points are expressed by latitude and longitude and ellipsoidal height. For example, the position of boundary surface, measurement point, calculation point, and integral move point (area element or volume element) are expressed by geodetic coordinates. The integral cell grid position is the geodetic coordinates of the center of the cell grid, and the integral radius is calculated by geodetic coordinates.

(3) The equipotential boundary surface

Most gravity field integral formulas are derived from Stokes boundary value theory, such as the Hotine integral, Vening Meinesz integral, radial gradient integral formula, etc. The solution of Stokes boundary value problem requires that the boundary surface is an equipotential surface, that is, the anomalous gravity field elements should be located on some an equipotential surface.

In PAGravf4.5, the accuracy of the ellipsoidal height employed as the boundary surface is not less than 10 m can meet most requirements. The boundary surface can be constructed from a 360-degree global geopotential coefficient model, which can also be replaced by the ellipsoidal height grid of normal or orthometric equiheight surface in near-Earth space.

1.5.3 Performance of gravity field approach using spherical radial basis functions

PAGravf4.5 proposes three key technical countermeasures to make the spherical radial basis functions (SRBF) approach algorithm independent of the observation error, avoid the spectral leakage of the undetermined target field element, and improve the analytical performance of the SRBF algorithm, to ensure the analytical approach of gravity field using SRBF.

(1) Using the edge effect suppression method to instead of normal equation regularization.

PAGravf4.5 proposes the algorithm to improve the performance of parameter estimation of spherical radial basis functions (SRBF) coefficients by suppressing edge effects. When the SRBF center is located at the margin of the calculation area, let that the SRBF coefficient is equal to zero as the observation equation to improve the stability and reliability of unknown SRBF coefficient estimation.

After the edge effect suppression method employed, the normal equation need not be regularized. Which can keep the analytical nature of the SRBF approach algorithm and the analytical nature of gravity field from being affected by the observation errors.

(2) Employing the cumulative SRBF approach method to achieve the best approach of the gravity field.

The target field elements are equal to the convolution of the observations and the filter SRBF. When the target field elements and the observations are of different types, it is difficult for one SRBF to effectively match the spectral center and bandwidth of the observations and the target field element at the same time, which would make the spectral leakage of the target field element. In addition, the SRBF type, the minimum and maximum degree of Legendre expansion and the SRBF center distribution also all affect the approach performance of the gravity field. Therefore, only the optimal estimation of the SRBF coefficient with the burial depth as the parameter is not enough to ensure the best approach of the gravity field.

PAGravf4.5 proposes a cumulative SRBF approach scheme according to the linear additivity of the gravity field to replace the optimal estimation scheme of SRBF coefficients with the burial depth as the parameter to solve the key problem above. Using the multiple cumulative SRBF approach scheme, it is not necessary to determine the optimal burial depth.

When each SRBF approach of gravity field employs a SRBF with different spectral figure, the cumulative SRBF approach can fully resolve the spectral domain signal of the target field element by combining multiple SRBF spectral centers and bandwidths, and then optimally restore the target field element in space domain.

The character of cumulative SRBF approach scheme of gravity field: the essence of each SRBF approach is to employ the previous approach results as the reference gravity field, and then refine the residual target field element by remove -restore scheme.

(3) Proposing the cofactor matrix diagonal standard deviation method.

PAGravf4.5 proposes a cofactor matrix diagonal standard deviation method to combinate different types of heterogeneous observations for estimation of the SRBF coefficients, instead of the common variance component estimation method. This method replaces the residual observations variance in the iterative process of the variance component estimation with the diagonal standard deviation of the cofactor matrix, so that the properties of the parameter estimation solution are only related to the space distribution of the observations from being affected by the observation errors. Which is conducive to combination of various types of observations with extreme differences in space distribution, such as a very small number of astronomical vertical deflections or GNSS-levelling data.

In this case, the normal equation does not also need to be iteratively calculated, which conducive to improve the analytical nature of the SRBF approach algorithm.

● The typical technical features of SRBF approach program in PAGrav4.5: ① The analytical function relationships between gravity field elements are strict, and the SRBF approach performance has nothing to do with the observation errors. ② Various heterogeneous observations in the different altitudes, cross-distribution, and land-sea coexisting cases can be directly employed to estimate the full element models of gravity field without reduction, continuation, and griding. ③ Can integrate very little astronomical vertical deflection or GNSS-levelling data, and effectively absorb the edge effect. ④ Has the strong capacity in detection of observation gross errors, measurement of external accuracy indexes, and control of computational performance.

1.5.4 Computation scheme of fine gravity prospecting by analytic fusion of heterogeneous observations

PAGravf4.5 has the high-precision analytical computation capacity of various terrain effects on any type of gravity field element. At the same time, it has the full-element analytic modelling function on gravity field from various heterogeneous observations. The combination of the two can effectively solve the fine computation problem of gravity exploration that can deeply fuse all the gravity field information in multi-source heterogeneous observations.

In any region of the world, you can accurately calculate the land-sea unified complete Bouguer gravity anomaly (disturbance), complete Bouguer vertical deflection, complete Bouguer gravity gradient, and classical Bouguer / isostatic gravity anomaly (disturbance) from heterogeneous observations such as gravity, gravity gradient, satellite altimetry, (astronomical) vertical deflection, GNSS-leveling etc. in the different altitudes, cross-distribution, and land-sea coexisting cases.

The general computation process for fine gravity prospecting from heterogeneous observations is as follows.

(1) Select the calculation area, target calculation surface (terrain equiheight surface is recommended here) for gravity exploration and obtain (or collect) all the gravity field and geodetic observations as much as possible.

(2) Call the related programs in the subsystem [Precision approach and full element modelling on Earth gravity field] to determine the high-resolution grid of gravity field element corresponding to the target prospecting model on the calculation surface.

(3) Call the related programs in the subsystem [Computation of various terrain effects on various field elements outside geoid] to determine the terrain effect grid on gravity field element corresponding to the target prospecting model on the calculation surface.

(4) By subtracting the terrain effect grid from the gravity field element grid directly, you can obtain the target gravity prospecting model which have deeply fuse all the gravity field information in heterogeneous observations.

If the SRBF approach method is employed in step (2), the whole gravity prospecting modelling process above are all strictly analytical, which can effectively avoid signal attenuation and distortion of gravity field and the problem of terrain effect difficult to control in traditional reduction, continuation, and gridding.

The computation of fine gravity prospecting with the analytic relations of gravity field as strong constraints by PAGravf4.5 can deeply fuse all the gravity field information from heterogeneous observations in the different altitudes, cross-distribution and land-sea coexisting cases, whose observation mode can be on terrestrial, marine, aviation and satellite.

1.5.5 Algorithm and computation scheme optimization

The algorithm system in PAGravf4.5 is scientific and rigorous, and there are several schemes to calculate the same terrain effect. Various gravity field elements can be solved from any type of field element outside. The field element type can be traditional or uncommon such as satellite tracking satellite and satellite orbit perturbation, and the applicable space can be on the geoid or in its outer space.

The approach of Earth gravity field is linear operation. The terrain effect and gravity field approach algorithms in PALGravf4.5 are all linear. Therefore, Any program in PAGravf4.5 can output the error distribution characters of the target field elements with the simulated spatial noises as observations. PAGravf4.5 has strong error analysis capacity, which can be the important means to optimize the gravity prospecting computation and gravity field approach scheme.

One mode of terrain effect, there are also multiple computation schemes to be chose. For example, to compute the land-sea complete Bouguer effect, PAGravf4.5 has three schemes and programs to be chose.

For the specific data processing and modelling purpose of gravity field, there are multiple PAGravf4.5 programs, different parameter settings or multiple schemes to be chose. In the actual application, you should investigate the gravity data situations and the nature of gravity field in the target region, carefully select, test and analyze the related PAGravf4.5 algorithms, parameters and schemes to design and optimize the computation technology route.

1.5.6 Performance testing and analysis of algorithm and parameter

(1) Performance test for terrain effect algorithm

The terrain effect optimization criterion proposed by PAGravf4.5 according to the basic principles of physical geodesy, can greatly reduce the complexity of terrain effect analysis, and provide a concrete and feasible technical route for effectively playing the key role of terrain effect in geophysical gravity exploration and gravity field approach.

The statistical properties of terrain effects vary significantly with the local terrain, gravity field nature and observations distribution in the computation area. PAGravf4.5 terrain effect computation subsystem present some cases in a difficult mountain area, and the ratio of the maximum and minimum values to the standard deviation of various terrain effects on various field elements was statistically analyzed. The statistical analysis results of these cases show that, when ignoring the observation distribution and gravity field nature, the local terrain effect is favorable for the gravity data processing, the terrain Helmert condensation is favorable for the processing of gravity gradient data, and the residual terrain effect is more favorable for the refinement of geoid.

Before the computation, it is necessary to comprehensively and carefully test and analyze the technical route of terrain effects according to the local terrain, gravity field nature and available gravity resources in the target area according to the quantitative criteria of terrain effect, so as to ensure that the terrain effect algorithm and parameter settings can be based on some evidence. Only then can the applicability and technical level of the terrain effect processing scheme be significantly improved.

(2) Performance test for gravity field approach algorithm

The performance of most gravity field approach algorithms and their parameter settings can be tested and verified with an ultra-high degree global geopotential coefficient model. Many program samples of PAGravf4.5 take the 2 to 540th degree EGM2008 geopotential coefficient model as the reference gravity field, and then employ the anomalous gravity field calculated by the 541 to 1800th degree model as the reference true values for testing and verification.

The test outline of the PAGravf4.5 program algorithm: Take some residual anomalous field elements calculated by the 541 to 1800th degree EGM2008 geopotential coefficients as the observations, call the PAGravf4.5 programs or functions to be verified, and obtain the calculated values of the target residual field elements. Then compare the difference between the calculated values and the target reference values calculated by the EGM2008 model, to evaluate the technical performance of the algorithm programs and their parameter settings in PAGravf4.5.

PAGravf4.5 can compute various types of field elements on the geoid or in its outer space from some a type of field and can also cyclically calculate the same type of field elements. Comparing the difference and similarity between the observed field elements and the calculated field elements obtained by the cyclical computation, the algorithm character and performance of the relevant programs and functions called in the computation process can be analyzed.

(3) Performance test for gravity field approach using SRBF

The best approach scheme of gravity field using SRBF is related to the observation situations, nature of gravity field and algorithm parameters. The program of [Gravity field approach using SRBFs in spectral domain and performance test] can be employed to comprehensively analyze the spectral center and bandwidth of the observation, target field element and SRBF in different parameters combination case. According to the principle of fully resolving the spectrum of the target field element, design and optimize the scheme and relevant parameters for SRBF approach of gravity field in advance.

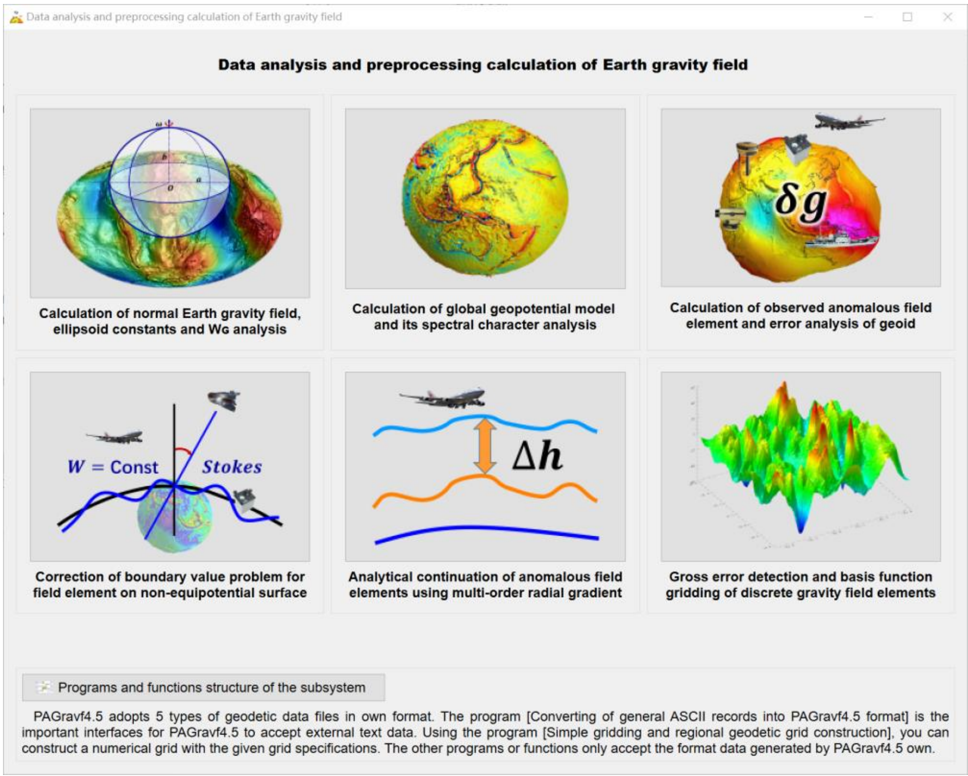
(4) Notes on the PAGravf4.5 program examples

In the examples of terrain effect programs of PAGravf4.5, a typical difficult mountainous area with an average altitude of 4000m and terrain relief of more than 3000m is selected to facilitate the display of the details of terrain effect and its algorithm characters. Similarly, in the gravity field integral examples of PAGravf4.5, the region with typical complex features where the short-wave signal of the gravity field is rich (residual gravity disturbance after the 540-degree reference model value removed, and the space variation exceeds 300mGal) is selected to facilitate the display of the detailed features of the local gravity field and its approach algorithm.

The statistical results of these samples roughly show the basic performance of the corresponding algorithms. Since the main purpose of these samples is to introduce the computation process, there is no statistical analysis and optimization of the algorithm itself and parameter settings. Therefore, there is still greater potential, which need be further explored by the user in combination with the specific situations.

2 Data analysis and preprocessing calculation of Earth gravity field

The subsystem of the data analysis and preprocessing calculation of the Earth gravity field is mainly employed for the analysis and calculation of the normal Earth gravity field and Earth ellipsoid constants, calculation of the geopotential coefficient model and its spectral feature, correction of gravity field elements for the geodetic boundary value problem, and analytical continuation, gross error detection and griding operation for gravity field observations.



2.1 Calculation of normal Earth gravity field, ellipsoid constants and W_G analysis

[Purpose] Calculate the normal gravity field parameters at the Earth space point, geometrical and physical constants of the normal Earth ellipsoid and gravimetric geoidal geopotential W_G according to the rigorous analytical algorithm of spherical harmonic expansion.

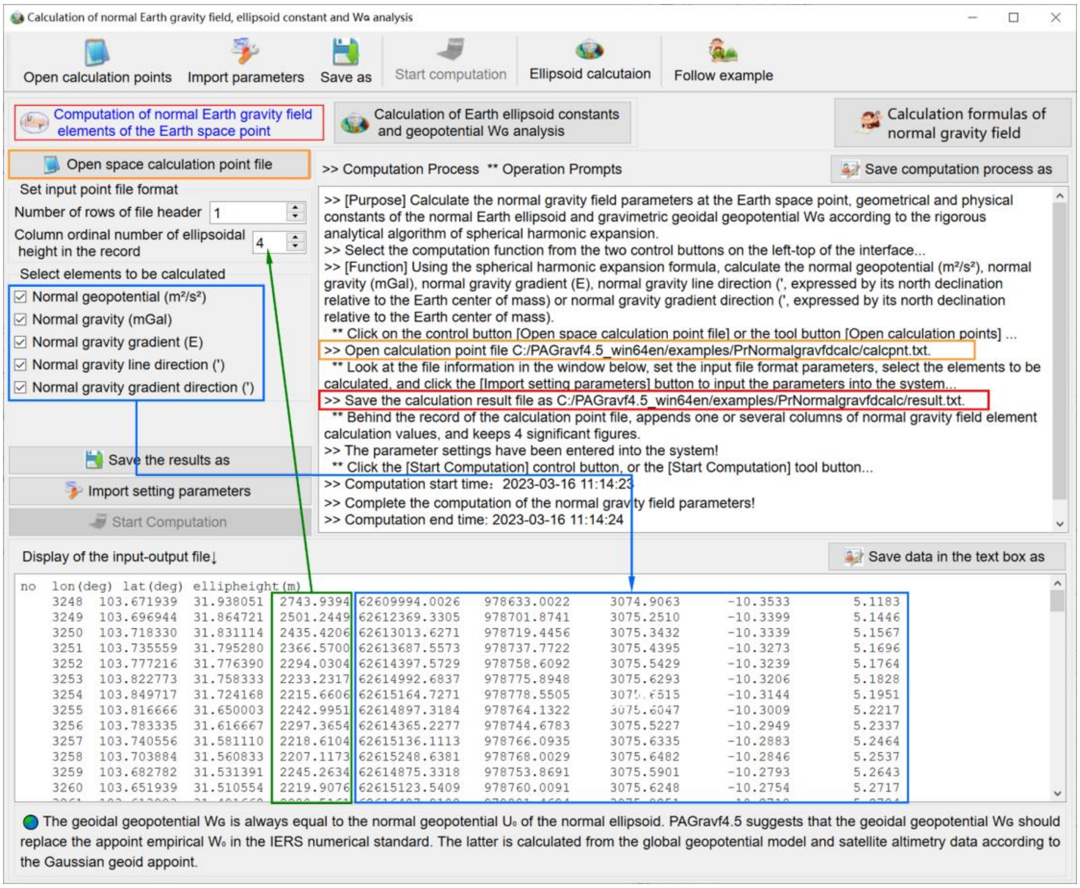
Essentially, the geoid determined according to the geodetic boundary value theory is the realization of the constant geopotential W_G in the Earth coordinate system, namely determining of the ellipsoidal height of the geoid. The geoidal geopotential W_G is always equal to the normal geopotential U_0 of the normal ellipsoid. PAGravf4.5 suggests that the geoidal geopotential W_G should replace the appoint empirical W_0 in the IERS numerical standard. The latter is calculated from the global geopotential model and satellite altimetry data according to the Gaussian geoid appoint.

2.1.1 Computation of normal Earth gravity field elements of the Earth space point

[Function] Using the spherical harmonic expansion formula, calculate the normal geopotential (m²/s²), normal gravity (mGal), normal gravity gradient (E), normal gravity line direction (', expressed by its north declination relative to the center of the Earth center of mass) or normal gravity gradient direction (', expressed by its north declination relative to the Earth center of mass).

[Input file] The space calculation point file.

The record format: ID (point no / point name), longitude (decimal degrees), latitude (decimal degrees), ellipsoidal height (m)......



[Parameter settings] Input the number of rows of the input file header, and column ordinal number of ellipsoidal height in the record, and select the normal gravity field elements to be calculated.

[Output file] The normal Earth gravity field element file.

Behind the record of the calculation point file, appends one or several columns of normal gravity field element calculation values, and keeps 4 significant figures.

2.1.2 Calculation of Earth ellipsoid constant and geopotential W_G analysis

[Function] From four basic parameters of the Earth ellipsoid, calculate the main geometric and physical derived constants of the Earth ellipsoid.

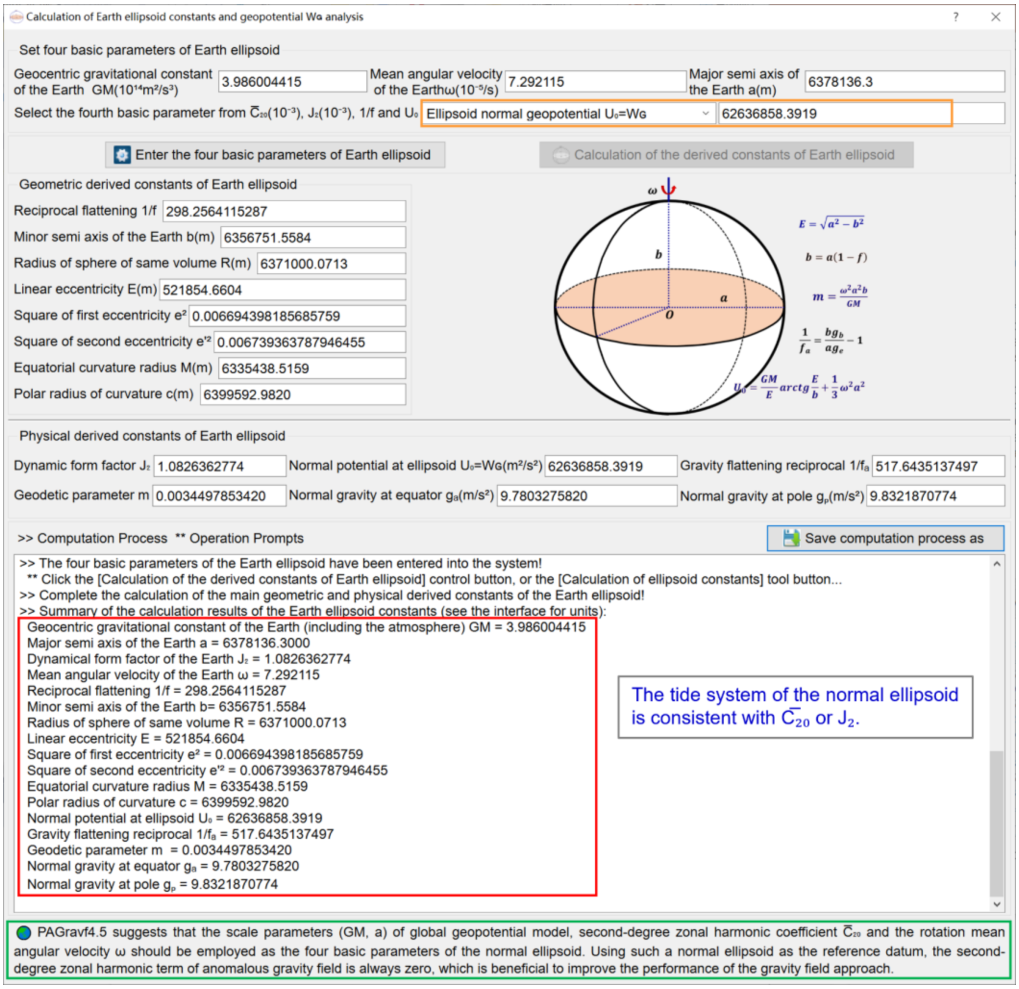
[Parameter settings] Select the four basic parameters of the Earth ellipsoid.

The fourth basic parameter can be selected from the second-degree zonal harmonic coefficient $\bar{C}_{20}$ from global geopotential model, dynamic form factor J_2, reciprocal 1/f of the ellipsoid flattening, and ellipsoid normal geopotential U_0.

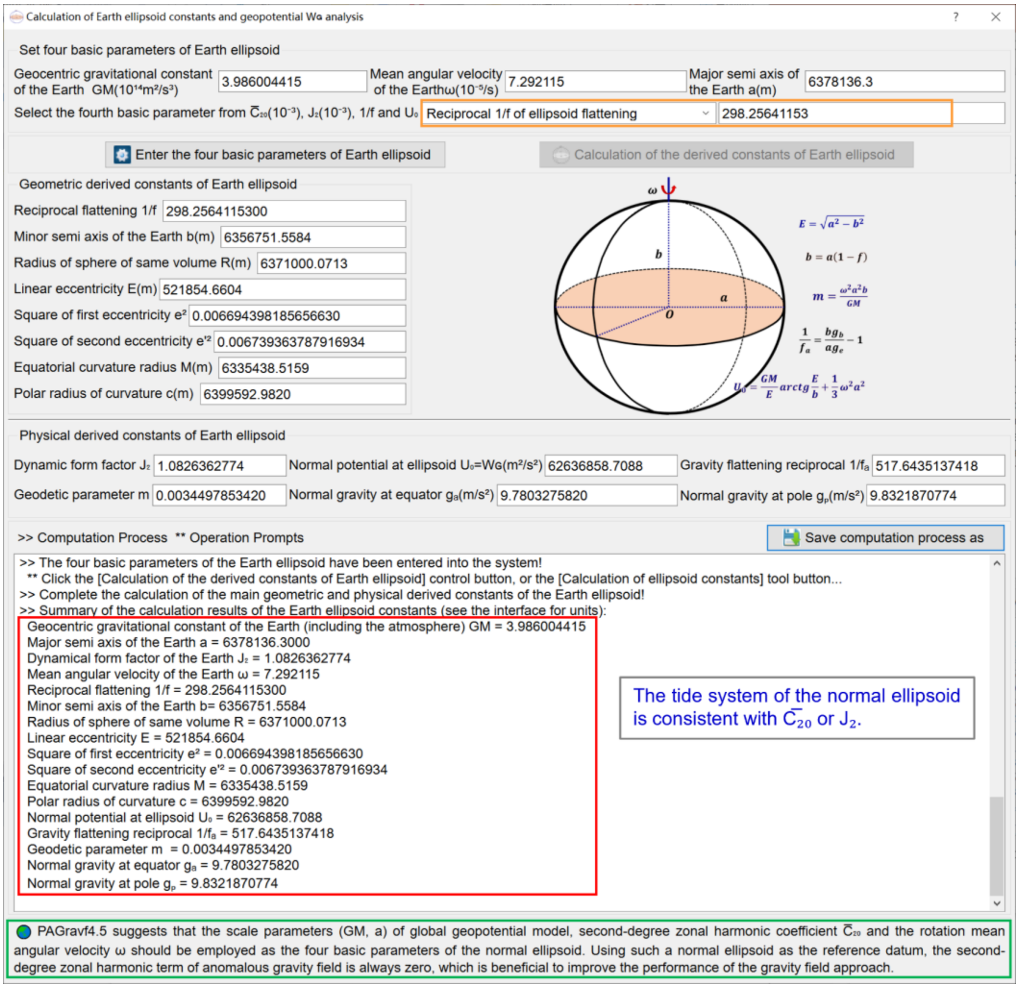
The coefficient $\bar{C}_{20}$ from EGM2008 global geopotential model is selected currently as the fourth basic parameter.

[Output] Interactively output the summary of the calculation results of the Earth ellipsoid constants in the program interface.

The tide system of the normal ellipsoid is consistent with $\bar{C}_{20}$ or J_2.



PAGravf4.5 suggests that the scale parameters (GM, a) of global geopotential model, second-degree zonal harmonic coefficient $\bar{C}_{20}$ and the rotation mean angular velocity ω should be employed as the four basic parameters of the normal ellipsoid. Using such a normal ellipsoid as the reference datum, the second-degree zonal harmonic term of anomalous gravity field is always zero, which is beneficial to improve the performance of the gravity field approach.



Geopotential model	Scale parameters		$\bar{C}_{20}\times10^{-4}$	W_G/U_0 (m^2/s^2)	Tide system
	$GM\times10^{14}m^2/s^3$	a(m)			
EGM2008	3.986004415	6378136.3	-4.84165143791	62636858.392	Tide free
EIGEN-6C4	3.986004415	6378136.46	-4.84165217061	62636856.834	zero tide
SGG-UGM2	3.986004415	6378136.3	-4.84168732275	62636858.644	zero tide
GOCO05c	3.986004415	6378136.3	-4.84169458843	62636858.694	zero tide
XGM2019	3.986004415	6378136.3	-4.84169494748	62636858.697	zero tide

2.2 Calculation of global geopotential model and its spectral character analysis

[Purpose] From the global geopotential coefficient model, calculate the model value of anomalous gravity field elements at any point in the Earth space, calculate the degree variance of the geopotential coefficients, error degree variance and cumulative error of anomalous field elements, then evaluate the performance of the geopotential model.

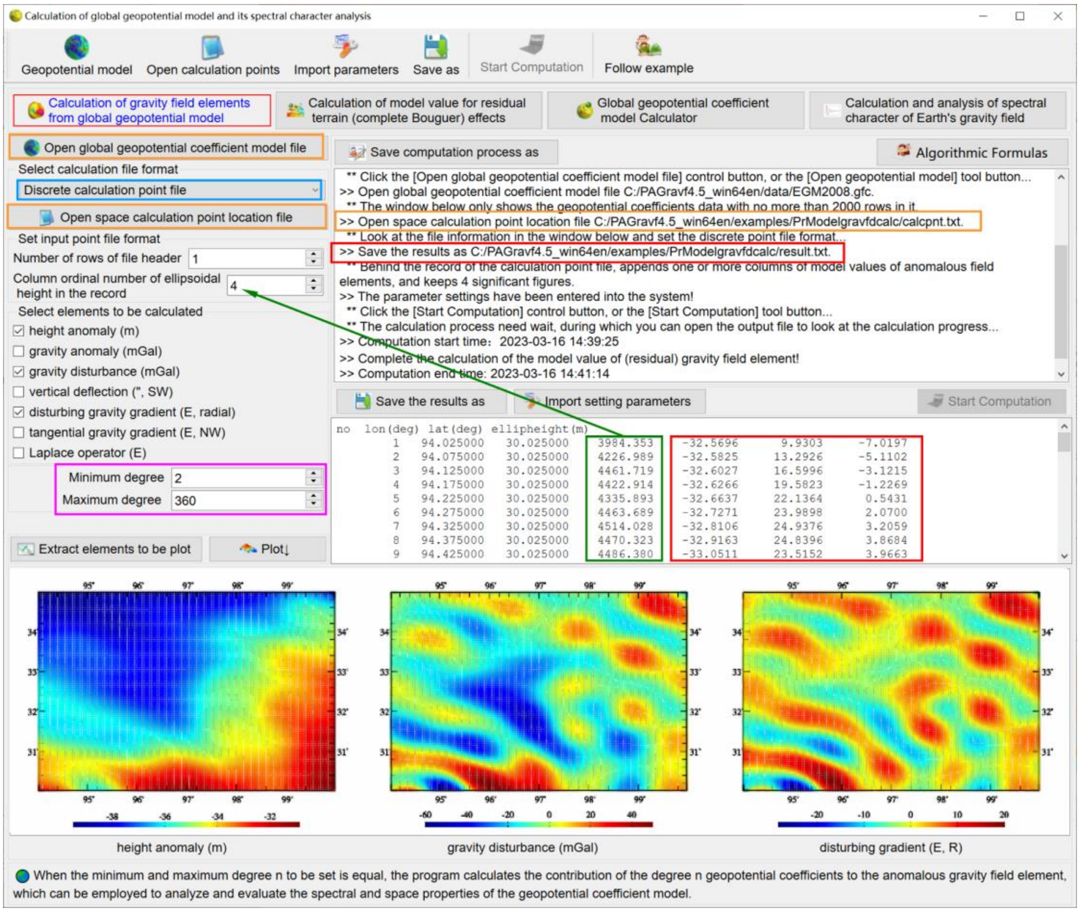
When the minimum and maximum degree n to be set is equal, the program calculates the contribution of the degree n geopotential coefficients to the anomalous gravity field element, which can be employed to analyze and evaluate the spectral and space properties of the geopotential coefficient model.

2.2.1 Calculation of gravity field elements from global geopotential model

[Function] From global geopotential coefficient model, calculate the model value of the (residual) height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection vector (", south, west), disturbing gravity gradient (E, radial), tangential gravity gradient vector (E, north, west), or Laplace operator (E).

[Input files] The global geopotential coefficient model file, and the space calculation point location file.

The first row of the geopotential model file is agreed to be the scale parameters of the geopotential coefficient model: the geocentric gravitational constant of the Earth GM ($10^{14}m^3/s^2$) and semi-major axis of the Earth a(m). Here, the surface harmonic functions in the geopotential model are defined on the spherical surface whose radius is equal to the semi-major axis a of the Earth.



The space calculation point location file may be a discrete calculation point file or an ellipsoidal height grid file of the calculation surface.

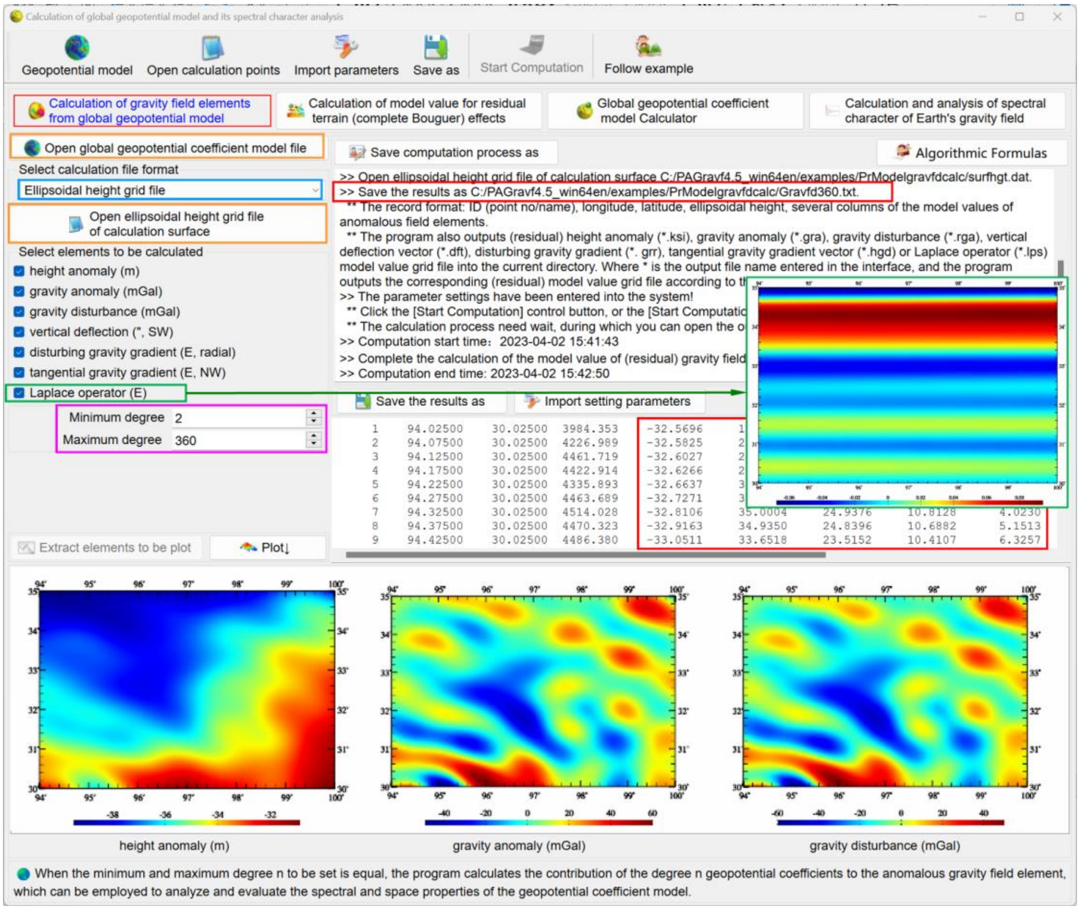
The record format of the calculation point file: ID (point no / point name), longitude (decimal degrees), latitude (decimal degrees), ellipsoidal height (m)......

[Parameter settings] Input the number of rows of the file header, column ordinal number of ellipsoidal height in the record, enter the minimum and maximum calculation degree of the geopotential model, and select the type of model gravity field elements to be calculated.

When the minimum calculation degree is equal to 2, the program calculates the model value of anomalous gravity field elements. And when the minimum calculation degree is greater than 2, the program calculates the model value of residual anomalous gravity field elements.

The program selects the minimum of the maximum degree of the global geopotential model and the input maximum degree as the calculation degree.

The calculation process need wait, during which you can open the output file to look at the calculation progress...



[Output file] The model value file of (residual) anomalous gravity field element.

When the discrete calculation point file input, the output file record format: Behind the record of the calculation point file, appends one or more columns of model values of anomalous field elements selected, and keeps 4 significant figures.

When the ellipsoidal height grid file input, the output file record format: point no/name, longitude, latitude, ellipsoidal height, several columns of the model values of anomalous field elements selected.

The program also outputs (residual) height anomaly (*.ksi), gravity anomaly (*.gra), gravity disturbance (*.rga), vertical deflection vector (*.dft), disturbing gravity gradient (*. grr), tangential gravity gradient vector (*.hgd) or Laplace operator (*.lps) model value grid file into the current directory. Where * is the output file name entered in the interface, and the program outputs the corresponding (residual) model value grid file according to the selected gravity field element type.

The theoretical value of the Laplace operator (E) of any degree n, cumulative n degree or n_1~n_2 degree are always equal to zero. By calculating the degree n, cumulative n degrees or n_1~n_2 degrees of Laplace operators from some a global geopotential model, the spectral domain and spatial domain performance of the model can be observed and evaluated.

2.2.2 Calculation of model value for residual terrain (complete Bouguer) effects

[Function] From the global land-sea terrain geopotential coefficient model (m, the first degree term is set to zero), calculate the model value of residual terrain (complete Bouguer) effects on the height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection vector (", south, west), disturbing gravity gradient (radial, E), tangential gravity gradient vector (E, north, west), or Laplace operator outside the geoid.

The global land-sea terrain geopotential coefficient model can be constructed by the function [Ultrahigh degree land-sea terrain spherical harmonic analysis and model construction].

[Input files] The global land-sea terrain geopotential coefficient model file, and the space calculation point location file.

The first row of the model file is agreed to be the scale parameters of the geopotential coefficient model: the geocentric gravitational constant of the Earth GM ($10^{14}m^3/s^2$) and semi-major axis of the Earth a(m).

The space calculation point location file may be a discrete calculation point file or an ellipsoidal height grid file of the calculation surface.

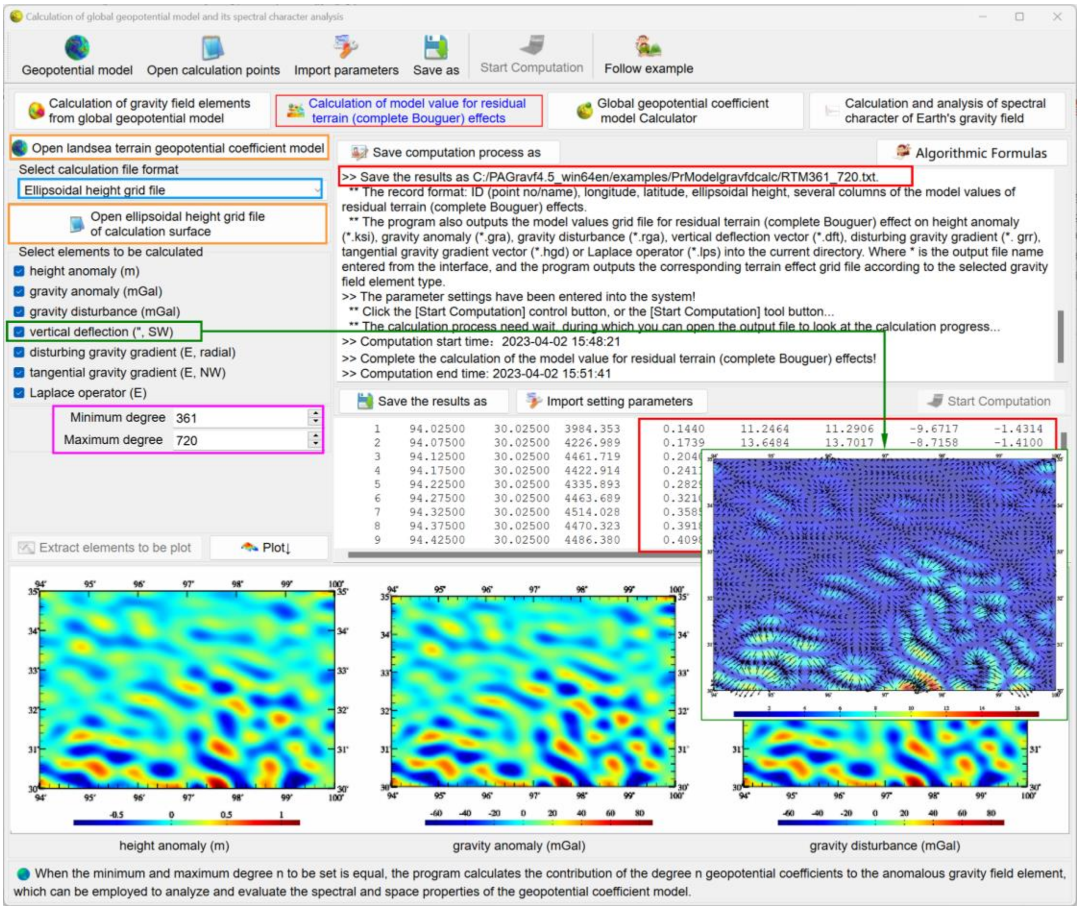
The record format of the calculation point file: point no / point name, longitude (decimal degrees), latitude (decimal degrees), ellipsoidal height (m)......

[Parameter settings] Input the number of rows of the file header, column ordinal number of ellipsoidal height in the record, input the minimum and the maximum calculation degree of the land-sea terrain geopotential model, and select the type of model residual terrain (complete Bouguer) effects to be calculated.

When the minimum calculation degree is equal to 2, the program calculates the model value of land sea complete Bouguer effects. And when the minimum calculation degree is greater than 2, the program calculates the model value of land sea residual terrain effects.

The program selects the minimum of the maximum degree of the model and the input maximum degree as the calculation degree.

The calculation process need wait, during which you can open the output file to look at the calculation progress...



[Output file] The model value file of residual terrain (complete Bouguer) effects.

When the discrete calculation point file input, the output file record format: Behind the record of the calculation point file, appends one or more columns of the model values of residual terrain (complete Bouguer) effects selected, and keeps 4 significant figures.

When the ellipsoidal height grid file input, the output file record format: point no/name, longitude, latitude, ellipsoidal height, several columns of the model values of residual terrain (complete Bouguer) effects selected.

The program also outputs the model value grid file for residual terrain (complete Bouguer) effect on height anomaly (*.ksi), gravity anomaly (*.gra), gravity disturbance (*.rga), vertical deflection vector (*.dft), disturbing gravity gradient (*. grr), tangential gravity gradient vector (*.hgd) or Laplace operator (*.lps) into the current directory. Where * is the output file name entered from the interface, and the program outputs the corresponding terrain effect grid file according to the selected gravity field element type.

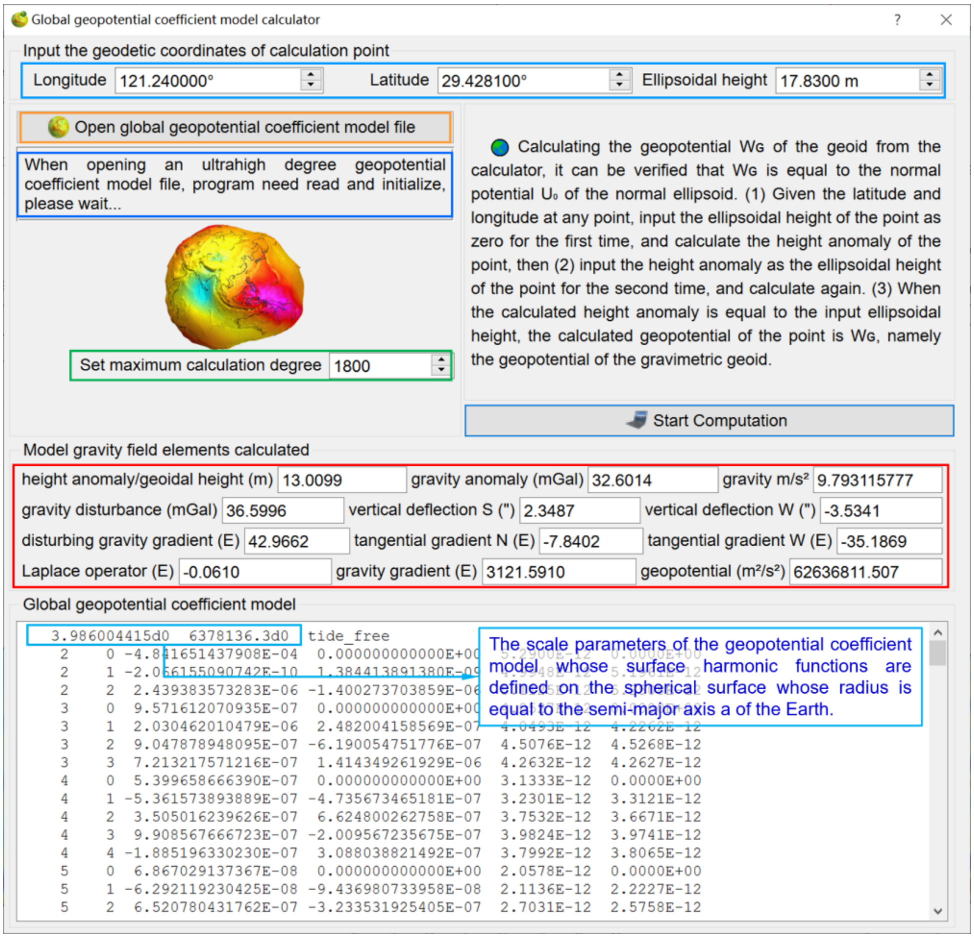
Similarly, by calculating the degree n, cumulative n degrees or $n_1 \sim n_2$ degrees

Laplace operators from some a global terrain geopotential model, the spectral domain and spatial domain performance of the model or that of residual terrain effect model values can be evaluated.

2.2.3 Global geopotential coefficient model calculator

[Function] Given the geodetic coordinates of the calculation point, from the global geopotential coefficient model file, calculate the model values of the height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection vector (", south, west), disturbing gravity gradient (radial, E), tangential gravity gradient vector (E, north, west), Laplace operator (E), gravity gradient (E) and geopotential (m²/s²). This program is suitable for classroom demonstrations.

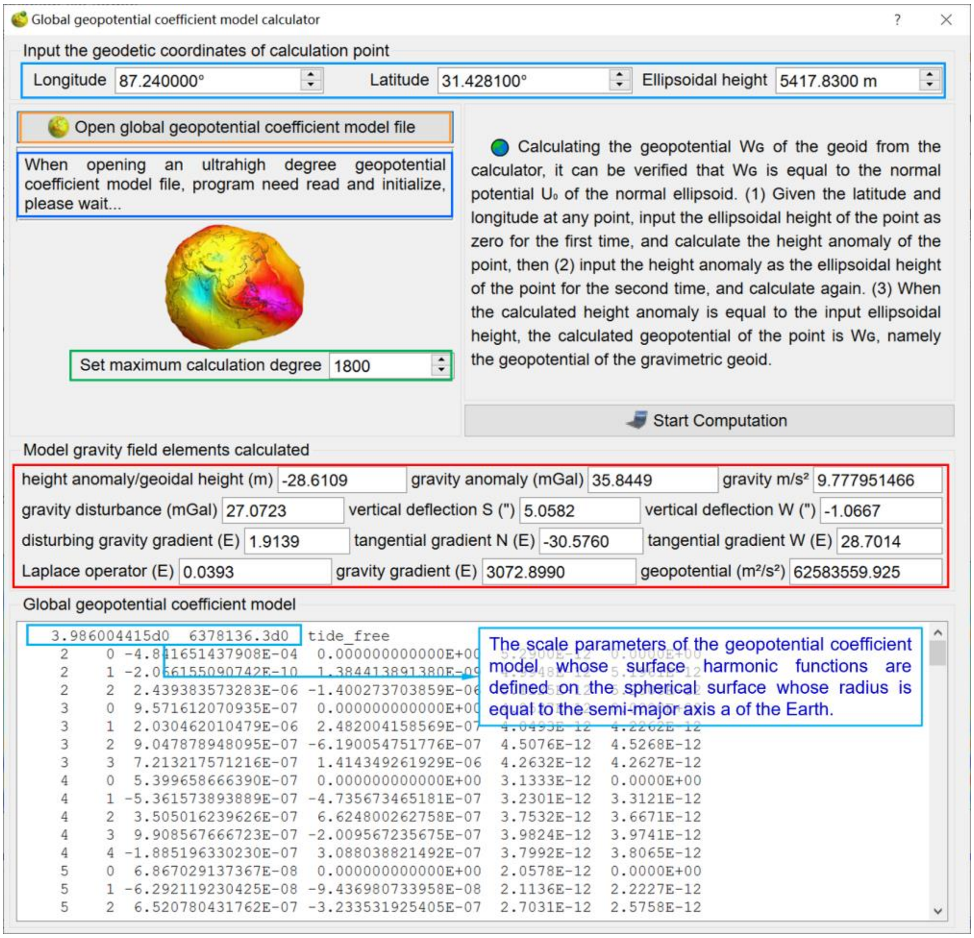
When opening an ultrahigh degree geopotential coefficient model file, program need read and initialize, please wait...



After the geopotential model input, the geodetic coordinates of the calculation point can be entered repeatedly, and the model values of various field elements at the calculation point can be calculated and displayed in time.

From the Laplace equation, the sum of the disturbing gravity gradient (radial) and two components of the tangential gravity gradient should theoretically be equal to zero.

Accordingly, the performance of the geopotential model can be evaluated.



Calculating the geopotential W_G of the geoid from the calculator, it can be verified that W_G is equal to the normal potential U_0 of the normal ellipsoid. Calculation process: Given the latitude and longitude at any point, input the ellipsoidal height of the point as zero for the first time, and calculate the height anomaly of the point, then input the height anomaly as the ellipsoidal height of the point for the second time, and calculate again. When the calculated height anomaly is equal to the input ellipsoidal height, the calculation point is on the geoid, and the calculated geopotential of the point is W_G, namely the geopotential of the gravimetric geoid.

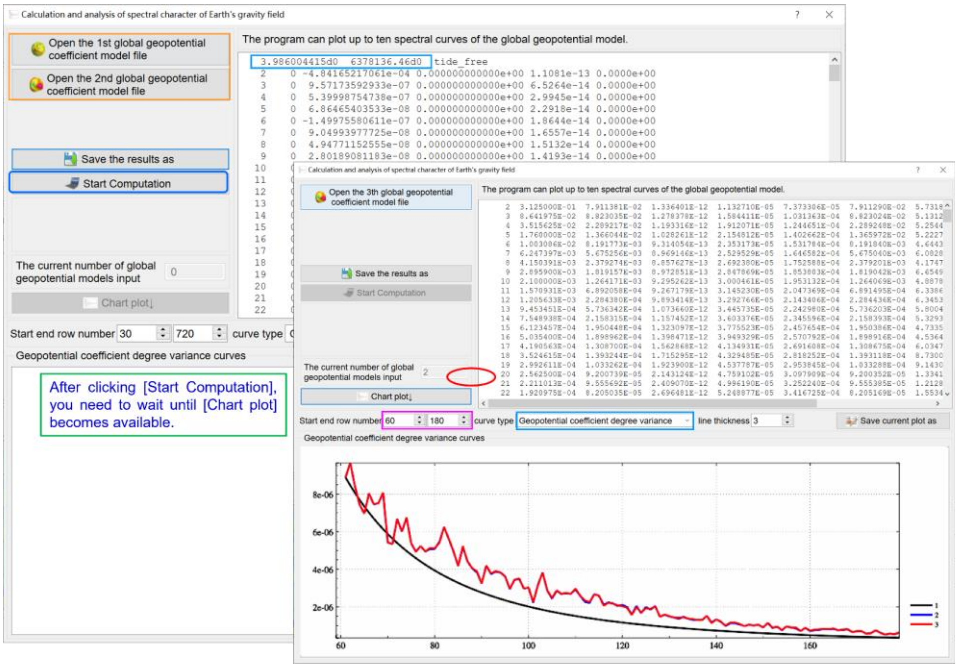
It is easy to verify that the gravity potential on the geoid at any point is always equal to W_G. Changing the latitude and longitude of the calculation point, adjust the ellipsoidal height of the calculation point according to the calculated value of the height anomaly, then you will find that the gravity potential of the geoid remains unchanged at any point all the word.

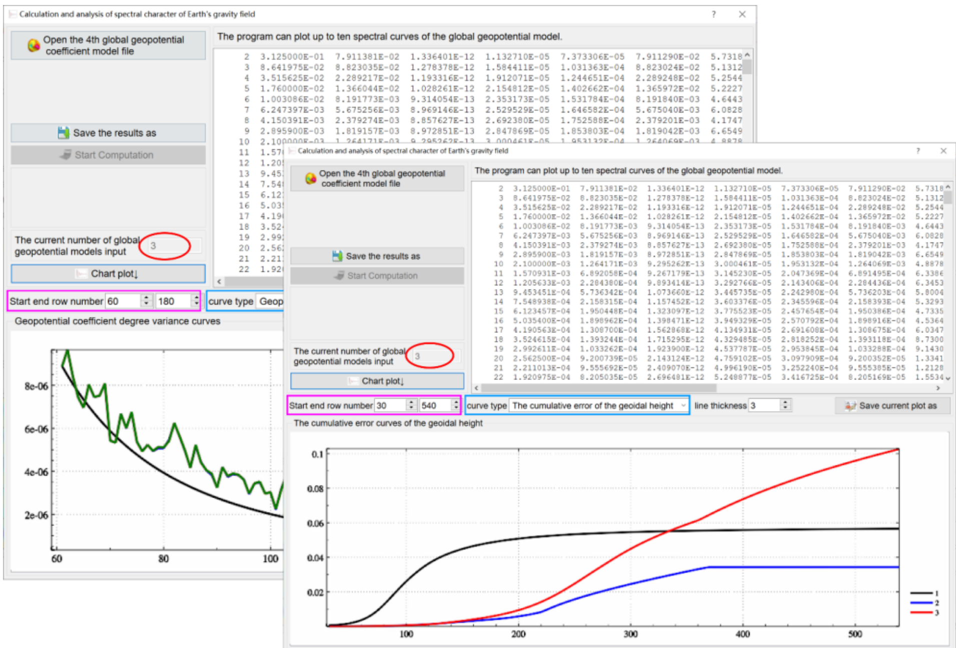
2.2.4 Calculation and analysis of spectral character of Earth's gravity field

[Function] From several global geopotential models, calculate the Kaula curve, geopotential coefficient degree variance, error degree variance, and cumulative error of the geoid and gravity anomaly. Compare the spectral properties of different geopotential coefficient models.

Firstly, open two global geopotential coefficient model files, can then continue to open other geopotential coefficient model files as needed. The program can plot up to ten spectral curves of the global geopotential model.

After clicking [Start Computation], you need to wait until [Chart plot] becomes available.





The interface shows the current number of global geopotential models input.

The geopotential coefficient degree variance curves, the error degree variance curves or the cumulative error curves of the geoid and gravity anomaly can be selected to plot.

2.3 Calculation of observed anomalous field element and error analysis of geoid

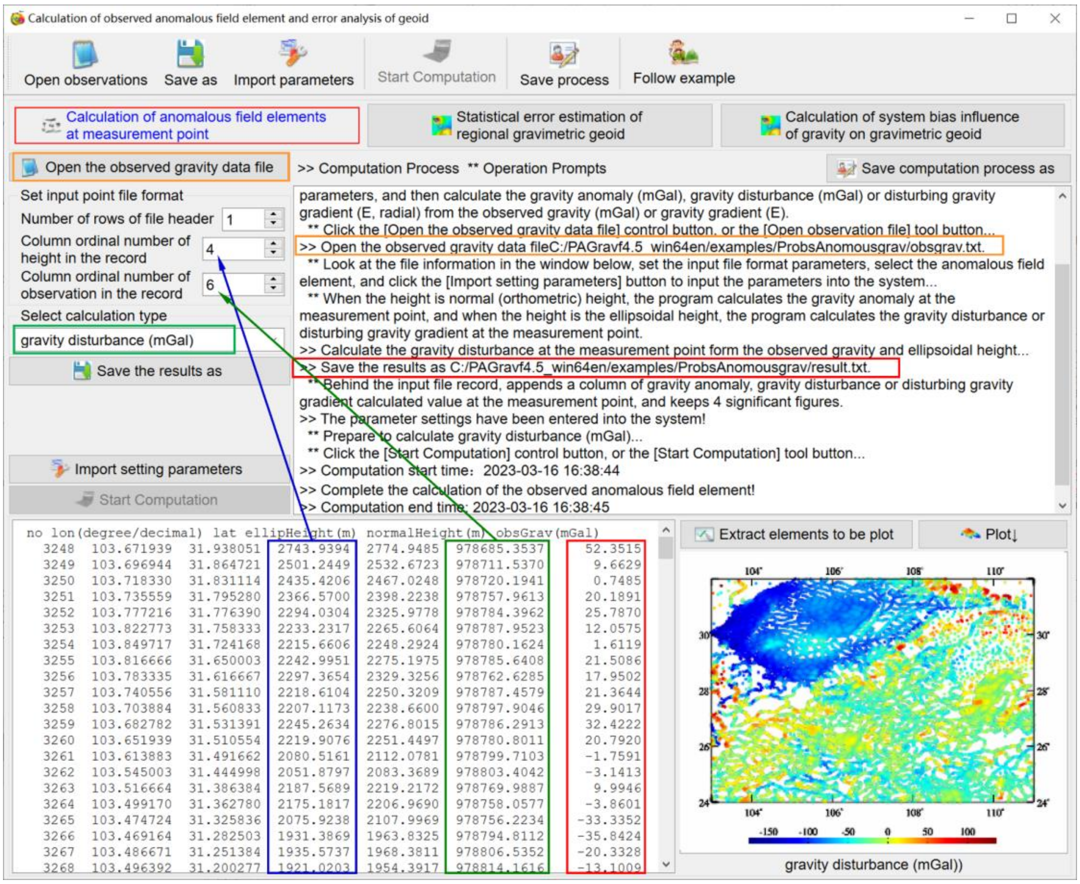
[Purpose] From the observed gravity data, calculate the gravity anomaly, gravity disturbance or disturbing gravity gradient at the measurement point, and estimate the influence of the grid mean gravity error and integral radius on regional gravimetric geoid.

2.3.1 Calculation of anomalous field elements at measurement point

[Function] Using the rigorous spherical harmonic expansion formula, calculate the normal gravity field parameters, and then calculate the gravity anomaly (mGal), gravity disturbance (mGal) or disturbing gravity gradient (E, radial) from the observed gravity (mGal) or gravity gradient (E).

[Input file] The observed gravity data file.

The record format: ID (point no / point name), longitude (decimal degrees), latitude (decimal degrees), height (m), …, observation, ...



[Parameter settings] Input the number of rows of the file header, column ordinal number of height attribute and observation in record.

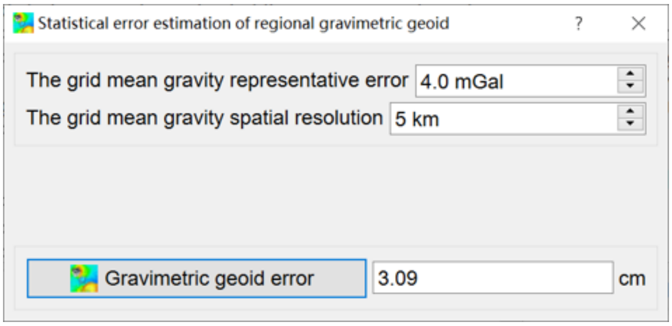
When the height is normal (orthometric) height, the program calculates the gravity anomaly at the measurement point, and when the height is the ellipsoidal height, the program calculates the gravity disturbance or disturbing gravity gradient at the measurement point.

[Output file] The observed anomalous field element file.

Behind the input file record, appends a column of gravity anomaly, gravity disturbance or disturbing gravity gradient calculated value at the measurement point, and keeps 4 significant figures.

2.3.2 Statistical error estimation of regional gravimetric geoid

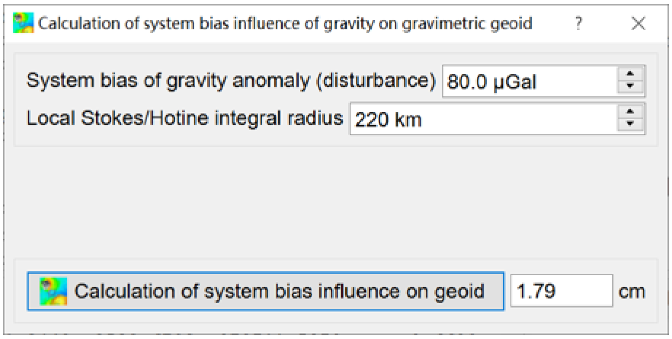
[Function] Estimate the error of the regional gravimetric geoid from the spatial resolution and error of the grid mean gravity. The regional gravimetric geoid error analysis is based on the Stokes/Hotine integral formula with a finite integral radius, ignoring the far-area effect of residual anomalous gravity field.



The grid mean gravity representative error here mainly refers to the terrain representative error, because in general, the gravity observation error during the griding process is much smaller than the influence of the terrain representative error.

2.3.3 Calculation of system bias influence of gravity on gravimetric geoid

[Function] Calculate the influence of the Stokes/Hotine integral radius on the gravimetric geoid from the systematic bias of the gravity anomaly or gravity disturbance.



The system bias of gravity anomaly or gravity disturbance. Whose possible causes are the inconsistency in normal gravity field, nonuniform in height datum, improper value of global geopotential W_0, or gravity datum error etc.

2.4 Correction of boundary value problem for gravity field element on non-equipotential surface

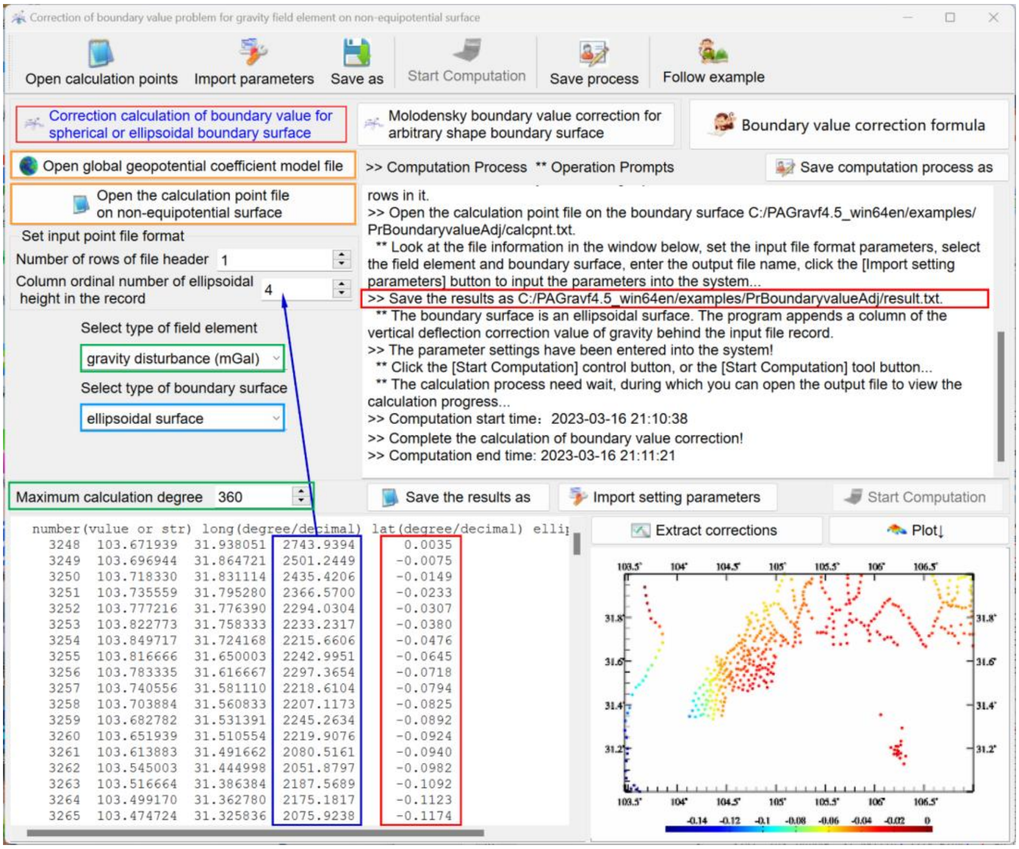
[Purpose] Correct the anomalous gravity field elements located on the spherical surface, ellipsoidal surface, or other shape of non-equipotential surface, to convert the Molodensky boundary value problem on non-equipotential boundary surface into the Stokes boundary value problem.

2.4.1 Correction calculation of boundary value for spherical or ellipsoidal boundary surface

[Function] With the geopotential model, calculate the correction value (mGal) of the gravity disturbance or gravity anomaly from the non-equipotential spherical or ellipsoidal boundary surface into the equipotential surface, thereby converting the Molodensky boundary value problem into the Stokes problem.

The ellipsoidal boundary value correction is mainly employed for ellipsoidal harmonic analysis of the global gravity field by FFT integral method. Spherical boundary value correction is commonly employed for the Bjerhammar boundary value problem.

[Input files] The global geopotential coefficient model file, the calculation point file on the boundary surface.



The first row of the geopotential model file is agreed to be the scale parameters of the geopotential coefficient model: the geocentric gravitational constant of the Earth GM

($10^{14}m^3/s^2$) and semi-major axis of the Earth a(m).

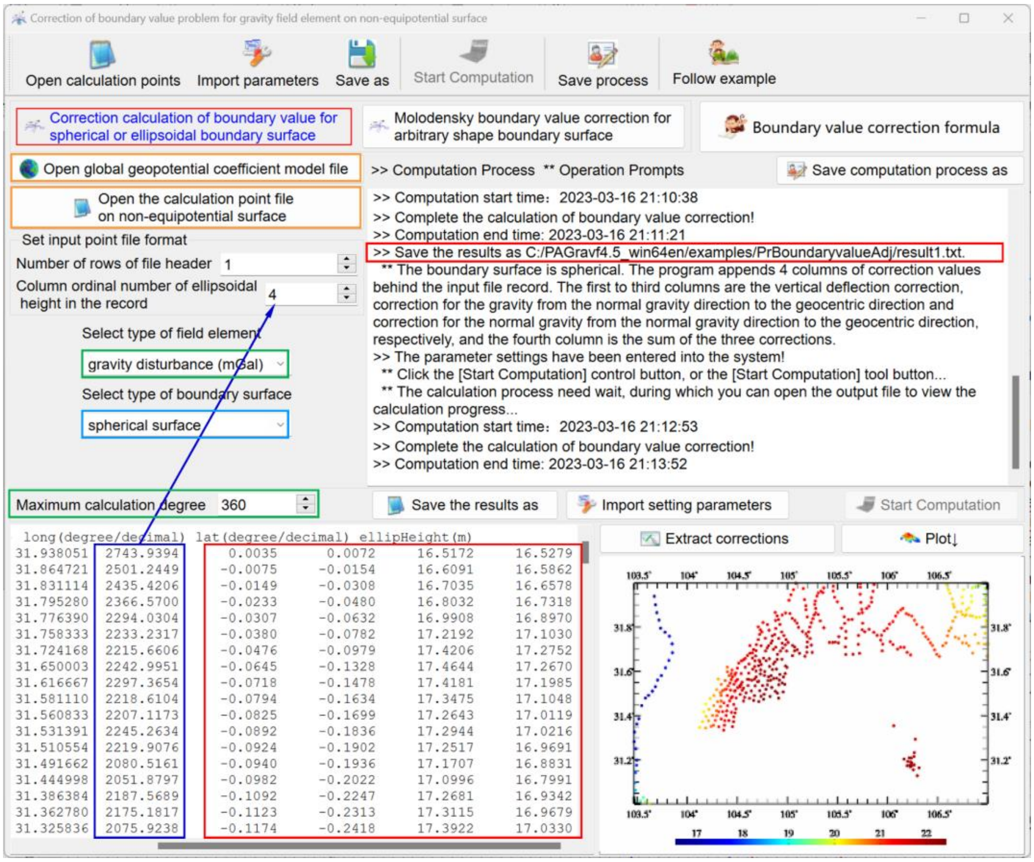
The record format of the calculation point file: ID (point no / point name), longitude (decimal degrees), latitude (decimal degrees), ellipsoidal height (m)......

[Parameter settings] Input the number of rows of the calculation point file header, and column ordinal number of height and observation attribute in record, select the type of field element and boundary surface, and enter the maximum calculation degree of geopotential coefficient model.

The program selects the minimum of the maximum degree of the global geopotential model and the input maximum degree as the calculation degree.

[Output file] The calculation result file.

When the boundary surface is an ellipsoidal surface, the program appends a column of the vertical deflection correction value of gravity behind the input calculation point file record.



When the boundary surface is spherical, the program appends 4 columns of correction values behind the input calculation point file record. The first to third columns are the vertical deflection correction, correction for the gravity from the normal gravity direction to the geocentric direction and correction for the normal gravity from the normal gravity direction to the geocentric direction, respectively, and the fourth column is the sum of the three corrections.

2.4.2 Molodensky boundary value correction for arbitrary shape boundary surface

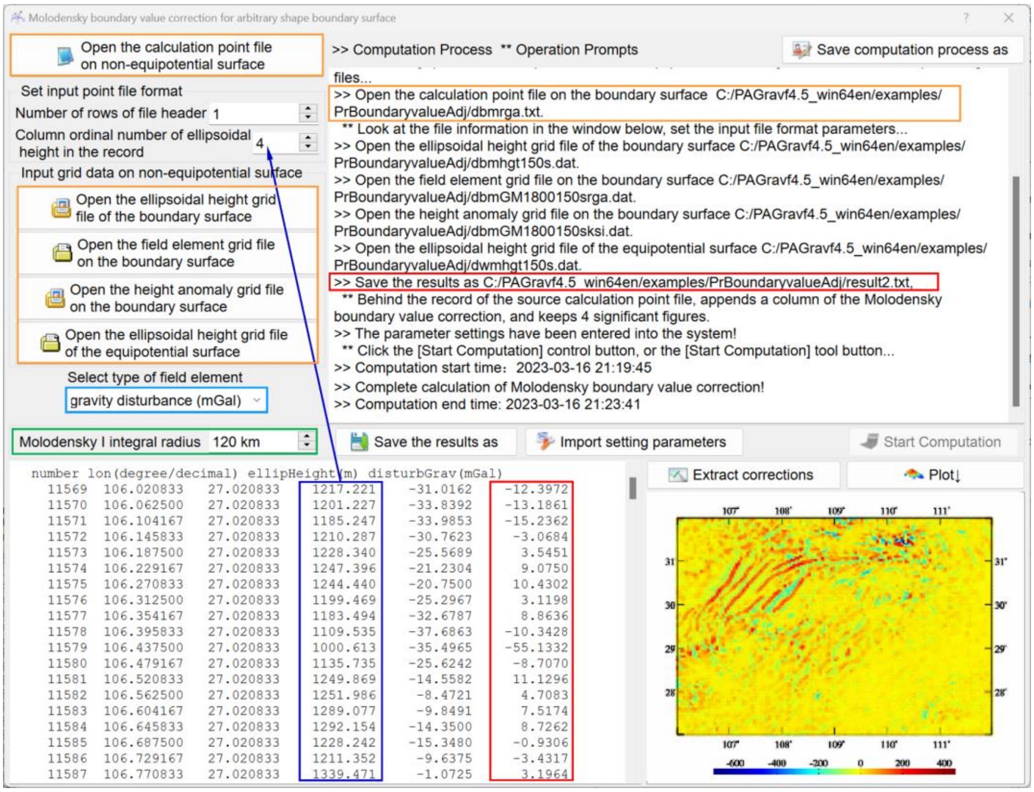
[Function] From the gravity anomaly or gravity disturbance (mGal) model grid, height anomaly model grid, ellipsoidal height grid of the boundary surface and ellipsoidal height grid of the equipotential surface, calculate the Molodensky I corrections of the gravity anomalies or gravity disturbances on the non-equipotential boundary surface on the ground or outside the Earth, thereby converting the Molodensky boundary value problem into a Stokes problem.

The boundary surface can be located at any altitude outside the geoid, and the shape of the boundary surface can be irregular.

[Input files] The calculation point file on non-equipotential surface, the ellipsoidal height grid file of the boundary surface, the field element grid file on the boundary surface, the height anomaly grid file on the boundary surface, and the ellipsoidal height grid file of the equipotential surface.

The grid specifications of the four input grid files are required to be identical, and the field element type is required to be the same as that selected in the selection box below.

The ellipsoidal height grid file of the boundary surface is employed to indicate the location of the non-equipotential boundary surface.



The ellipsoidal height grid file of the equipotential surface is employed to indicate the location of the reference equipotential surface.

[Parameter settings] Input the number of rows of the calculation point file header, column ordinal number of ellipsoidal height attribute in record, select the type of field element, and enter the Molodensky I integral radius.

[Output file] The Molodensky boundary value correction file.

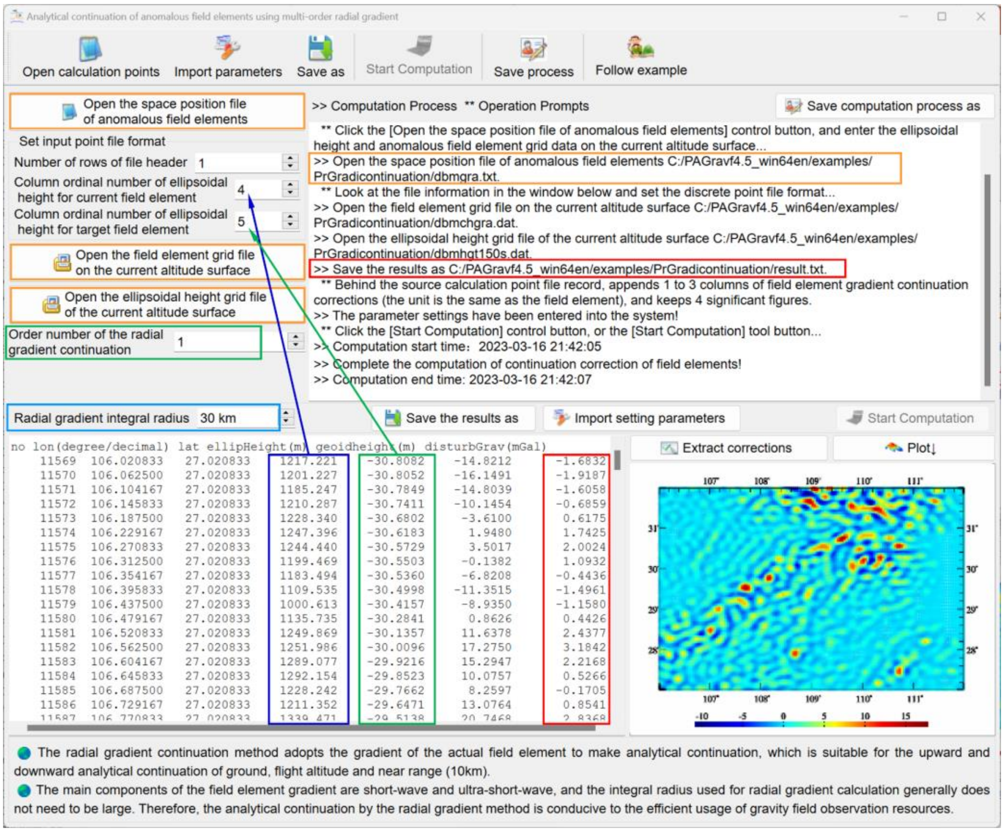
Behind the record of the source calculation point file, appends a column of the Molodensky boundary value correction, and keeps 4 significant figures.

When the boundary surface is the ground surface and the reference equipotential surface is the geoid, the program calculates the classical Molodensky I (mGal).

2.5 Analytical continuation of anomalous field elements using multi-order radial gradient

[Function] From the ellipsoidal height grid of the current altitude surface and anomalous field element grid on the surface, calculate the 1st to 3rd order radial gradients by the rigorous radial gradient integral formula, and then calculate the continuation corrections of the field elements from the current altitude to the target altitude.

[Input files] The anomalous field element grid file on the current altitude surface, the ellipsoidal height grid file of the current altitude surface, and the space position file of anomalous field elements.



The space position file record format: ID (Point no/point name), longitude (decimal degrees), latitude (decimal degrees), ...

[Parameter settings] Input the number of rows of the calculation point file header, column ordinal number of ellipsoidal height attribute in record, and enter the order number and integral radius of the radial gradient.

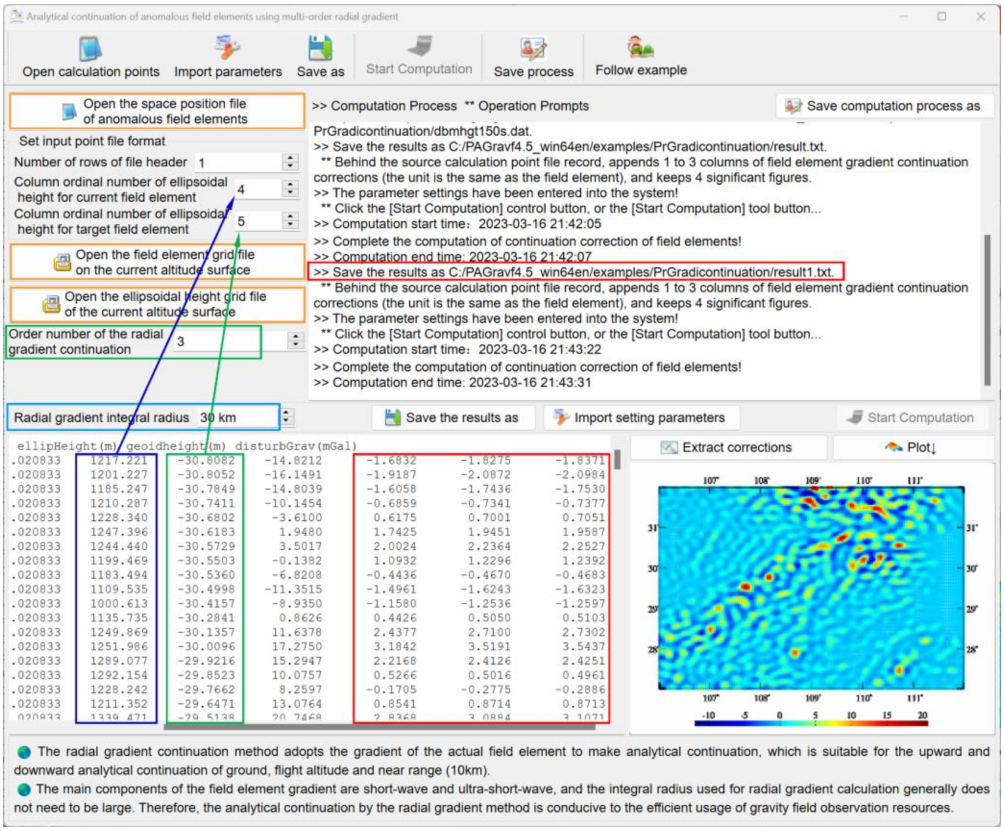
The larger the integral radius, the greater the edge effect. The integral radius should not be less than three times of the continuation height difference.

[Output file] The analytical continuation result file.

Behind the source calculation point file record, appends 1 to 3 columns of field element gradient continuation corrections (the unit is the same as the field element), and keeps 4 significant figures.

The radial gradient continuation method adopts the gradient of the actual field element to make analytical continuation, which is suitable for the upward and downward analytical continuation of ground, flight altitude and near range (10km).

The main components of the field element gradient are short-wave and ultra-short-wave, and the integral radius used for radial gradient calculation generally does not need to be large. Therefore, the analytical continuation by the radial gradient method is conducive to the efficient usage of gravity field observation resources.



PAGravf4.5 recommends a remove-continuation-restore scheme combined with ultra-high-degree geopotential model and residual field element radial continuation. Firstly, remove the ultra-high-degree model values of field elements at the current

altitude to obtain the residual field elements, and then conduct analytical continuation of residual field elements, and finally restore the ultra-high-degree model values of field elements at target altitude.

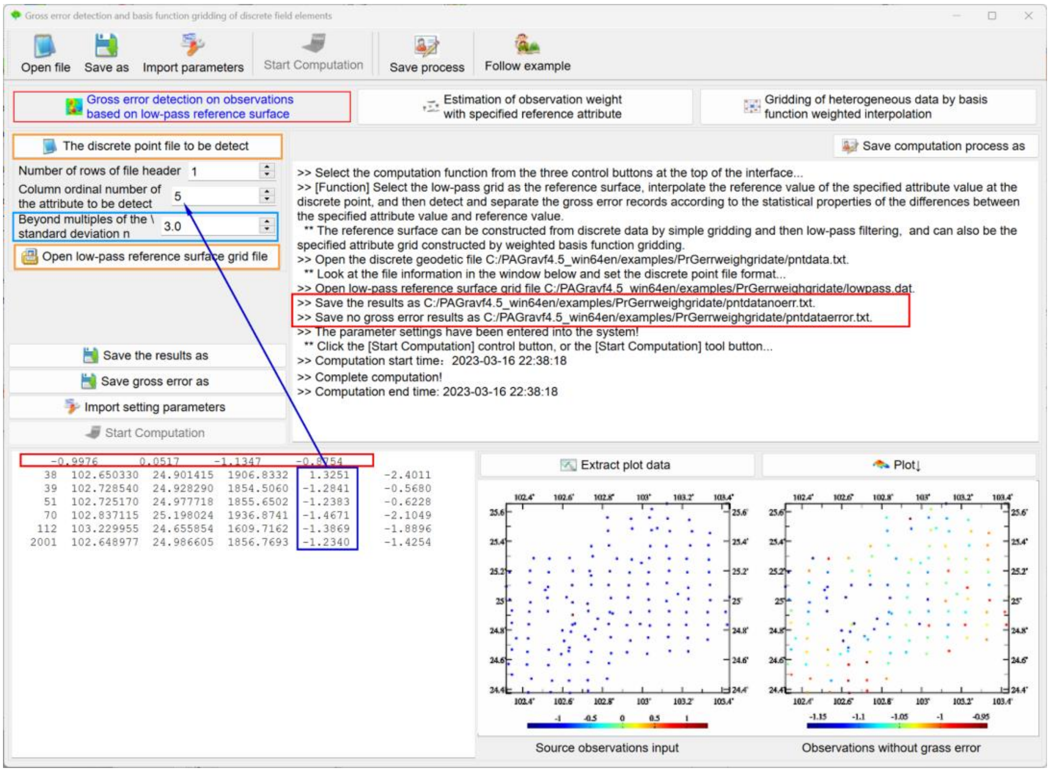
2.6 Gross error detection and basis function gridding of discrete field elements

2.6.1 Gross error detection on observations based on low-pass reference surface

[Function] Select the low-pass grid as the reference surface, interpolate the reference value of the specified attribute value at the discrete point, and then detect and separate the gross error records according to the statistical properties of the differences between the specified attribute value and reference value.

[Input files] The discrete geodetic point file to be detect, the low-pass reference surface grid file.

The reference surface can be constructed from discrete data by simple gridding and then low-pass filtering, and can also be the specified attribute grid constructed by weighted basis function gridding. When the zero-value grid of the measuring point coverage area is employed as the reference surface, that is, no reference surface support, the program performs simple gross error detection.



[Parameter settings] Enter number of rows of the discrete geodetic point file header, column ordinal number of the attribute to be detect in the record, and beyond multiples of the standard deviation.

When the absolute value of the difference between the attribute and its mean is greater than n times the attribute standard deviation, the record in which attribute is a gross error record.

[Output files] The discrete geodetic point file without gross error, whose format is the same with the input discrete point file. The gross error point file, whose file header include the mean, standard deviation, minimum and maximum of the differences.

2.6.2 Estimation of observation weight with specified reference attribute

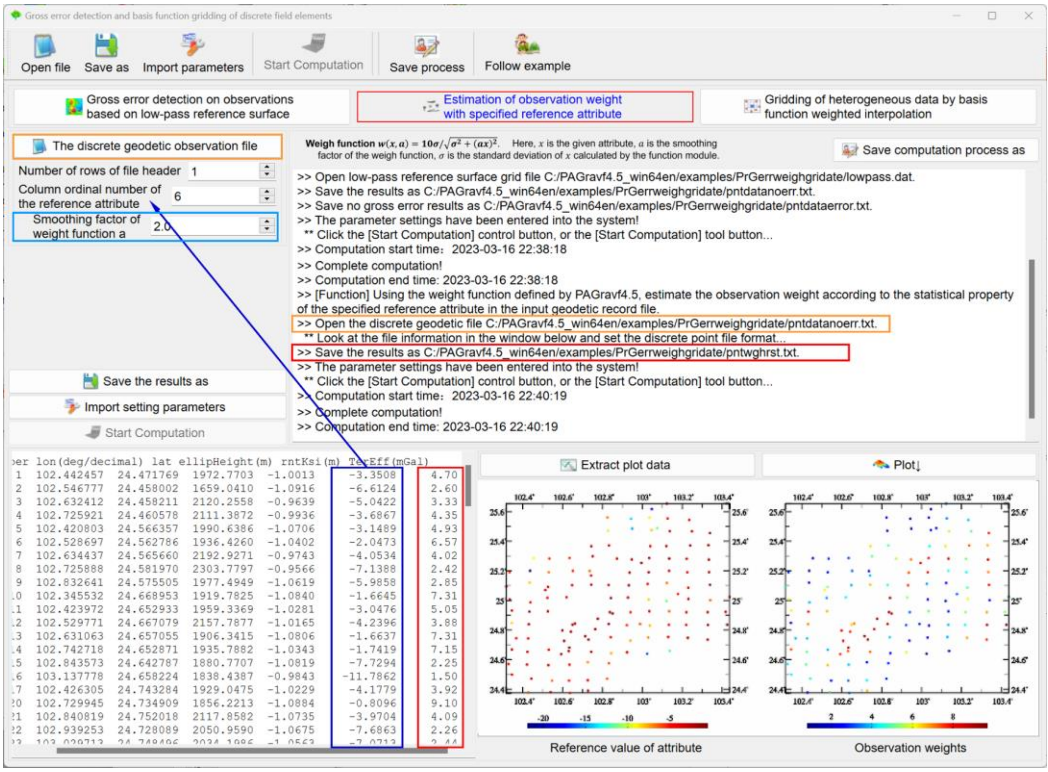
[Function] Using the weight function defined by PAGravf4.5, estimate the observation weight according to the statistical property of the specified reference attribute in the input geodetic record file.

[Input file] The discrete geodetic observation file.

Weight function defined by PAGravf4.5 $w(x,a)=10\sigma\sqrt{\sigma^2+(ax)^2}$, here x is the reference attribute, a is the given smoothing factor of the weight function, σ is the standard deviation of x calculated automatically by the program.

[Parameter settings] Enter number of rows of the discrete geodetic observation file header, column ordinal number of the reference attribute in the record, and the smoothing factor of the weight function.

The larger the weight function smoothing factor a, the slower the weight function w decays with distance.



2.6.3 Gridding of heterogeneous data by basis function weighted interpolation

[Function] According to the given grid specifications (grid range and spatial resolution), and specified basis function, grid the specified attribute in the input discrete geodetic point file by the weighted basis function interpolation method.

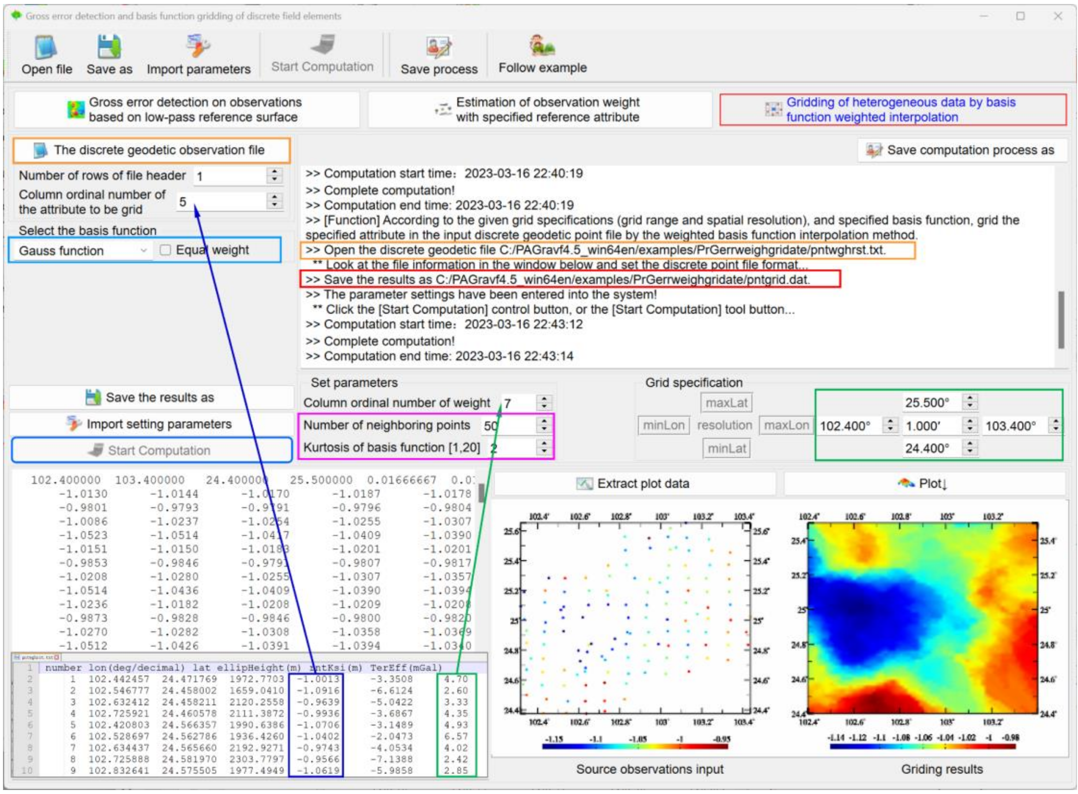
[Input file] The discrete geodetic observation file.

[Parameter settings] Enter number of rows of the discrete point file header, column ordinal number of the attribute to be grid in the file record, and grid specification parameters. Select the base function and set its parameters.

The smaller the kurtosis is (the slower the basis function value reduces with distance), the larger the number of neighboring points for the interpolation, the smoother the interpolation result, the weaker the edge effect and the stronger the interpolation capacity for sparse data.

The interpolation weight is equal to the product of the attribute weight and base function value.

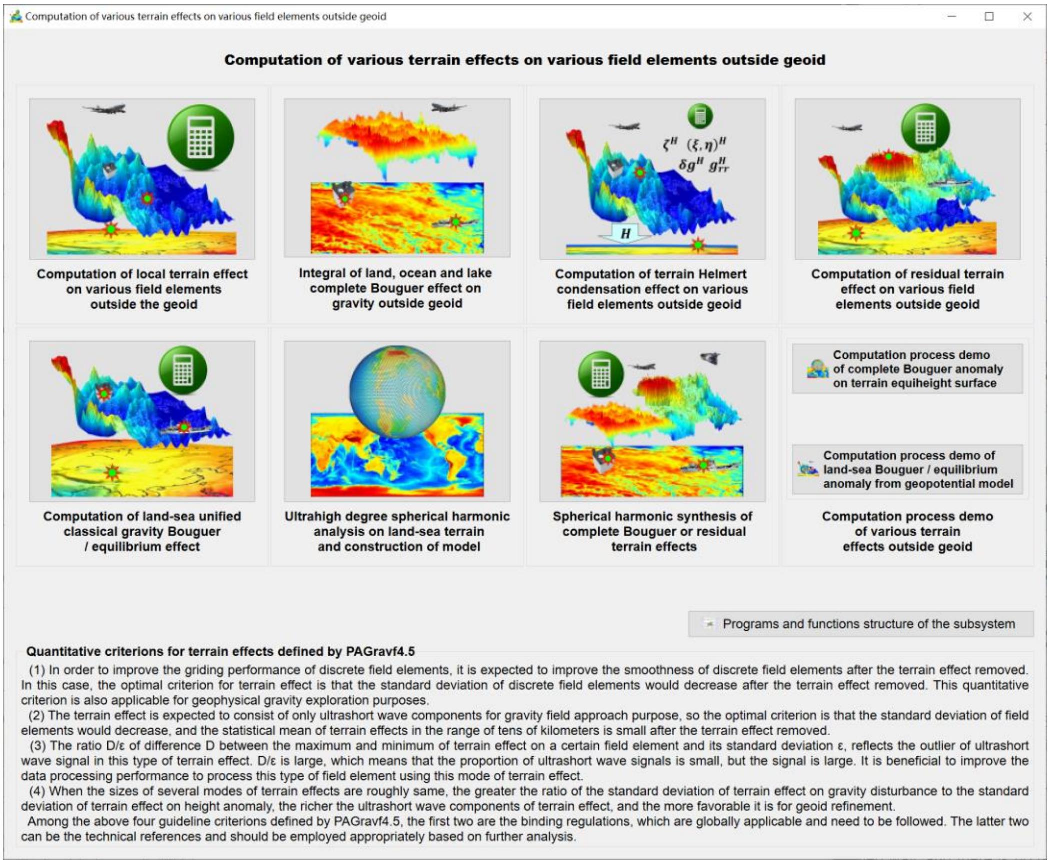
[Output file] The geodetic grid file.



The program is specially designed by PAGravf4.5 based on the properties of general geophysical field, and it is suitable for griding of single types of multi-source heterogeneous geophysical field.

3 Computation of various terrain effects on various field elements outside geoid

PAGravf4.5 develops the unified analytical algorithm system for various modes of terrain effects on different types of gravity field elements outside the geoid (on the geoid or in its outer space) to improve the geophysical gravity exploration and gravity field data processing, which can be employed uniformly to deal with various terrestrial, marine, and airborne gravity field data.



From the terrain data of typical difficult mountainous areas with a mean altitude of 4000m and terrain undulation of more than 3000m, we verify and analyze the performance of various terrain effect algorithms in this group of programs to facilitate and quickly grasp the characteristics and usage of terrain effects.

3.1 Computation of local terrain effect on various field elements outside the geoid

[Purpose] With the rigorous numerical integral method or FFT algorithm, from the ground digital elevation model and ground ellipsoidal height grid, compute the local terrain effects on the height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection (", to south, to west) or disturbing gravity gradient (E, radial) on the geoid or in its outer space.

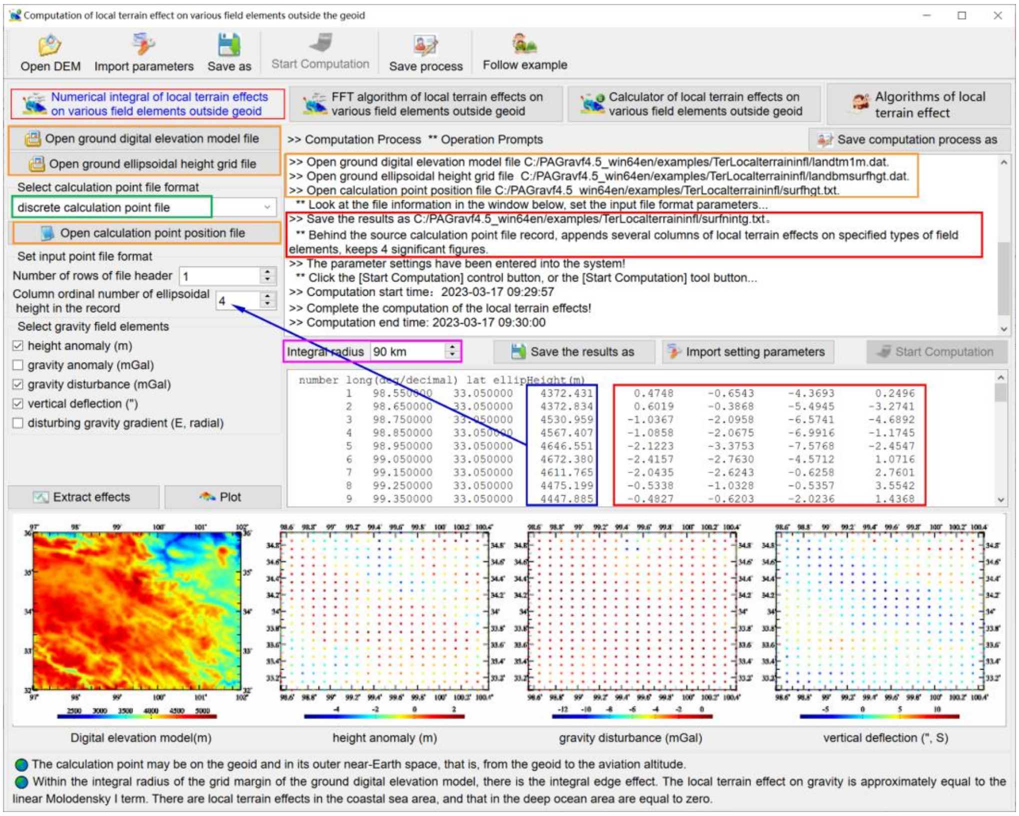
The terrain effect on field element is equal to the negative value of its classic terrain correction, such as the local terrain effect is equal to the negative local terrain correction. Since the normal gravity field keeps unchanged, the terrain effect on the gravity disturbance and gravity anomaly is always equal to the terrain effect on gravity.

3.1.1 Numerical integral of local terrain effects on various field elements outside geoid

[Function] With the rigorous numerical integral method, from the ground digital elevation model and ground ellipsoidal height grid, compute the local terrain effects on the height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection (", to south, to west) or disturbing gravity gradient (E, radial) on the geoid or in its outer space.

[Input files] The ground digital elevation model file and ground ellipsoidal height grid file with the same grid specifications, and the calculation point position file or ellipsoidal height grid file of the calculation surface.

The ground digital elevation model (normal /orthometric height) is employed to indicate terrain relief, and the ground ellipsoidal height grid represents the ground position employed to calculate the integral distance.



[Parameter settings] Set the input file format parameters, select the gravity field element type, and enter the integral radius.

[Output file] The local terrain effect result file.

When the discrete calculation point file input, the output file record format: Behind the source calculation point file record, appends several columns of local terrain effects on specified types of field elements, keeps 4 significant figures.

When the ellipsoidal height grid file input, the output file record format: Point no, longitude, latitude, ellipsoidal height, several columns of local terrain effects on specified types of field elements, keeps 4 significant figures.

At the same time, the program also outputs the local terrain effects on height anomaly (*.ksi), gravity anomaly (*.gra), gravity disturbance (*.rga), vertical deflection (*.dft) or disturbing gravity gradient (*.grr) grid files into the current directory. Where * is the output file name entered from the interface. The program outputs the local terrain effect grid file on the specified types of elements.

In this example, the 1′ digital elevation model is used, the integral radius is 90km, the terrain surface (ground) is selected as the calculation surface which is represented by the ground ellipsoidal height grid, and the local terrain effects on various ground field elements are calculated. After the 1° area of the grid margin with integral edge effect deducted, the statistical analysis on the local terrain effects on various field elements are shown in the table below.

Let D be the difference between the maximum value and the minimum value, ε is the standard deviation, and a large D/ε indicates that the number of ultrashort wave signals accounts for a small proportion but the amplitude is prominent. In general, if a certain type of terrain effect on a certain type of field element is larger, the more favorable it is to process the data of this type of field element.

In this example, the D/ε of the local terrain effect on gravity disturbance is large, showing that the local terrain effect is suitable to process the gravity disturbance data. However, the D/ε of the local terrain effect on the vertical deflection is the smallest, which shows that it is not a good choice to employ the local terrain effect to process the vertical deflection data.

Field element type /unit	mean	standard deviation ε	minimum	maximum	D/ε
height anomaly /m	-0.2233	1.4995	-8.9709	4.9249	9.3
gravity disturbance /mGal	-0.5364	0.7599	-14.3238	0.5061	19.5
vertical deflection /S″	1.0139	3.2809	-8.7789	13.2512	6.7
vertical deflection /W″	1.7161	3.2445	-6.8166	14.4212	6.5
disturbing gravity gradient /E	1.3980	83.1923	-368.3504	493.4792	10.4

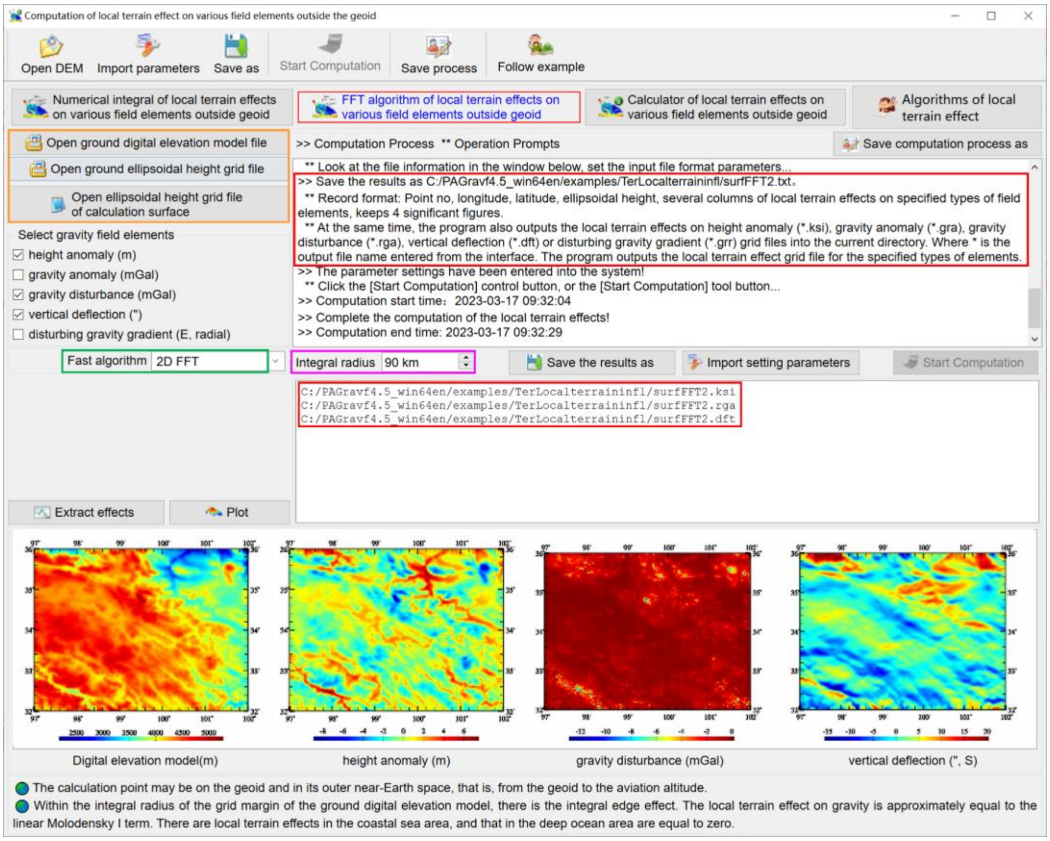
3.1.2 FFT algorithm of local terrain effects on various gravity field elements outside geoid

[Function] With the FFT integral algorithm, from the ground digital elevation model and ground ellipsoidal height grid, compute the local terrain effects on the height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection (", to south, to west) or disturbing gravity gradient (E, radial) on the geoid or in its outer space.

[Input files] The ground digital elevation model file, the ground ellipsoidal height grid file, and the ellipsoidal height grid file of the calculation surface with the same grid specifications.

The ground digital elevation model (normal /orthometric height) is employed to indicate terrain relief. The ground ellipsoidal height grid represents the ground position employed to calculate the integral distance.

[Parameter settings] Select the gravity field element type and the integral fast algorithm and enter the integral radius.

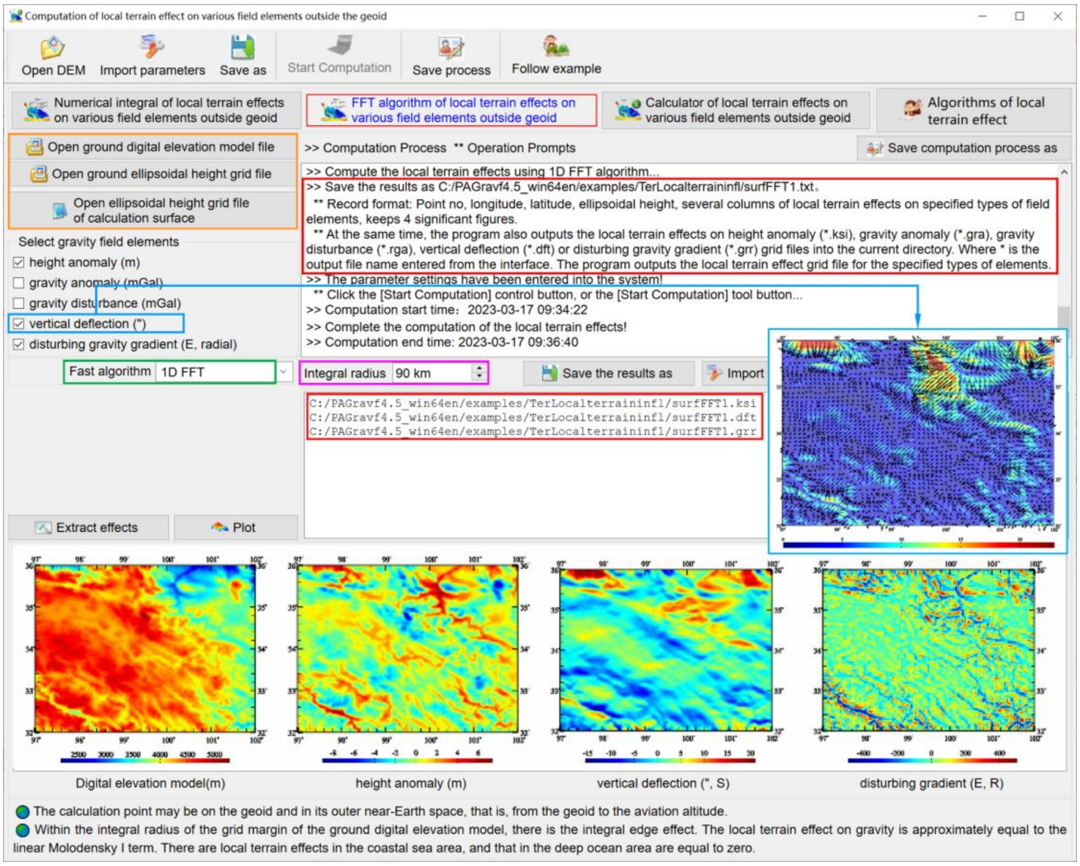


[Output file] The local terrain effect file.

Record format: Point no, longitude, latitude, ellipsoidal height, several columns of local terrain effects on specified types of field elements, keeps 4 significant figures.

At the same time, the program also outputs the local terrain effects on height anomaly (*.ksi), gravity anomaly (*.gra), gravity disturbance (*.rga), vertical deflection (*.dft) or disturbing gravity gradient (*.grr) grid files into the current directory. Where * is the output file name entered from the interface. The program outputs the local terrain effect grid file on the specified types of elements.

Using the exact same parameters as the numerical integral, compute the local terrain effects on various field elements by the FFT algorithm, and statistically analyze the difference between the FFT result and the numerical integral result.



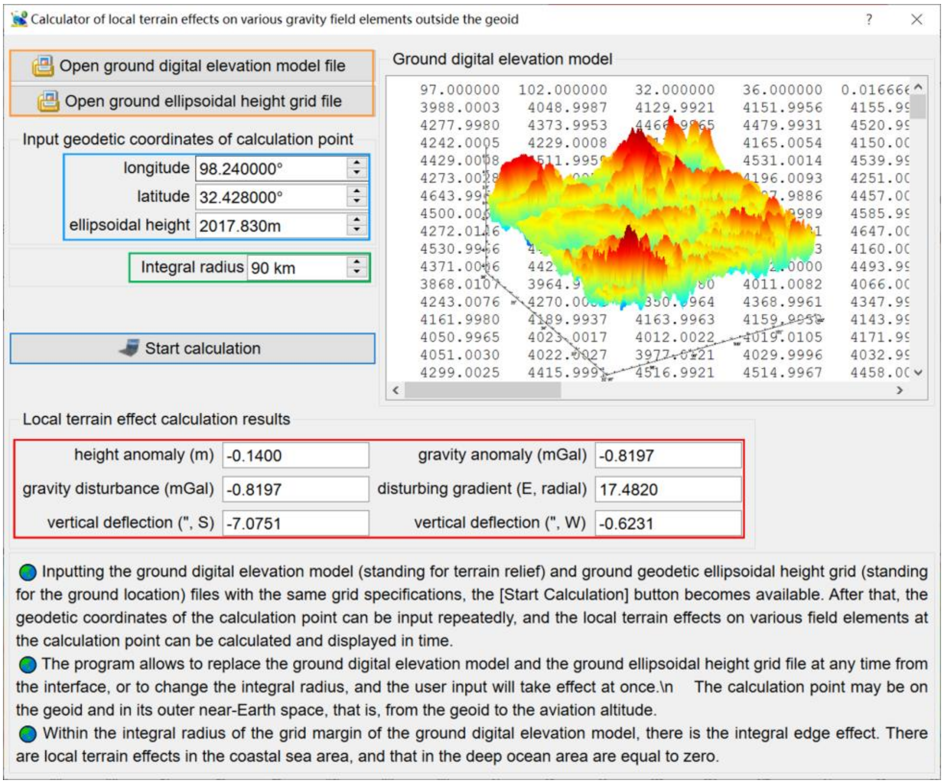
FFT − numerical integral		mean	standard deviation	minimum	maximum
height anomaly /m	FFT2	-0.0001	0.0138	-0.0686	0.0883
	FFT1	0.0012	0.0027	-0.0115	0.0110
gravity disturbance /mGal	FFT2	0.0352	0.2316	-0.7410	1.4406
	FFT1	0.0341	0.2298	-0.7602	1.3356
vertical deflection /S″	FFT2	-0.0082	0.0197	-0.1176	0.1275
	FFT1	-0.0026	0.0138	-0.1616	0.1538
vertical deflection /W″	FFT2	0.0018	0.0400	-0.3873	0.1555
	FFT1	0.0051	0.0198	-0.4444	0.2844
disturbing gravity gradient /E	FFT2	0.0042	0.5176	-19.6242	10.5354
	FFT1	0.0036	0.5859	-21.2847	12.1181

There is little difference between the computation results of the rigorous numerical integral and the fast FFT algorithm, and there is also no obvious difference between the results of the one-dimensional FFT and the two-dimensional FFT.

3.1.3 Calculator of local terrain effects on various gravity field elements outside the geoid

[Function] From the ground digital elevation model and ground ellipsoidal height grid file, given the calculation point geodetic coordinates on the geoid or in its outer space, compute the local terrain effects on the height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection (", to south, to west) and disturbing gravity gradient (E, radial).

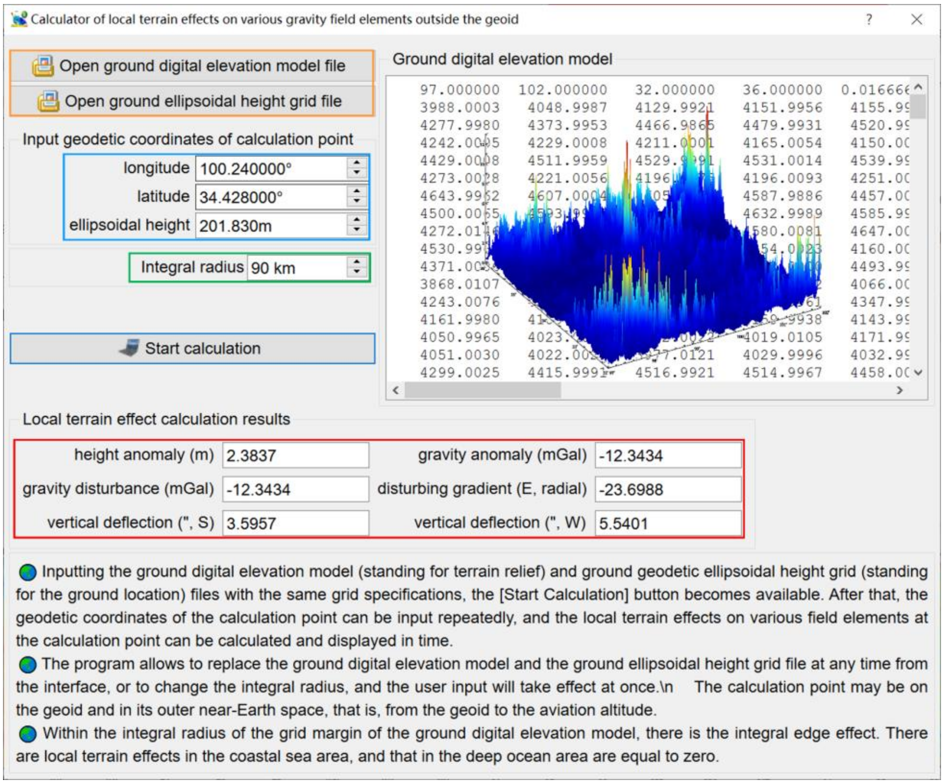
Inputting the ground digital elevation model (standing for terrain relief) and ground geodetic ellipsoidal height grid (standing for the ground location) files with the same grid specifications, the [Start Calculation] button becomes available. After that, the geodetic coordinates of the calculation point can be input repeatedly, and the local terrain effects on various field elements at the calculation point can be computed and displayed in time.



The program allows to replace the ground digital elevation model and the ground ellipsoidal height grid file at any time from the interface, or to change the integral radius, and the user input will take effect at once.

The calculation point may be on the geoid and in its outer near-Earth space, that is, from the geoid to the aviation altitude.

Within the integral radius of the grid margin of the ground digital elevation model, there is the integral edge effect. The local terrain effect on gravity is approximately equal to the linear Molodensky I term. There are local terrain effects in the coastal sea area, and that in the deep ocean area are equal to zero.



3.2 Computation of land, ocean, and lake complete Bouguer effect on gravity outside geoid

[Purpose] Using the rigorous numerical integral method or FFT algorithm, from the land-sea terrain model and ellipsoidal height grid file of the land ground and sea surface, compute the land-sea unified complete Bouguer effect on the gravity anomaly (mGal) or gravity disturbance (mGal) on the geoid or in near-Earth space.

The complete Bouguer effect on the anomalous field element is much larger than the field element itself, which is usually employed in geophysics to detect the geometric structure of the gravity field. Physical geodesy pays attention to the requirement of quantitative accuracy, and would not directly use the complete Bouguer effect, which is mainly employed to compute the residual terrain effect.

Since the normal gravity field keeps unchanged, the complete Bouguer effect on the gravity disturbance and gravity anomaly is always equal to the complete Bouguer effect on gravity.

The program is suitable for the unified computation of the complete Bouguer effect on gravity, gravity anomaly and gravity disturbance in land, land-sea junction, and sea area. The calculation point may be on the geoid and its outer near-Earth space.

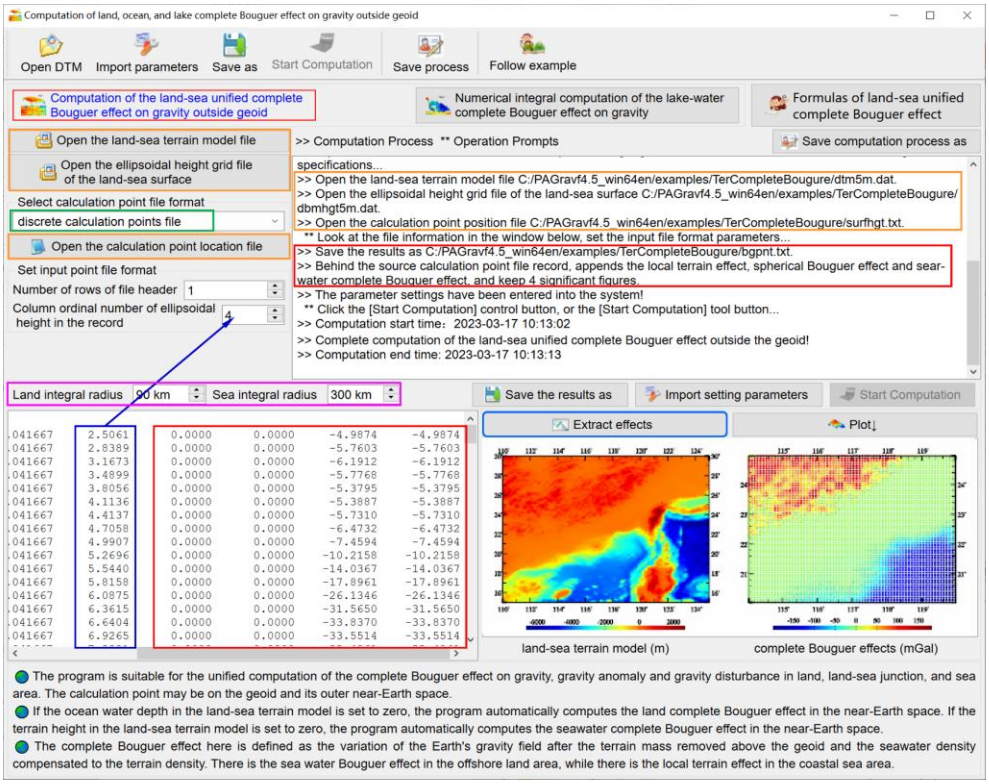
If the ocean water depth in the land-sea terrain model is set to zero, the program automatically computes the land complete Bouguer effect in the near-Earth space. If the terrain height in the land-sea terrain model is set to zero, the program automatically computes the seawater complete Bouguer effect in the near-Earth space.

The complete Bouguer effect here is defined as the variation of Earth gravity field because of the terrain mass above the geoid removed and the seawater density compensated to the terrain density. There is the sea water Bouguer effect in the offshore land area, while there is the local terrain effect in the coastal sea area.

3.2.1 Computation of the land-sea unified complete Bouguer effect on gravity outside geoid

[Function] Using the rigorous numerical integral method or FFT algorithm, from the land-sea terrain model and ellipsoidal height grid file of the land-sea surface, compute the land-sea unified complete Bouguer effect on the gravity (mGal) on the geoid or in near-Earth space.

[Input files] The land-sea terrain model file and the ellipsoidal height grid file of the land-sea surface with the same grid specifications, and the calculation point position file or the ellipsoidal height grid file of the calculation surface.



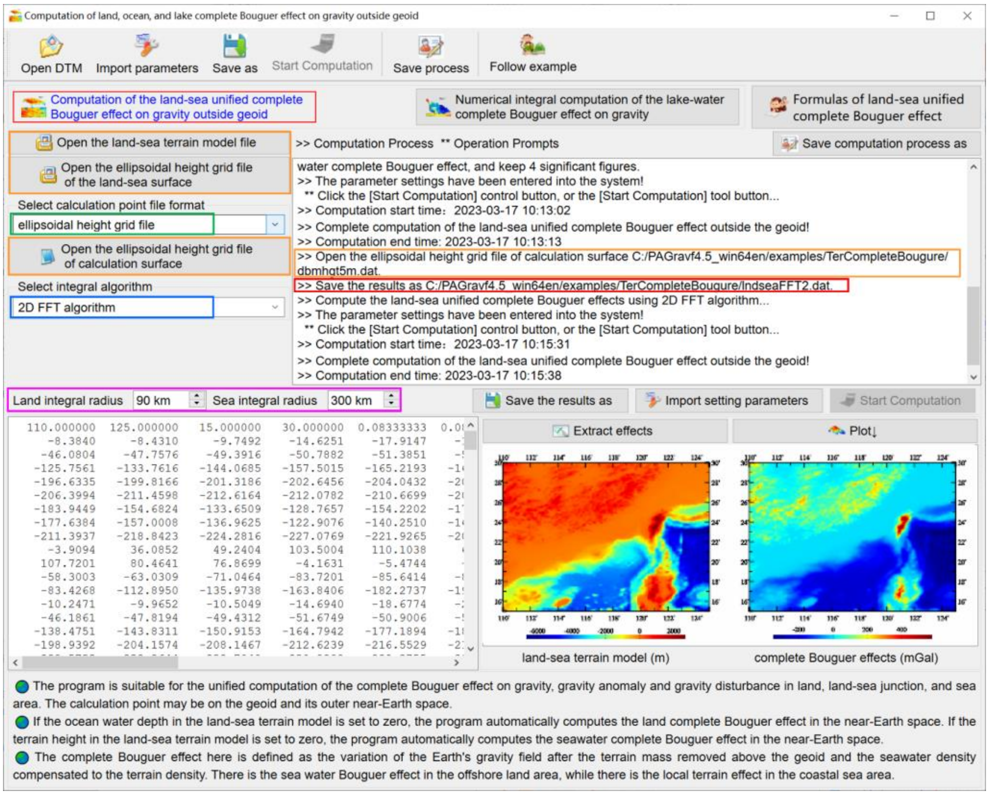
In the land-sea terrain model, the land terrain height is greater than zero while the seafloor water depth is less than zero.

The ellipsoidal height grid of the land-sea surface stands for the land-sea surface position employed to calculate the integral distance.

[Parameter settings] Set the input file format parameters, select the integral algorithm, and enter the land integral radius and sea integral radius.

[Output file] The land-sea unified complete Bouguer effect file.

When the discrete calculation points file input, the output file record format: Behind the source calculation point file record, appends the local terrain effect, spherical Bouguer effect and sear-water complete Bouguer effect, and keep 4 significant figures.



In this example, the 5′ land-sea terrain model is employed, the integral radius is 90km, the integral radius for sear-water complete Bouguer effect is 300km, the land-sea surface is the calculation surface which is represented by the land-sea surface ellipsoidal height grid, and the land-sea unified complete Bouguer effects on the gravity are computed. After the 3° area of the grid margin with integral edge effect deducted, the statistical analysis on the complete Bouguer effects are shown in the table below.

mGal	mean	standard deviation	minimum	maximum
numerical integral	-17.2622	110.3575	-260.5460	562.1404
FFT2 − numerical integral	1.7056	1.8637	0.0265	19.7512
FFT1 − numerical integral	2.0864	2.2786	0.0268	19.3779
FFT2 − FFT1	-0.3808	0.7884	-3.3353	0.8730

Due to the large the complete Bouguer effect on various field elements, the error caused by the third-order approximation of terrain relief sometimes even exceeds the anomalous field element itself. PAGravf4.5 therefore recommends that, in addition to the complete Bouguer effect on gravity, the effects on other types of field elements should be computed by the remove-restore scheme with the global land-sea terrain mass spherical harmonic coefficient model as the reference terrain field. For the specific computation process, please refer to [Computation process demo of complete Bouguer anomaly outside geoid].

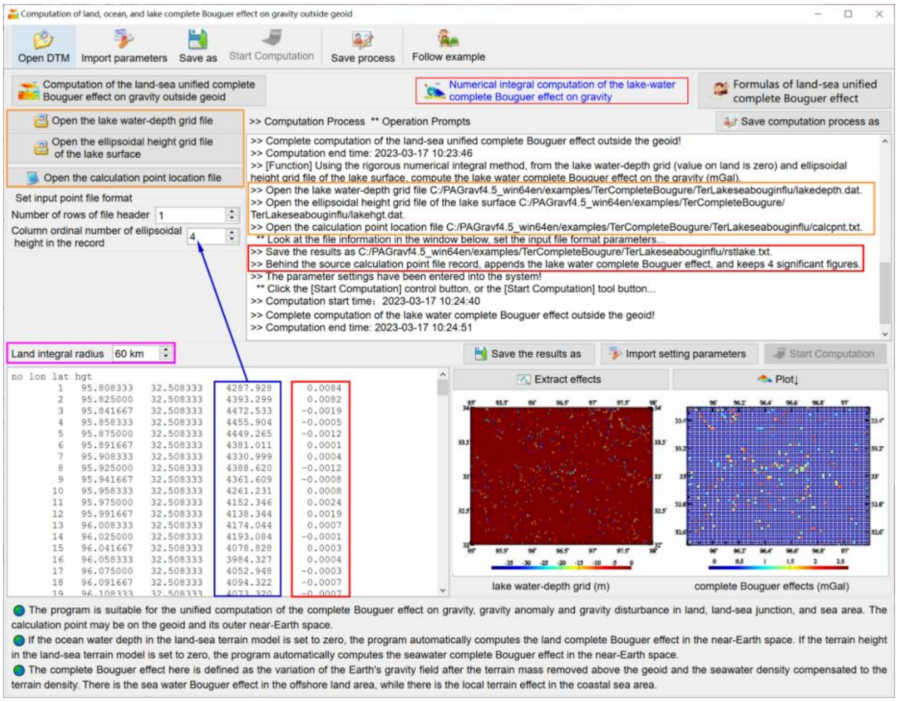
3.2.2 Numerical integral computation of the lake-water complete Bouguer effect on gravity

[Function] Using the rigorous numerical integral method, from the lake water-depth grid (value on land is zero) and ellipsoidal height grid file of the lake surface, compute the lake water complete Bouguer effect on the gravity (mGal).

[Input files] The lake water-depth grid file and ellipsoidal height grid file of the lake surface with the same grid specifications, and the calculation point position file or ellipsoidal height grid file of the calculation surface.

The ellipsoidal height grid of the land-sea surface stands for the land-sea surface position employed to calculate the integral distance.

[Parameter settings] Set the input file format parameters and integral radius.



[Output file] The lake water complete Bouguer effect file.

When the discrete calculation points file input, the output file record format: Behind the source calculation point file record, appends the lake water complete Bouguer effect, and keeps 4 significant figures.

3.3 Computation of terrain Helmert condensation effect on various field elements outside geoid

[Purpose] Using the rigorous numerical integral method or FFT algorithm, from the ground digital elevation model and ground ellipsoidal height grid, compute the terrain Helmert condensation effects on the height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection (", to south, to west) or disturbing gravity gradient (E, radial) on the geoid or in its outer space.

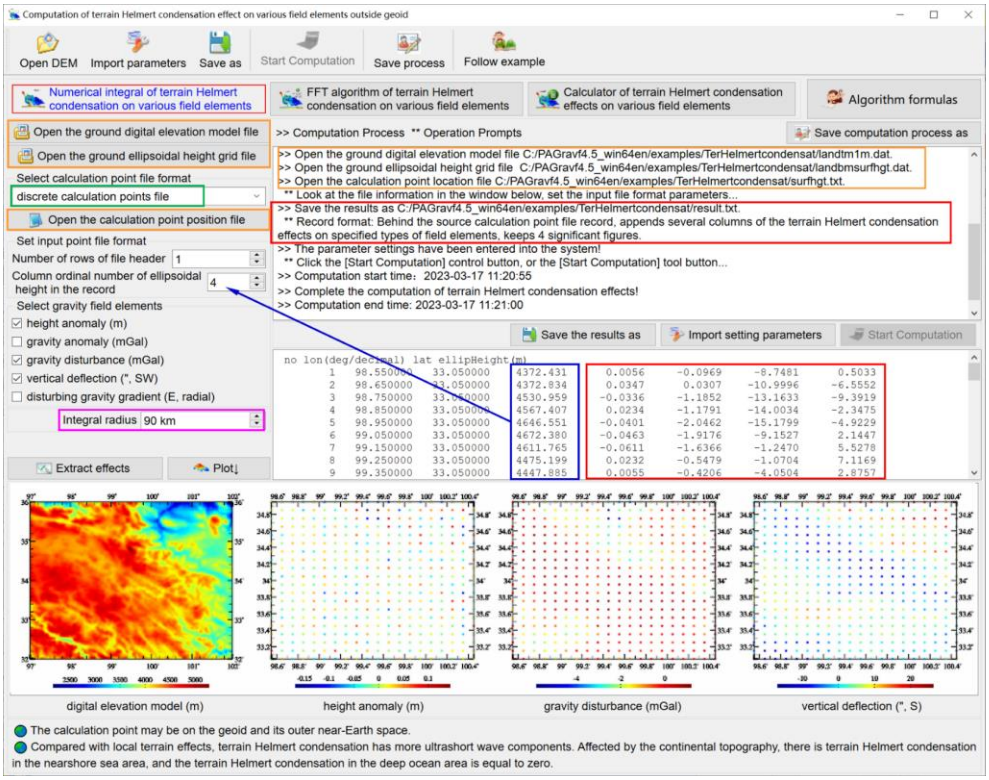
Since the normal gravity field keeps unchanged, the terrain Helmert condensation effect on the gravity disturbance and gravity anomaly is always equal to the terrain Helmert condensation effect on gravity.

The calculation point may be on the geoid and its outer near-Earth space.

3.3.1 Numerical integral of terrain Helmert condensation effects on various field elements

[Function] Using the rigorous numerical integral method, from the ground digital elevation model and ground ellipsoidal height grid, compute the terrain Helmert condensation effects on the height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection (", to south, to west) or disturbing gravity gradient (E, radial) on the geoid or in its outer space.

[Input files] The ground digital elevation model file and ground ellipsoidal height grid file with the same grid specifications, and the calculation point location file or the ellipsoidal height grid file of the calculation surface.



The ground digital elevation model (normal /orthometric height) is used to indicate terrain relief, and the ground ellipsoidal height grid stands for the ground position employed to calculate the integral distance.

[Parameter settings] Set the input file format parameters, select the gravity field element type, and enter the integral radius.

[Output file] The terrain Helmert condensation effect result file.

When the discrete calculation points file input, the output file record format: Behind the source calculation points file record, appends several columns of terrain Helmert condensation effects on specified types of field elements, keeps 4 significant figures.

When the ellipsoidal height grid file input, the output file record format: Point no, longitude, latitude, ellipsoidal height, several columns of terrain Helmert condensation effects on specified types of field elements, keeps 4 significant figures.

At the same time, the program also outputs the terrain Helmert condensation effects on height anomaly (*.ksi), gravity anomaly (*.gra), gravity disturbance (*.rga), vertical deflection (*.dft) or disturbing gravity gradient (*.grr) grid files into the current directory. Where * is the output file name entered from the interface. The program outputs the terrain Helmert condensation effect grid file on the specified types of elements.

In this example, the 1′ digital elevation model is employed, the integral radius is 90km, the ground is selected as the calculation surface which is represented by the ground ellipsoidal height grid, and the terrain Helmert condensation effects on various ground field elements are computed. After the 1° area of the grid margin with integral edge effect deducted, the statistical analysis on the terrain Helmert condensation effects on various field elements are shown in the table below.

The terrain Helmert condensation has more ultrashort wave components. If the actual gravity field structure and the distribution of gravity observations are not considered, only by comparing D/ε, the terrain Helmert condensation is more suitable for processing disturbing gravity gradient data than the local terrain effect.

Field element type /unit	mean	standard deviation ε	minimum	maximum	D/ε
height anomaly /m	-0.0005	0.0029	-0.0134	0.0180	1.6
gravity disturbance /mGal	0.2932	0.4168	-0.4313	3.1693	8.6
vertical deflection /S″	2.0301	6.5718	-17.5982	26.6152	6.8
vertical deflection /W″	3.4358	6.5000	-13.6625	28.9665	6.8
disturbing gravity gradient /E	-0.3677	16.5856	-257.5543	112.1249	22.3

3.3.2 FFT algorithm of terrain Helmert condensation effects on various field elements

[Function] Using the FFT integral algorithm, from the ground digital elevation model and ground ellipsoidal height grid, compute the terrain Helmert condensation effects on the height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection (", to south, to west) or disturbing gravity gradient (E, radial) on the geoid or in its outer space.

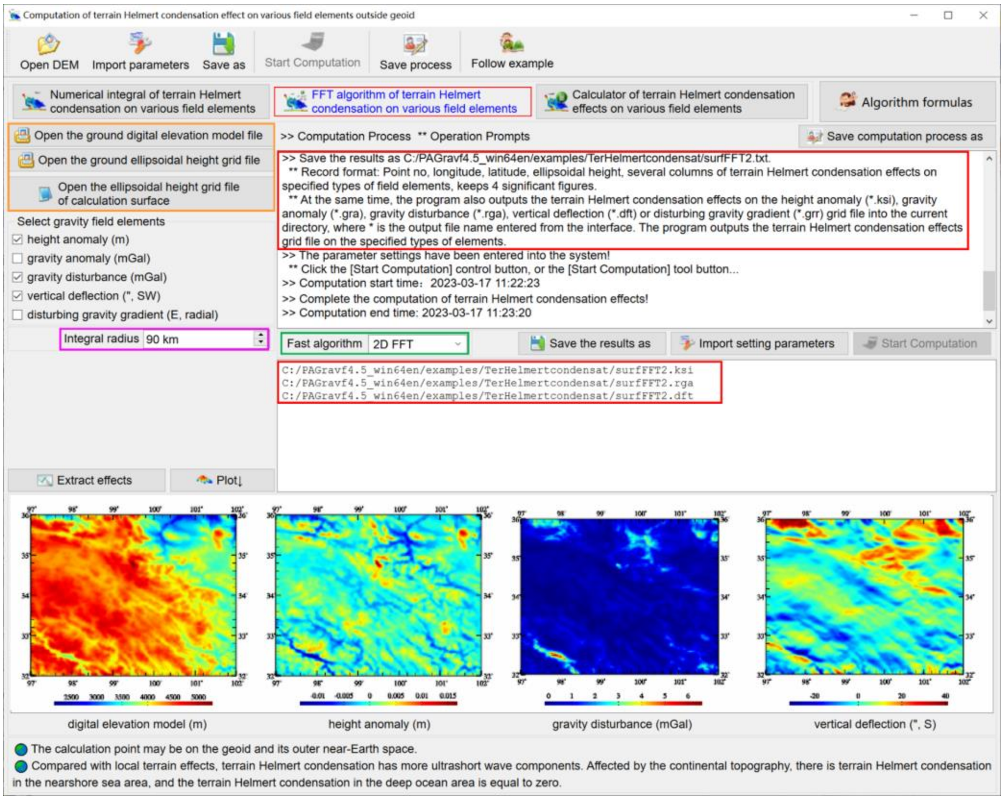
[Input files] The ground digital elevation model file, ground ellipsoidal height grid file, and ellipsoidal height grid file of the calculation surface with the same grid specifications.

The ground digital elevation model (normal /orthometric height) is employed to indicate terrain relief and the ground ellipsoidal height grid represents the ground location employed to calculate the integral distance.

[Parameter settings] Select the gravity field element type and the integral fast algorithm and enter the integral radius.

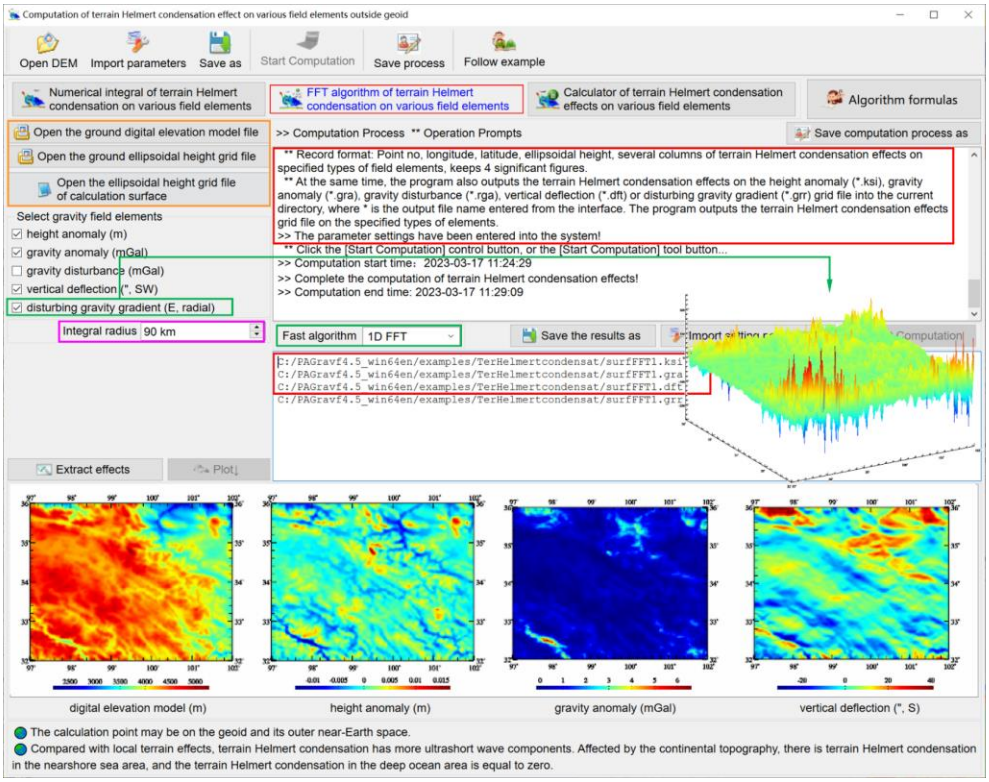
[Output file] The local terrain effects file.

Record format: Point no, longitude, latitude, ellipsoidal height, several columns of terrain Helmert condensation effects on specified types of field elements, keeps 4 significant figures.



At the same time, the program also outputs the terrain Helmert condensation effects on the height anomaly (*.ksi), gravity anomaly (*.gra), gravity disturbance (*.rga), vertical deflection (*.dft) or disturbing gravity gradient (*.grr) grid files into the current directory, where * is the output file name entered from the interface. The program outputs the terrain Helmert condensation effect grid file on the specified types of elements.

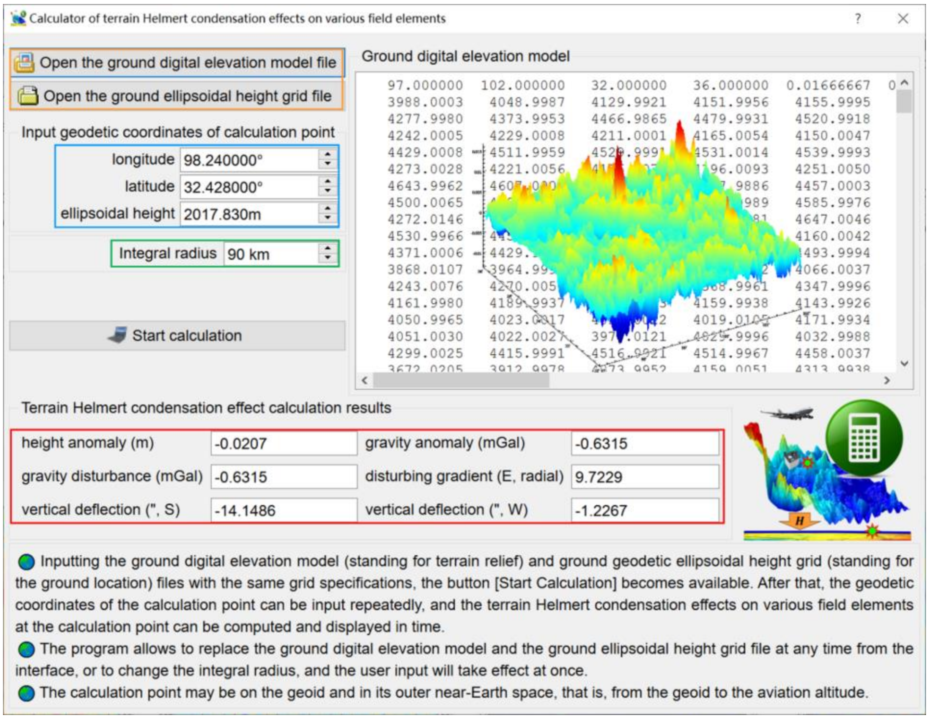
Using the exact same parameters as the numerical integral, compute the terrain Helmert condensation effects on various field elements by the FFT algorithm, and statistically analyze the difference between the FFT result and the numerical integral result. Compared with local terrain effects, terrain Helmert condensation has more ultrashort wave components. Affected by the continental topography, there is terrain Helmert condensation in the nearshore sea area, and the terrain Helmert condensation in the deep ocean area is equal to zero.

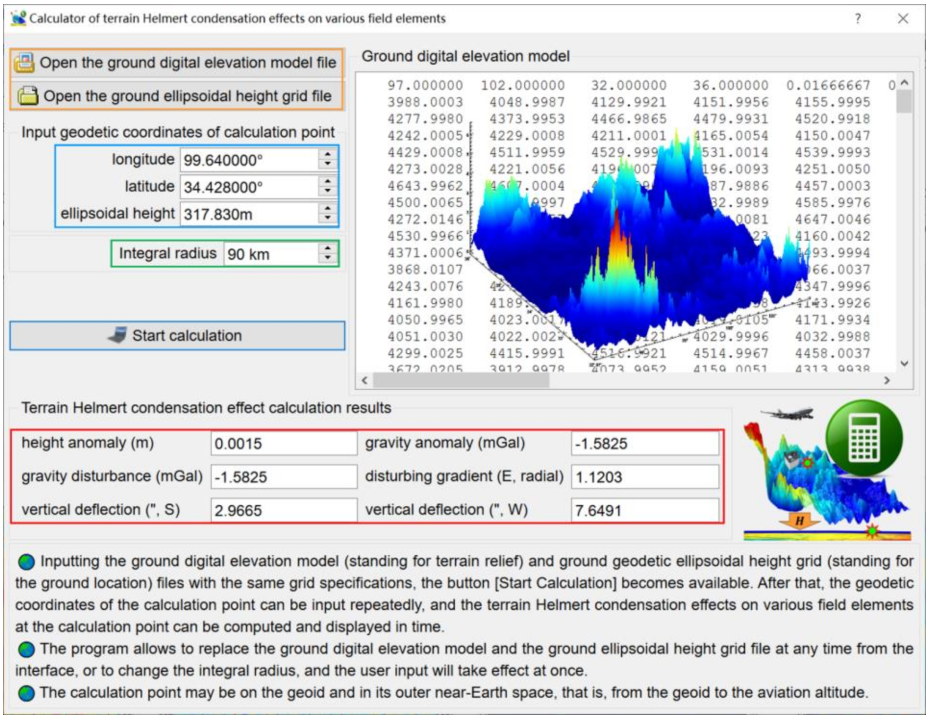


FFT – numerical integral		mean	standard deviation	minimum	maximum
height anomaly /m	FFT2	0.0008	0.0015	-0.0026	0.0064
	FFT1	0.0007	0.0015	-0.0027	0.0063
gravity disturbance /mGal	FFT2	0.0009	0.1262	-0.5535	0.6116
	FFT1	0.0025	0.1255	-0.5085	0.6768
vertical deflection /S″	FFT2	-0.0174	0.0404	-0.2220	0.1438
	FFT1	-0.0062	0.0240	-0.2359	0.2179
vertical deflection /W″	FFT2	0.0025	0.0804	-0.5046	0.2902
	FFT1	0.0093	0.0306	-0.6211	0.4129
disturbing gravity gradient /E	FFT2	0.0062	0.4810	-17.1773	9.8987
	FFT1	0.0062	0.4799	-16.8811	9.8312

3.3.3 Calculator of terrain Helmert condensation effects on various field elements

[Function] From the ground digital elevation model and ground ellipsoidal height grid file, given the calculation point geodetic coordinates on the geoid or in its outer space, compute the terrain Helmert condensation effects on the height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection (", to south, to west) and disturbing gravity gradient (E, radial).





Inputting the ground digital elevation model (standing for terrain relief) and ground geodetic ellipsoidal height grid (standing for the ground location) files with the same grid specifications, the button [Start Calculation] becomes available. After that, the geodetic coordinates of the calculation point can be input repeatedly, and the terrain Helmert condensation effects on various field elements at the calculation point can be computed and displayed in time.

The program allows to replace the ground digital elevation model and the ground ellipsoidal height grid file at any time from the interface, or to change the integral radius, and the user input will take effect at once.

The calculation point may be on the geoid and in its outer near-Earth space, that is, from the geoid to the aviation altitude.

3.4 Computation of residual terrain effect on various field elements outside geoid

[Purpose] Using the rigorous numerical integral method or FFT algorithm, from the high-resolution land-sea terrain model, low-pass land-sea terrain model and ellipsoidal height grid of the land-sea surface, compute the residual terrain effects on the height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection (", to south, to west) or disturbing gravity gradient (E, radial) on geoid or in its outer space.

The land-sea residual terrain effect is defined as the short-wave and ultra-short-wave components of the land-sea complete Bouguer effect.

Since the normal gravity field keeps unchanged, the residual terrain effect on the gravity disturbance and gravity anomaly is always equal to the residual terrain effect on gravity.

The low-pass land-sea terrain model can be generated by low-pass filtering the high-resolution land-sea terrain model, or can be also calculated from a global land-sea terrain mass spherical harmonic coefficient model by the spherical harmonic synthesis method.

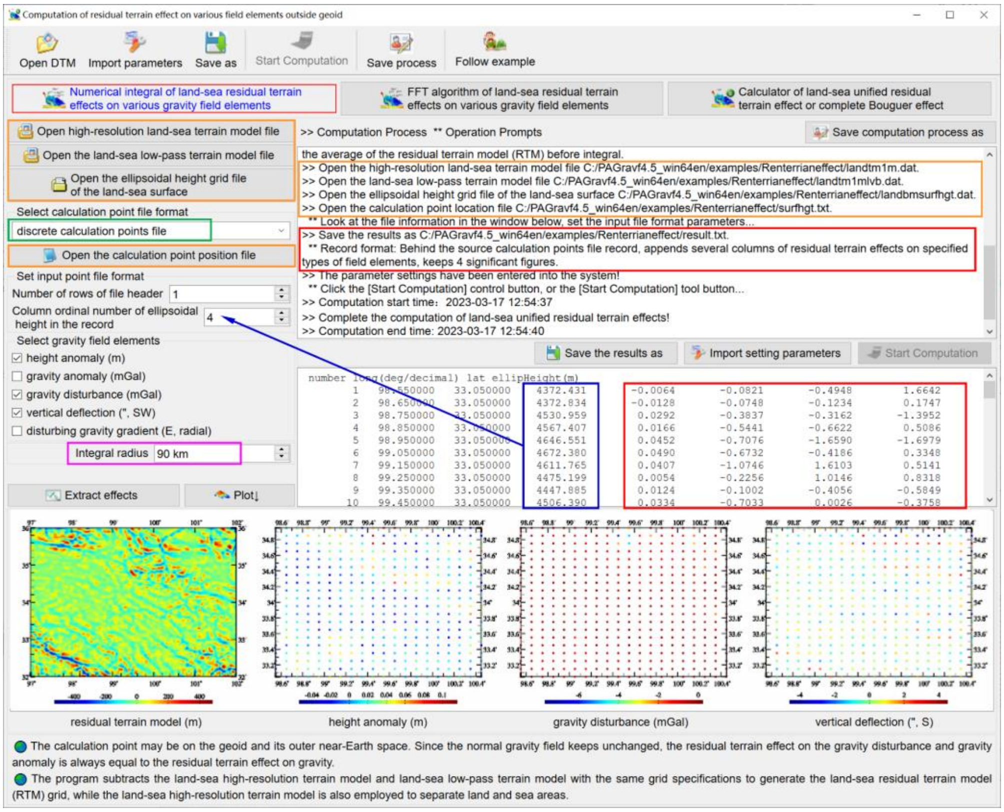
It is recommended that the land-sea low-pass terrain model is constructed by the spherical harmonic synthesis method from a global land-sea terrain mass spherical harmonic coefficient model to effectively improve the approach performance of gravity field.

3.4.1 Numerical integral of land-sea residual terrain effects on various gravity field elements

[Function] Using the rigorous numerical integral, from the high-resolution land-sea terrain model, low-pass land-sea terrain model and ellipsoidal height grid of the land-sea surface, compute the residual terrain effects on the height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection (", to south, to west) or disturbing gravity gradient (E, radial) on the geoid or in its outer space.

The program subtracts the land-sea high-resolution terrain model and land-sea low-pass terrain model with the same grid specifications to generate the land-sea residual terrain model (RTM) grid, while the land-sea high-resolution terrain model is also employed to separate land and sea areas. Since the finite radius integral cannot deal with terrain zero-degree term, the program removes the average of the residual terrain model (RTM) before integral.

[Input files] The high-resolution land-sea terrain model file, land-sea low-pass terrain model file and ellipsoidal height grid file of land-sea surface with the same grid specifications, and the calculation point position file or ellipsoidal height grid file of the calculation surface.



In the land-sea terrain model, the land terrain height is greater than zero while the seafloor water depth is smaller than zero.

The ellipsoidal height grid of the land-sea surface stands for the land-sea surface location employed to calculate the integral distance.

[Parameter settings] Set the input file format parameters, select the gravity field element type, and enter the integral radius.

[Output file] The residual terrain effect result file.

When the discrete calculation points file input, the output file record format: Behind the source calculation points file record, appends several columns of residual terrain effects on specified types of field elements, keeps 4 significant figures.

When the ellipsoidal height grid file input, the output file record format: Point no, longitude, latitude, ellipsoidal height, several columns of residual terrain effects on specified types of field elements, keeps 4 significant figures.

At the same time, the program also outputs the residual terrain effects on height anomaly (*.ksi), gravity anomaly (*.gra), gravity disturbance (*.rga), vertical deflection

(*.dft) or disturbing gravity gradient (*.grr) grid files into the current directory, where * is the output file name entered from the interface. The program outputs residual terrain effect grid file on the specified types of elements.

In this example, the 1′ digital elevation model is employed, the low-pass terrain model is constructed from the 1440-degree global land-sea terrain mass spherical harmonic coefficient model, the integral radius is 90km, and the residual terrain effects on various ground field elements are computed. After the 1° area of the grid margin with integral edge effect deducted, the statistical analysis on the residual terrain effects on various field elements are shown in the table below.

Field element type /unit	mean	standard deviation ε	minimum	maximum	D/ε
height anomaly /m	0.0188	0.0566	-0.2151	0.3835	10.2
gravity disturbance /mGal	0.5342	1.0570	-1.6605	13.2024	14.1
vertical deflection /S″	-0.0000	0.8730	-4.7838	4.6030	10.8
vertical deflection /W″	-0.0075	0.8157	-6.2512	5.1511	14.0
disturbing gravity gradient /E	-7.0310	300.1898	-928.6079	884.6364	6.0

When the zero-value grid is employed as the low-pass land-sea terrain model, the program computes the land-sea unified complete Bouguer effect on various field elements on the geoid or in its outer space but without the integral far-zone effect. In this case, the integral radius is generally not less than 250km.

The spectral domain properties of the residual terrain model (RTM) can be regulated by the land-sea low-pass terrain models.

If the gravity field structure and the distribution of gravity observations are not considered, only compared the D/ε with the local terrain effect and terrain Helmert condensation, the residual terrain effect is more conducive to modelling of geoid (height anomaly), suitable for processing of vertical deflection data such as for satellite altimetry data, but not conducive to processing of disturbing gravity gradient data.

It is not difficult to find that whether in the land area or in the sea area, the residual terrain could be positive or negative.

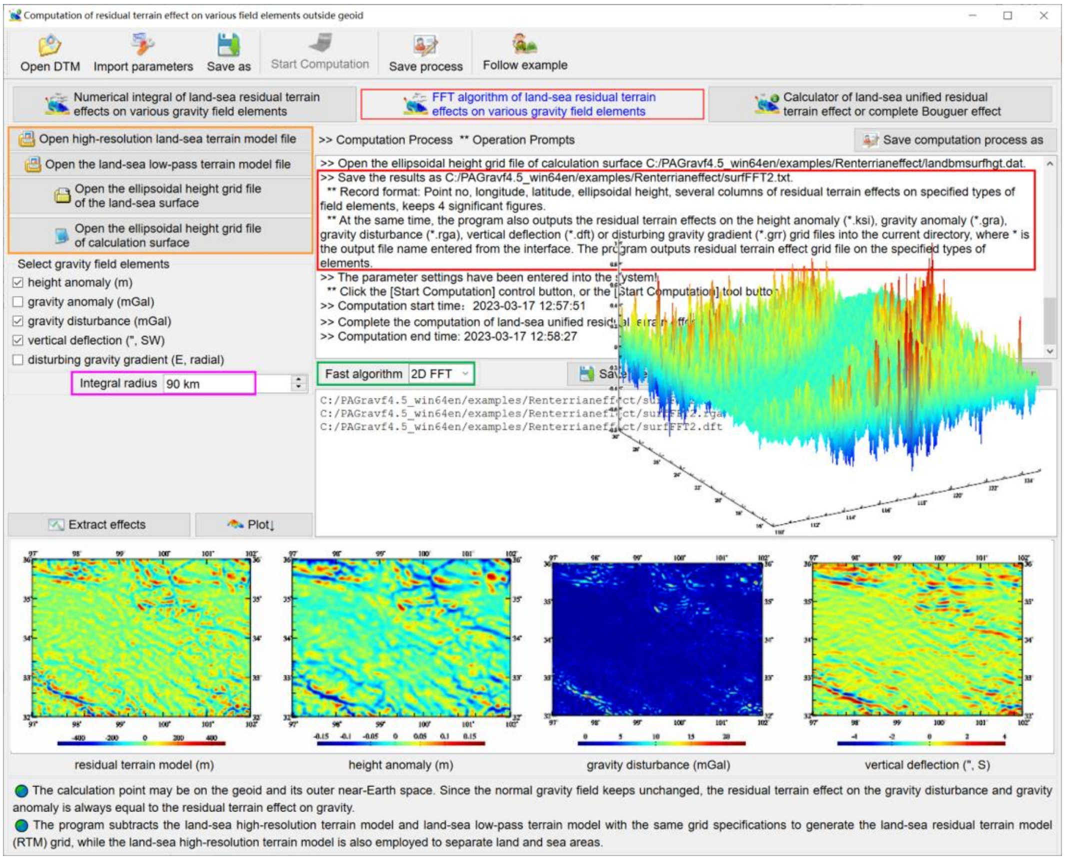
3.4.2 FFT algorithm of land-sea residual terrain effects on various gravity field elements

[Function] Using the FFT integral algorithm, from the high-resolution land-sea terrain model, low-pass land-sea terrain model and ellipsoidal height grid of the land-sea surface, compute the residual terrain effects on the height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection (", to south, to west) or disturbing gravity gradient (E, radial) on the geoid or in its outer space.

[Input files] The high-resolution land-sea terrain model file, land-sea low-pass terrain model file, ellipsoidal height grid file of land-sea surface, and ellipsoidal height grid file of the calculation surface with the same grid specifications.

The ellipsoidal height grid of the land-sea surface stands for the land-sea surface position employed to calculate the integral distance.

[Parameter settings] Select the gravity field element type and the integral fast algorithm and enter the integral radius.



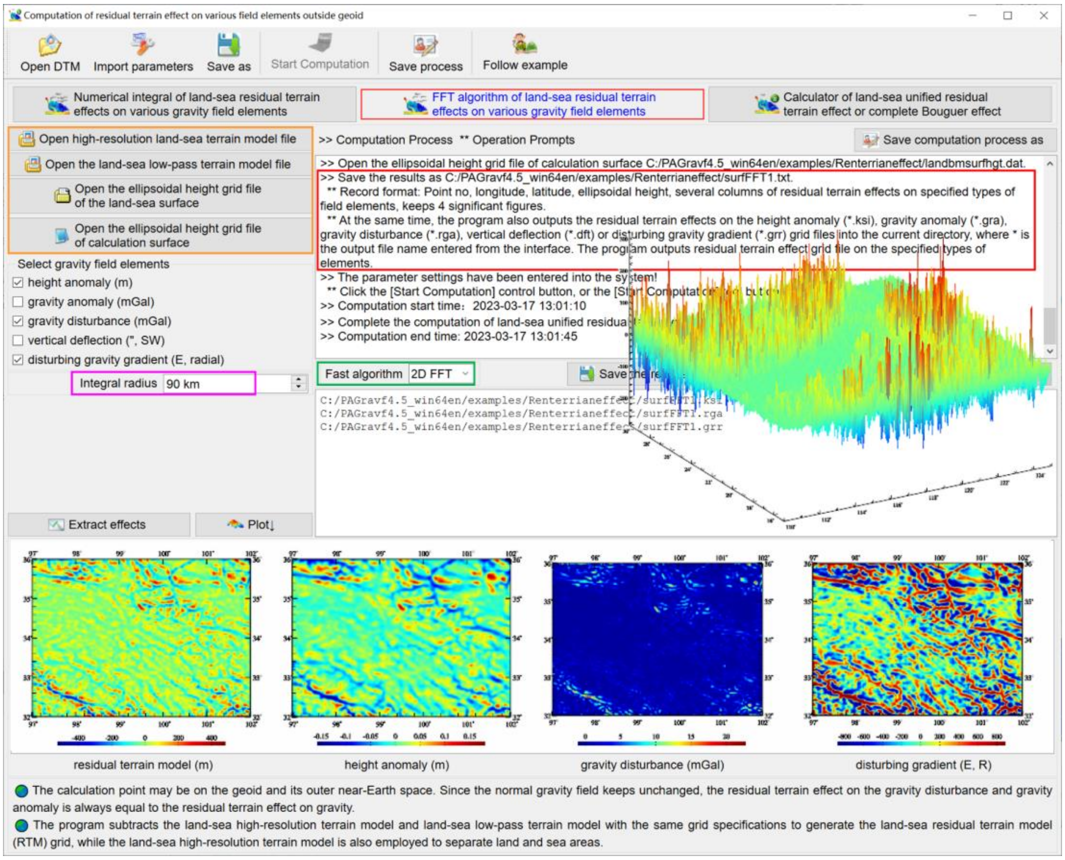
[Output file] The land-sea residual terrain effect file.

Record format: Point no, longitude, latitude, ellipsoidal height, several columns of land-sea residual terrain effects on specified types of field elements, keeps 4 significant figures.

Using the exact same parameters as the numerical integral, compute the residual terrain effects on various field elements according to the FFT algorithm, and statistically analyze the difference between the FFT result and the numerical integral result.

FFT – numerical integral		mean	standard deviation	minimum	maximum
height anomaly /m	FFT2	-0.0129	0.0335	-0.2051	0.1237
	FFT1	-0.0096	0.0242	-0.1514	0.0870
gravity disturbance /mGal	FFT2	0.1031	0.2585	-0.4320	3.0334
	FFT1	0.0328	0.2266	-0.7472	1.3431

vertical deflection /S″	FFT2	0.0001	0.3380	-1.7813	1.8500
	FFT1	0.0002	0.0021	-0.0248	0.0334
vertical deflection /W″	FFT2	0.0029	0.3145	-1.9819	2.3883
	FFT1	0.0001	0.0040	-0.1178	0.0822
disturbing gravity gradient /E	FFT2	0.4968	15.1962	-67.4822	97.0052
	FFT1	-0.0239	0.3995	-10.3352	4.9832

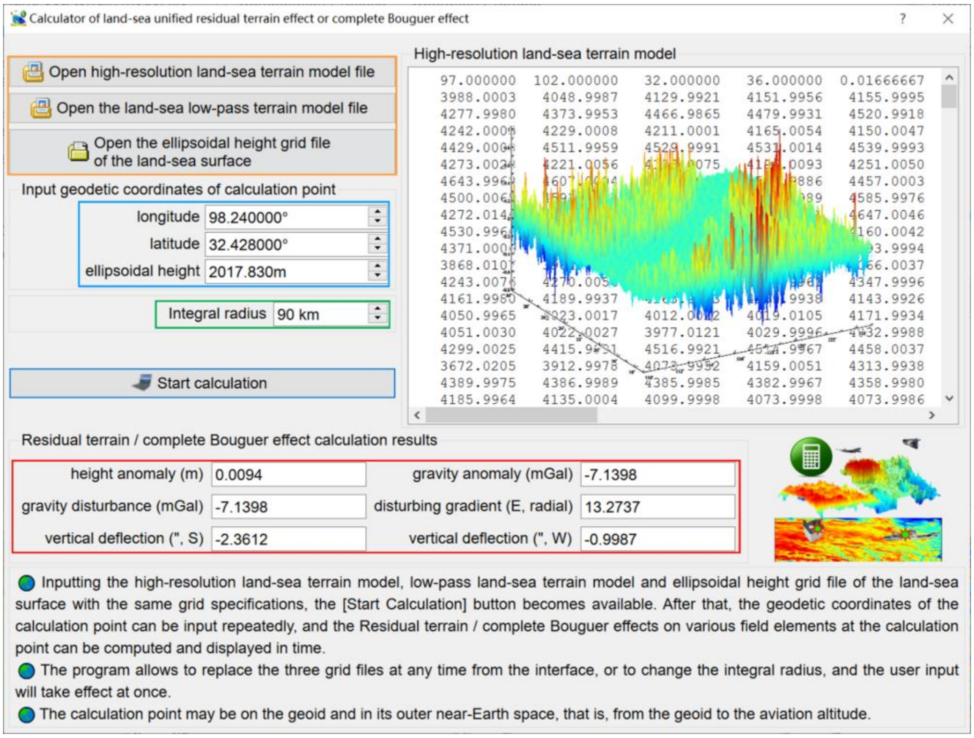


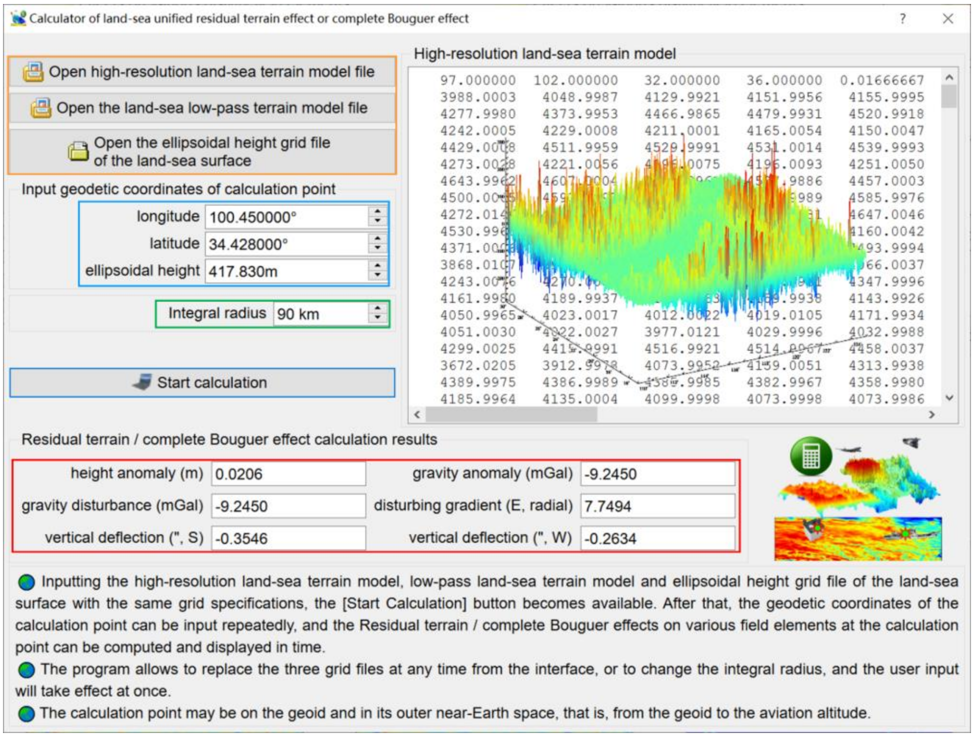
3.4.3 Calculator of land-sea unified residual terrain effect or complete Bouguer effect

[Function] From the high-resolution land-sea terrain model, low-pass land-sea terrain model and ellipsoidal height grid file of the land-sea surface with the same grid specifications, given the calculation point geodetic coordinates on the geoid or in its outer space, compute the land-sea unified residual terrain effects or complete Bouguer effects on the height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection (", to south, to west) and disturbing gravity gradient (E, radial).

Inputting the high-resolution land-sea terrain model, low-pass land-sea terrain model and ellipsoidal height grid file of the land-sea surface with the same grid specifications, the button [Start Calculation] becomes available. After that, the geodetic coordinates of the calculation point can be input repeatedly, and the residual terrain /

complete Bouguer effects on various field elements at the calculation point can be computed and displayed in time.





The program allows to replace the three grid files at any time from the interface, or to change the integral radius, and the user input will take effect at once.

The calculation point may be on the geoid and in its outer near-Earth space, that is, from the geoid to the aviation altitude.

3.5 Computation of land-sea unified classical gravity Bouguer / equilibrium effect

[Purpose] From the land-sea terrain model and ellipsoidal height grid of the land-sea surface, compute the land-sea unified classical Bouguer / equilibrium effects on land-sea surface gravity.

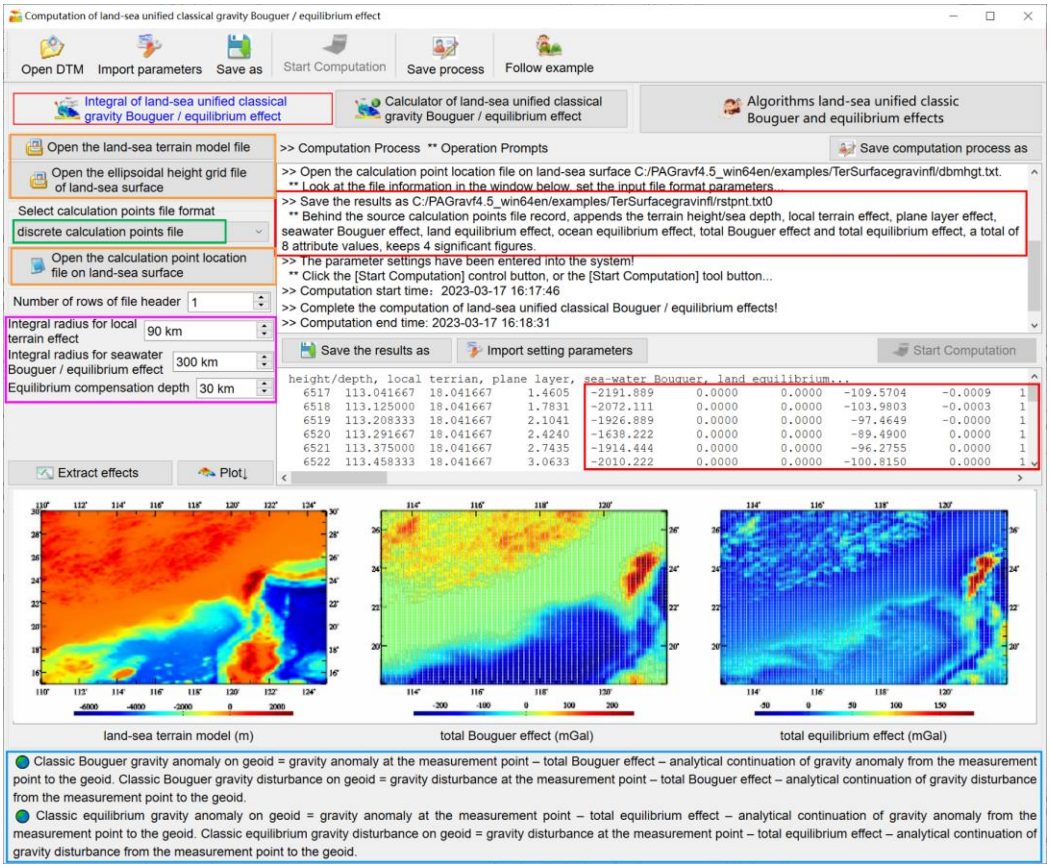
The terrain effect is equal to the negative value of the classical terrain correction. For example, the plane layer effect is equal to the negative layer correction, and the seawater Bouguer effect is equal to the negative seawater Bouguer correction.

3.5.1 Integral of land-sea unified classical gravity Bouguer / equilibrium effect

[Function] From the land-sea terrain model and ellipsoidal height grid of the land-sea surface, compute the land-sea unified classical Bouguer / equilibrium effect on land-sea surface gravity (mGal).

In the land-sea terrain model, the land terrain height is greater than zero while the seafloor water depth is smaller than zero.

The ellipsoidal height grid of the land-sea surface stands for the land-sea surface position employed to calculate the integral distance.



[Parameter settings] Set the input file format parameters, select the integral algorithm, and enter the land integral radius and sea integral radius.

The local terrain effect is the ultrashort wave effect of the terrain, and the integral radius of about 100km is suitable.

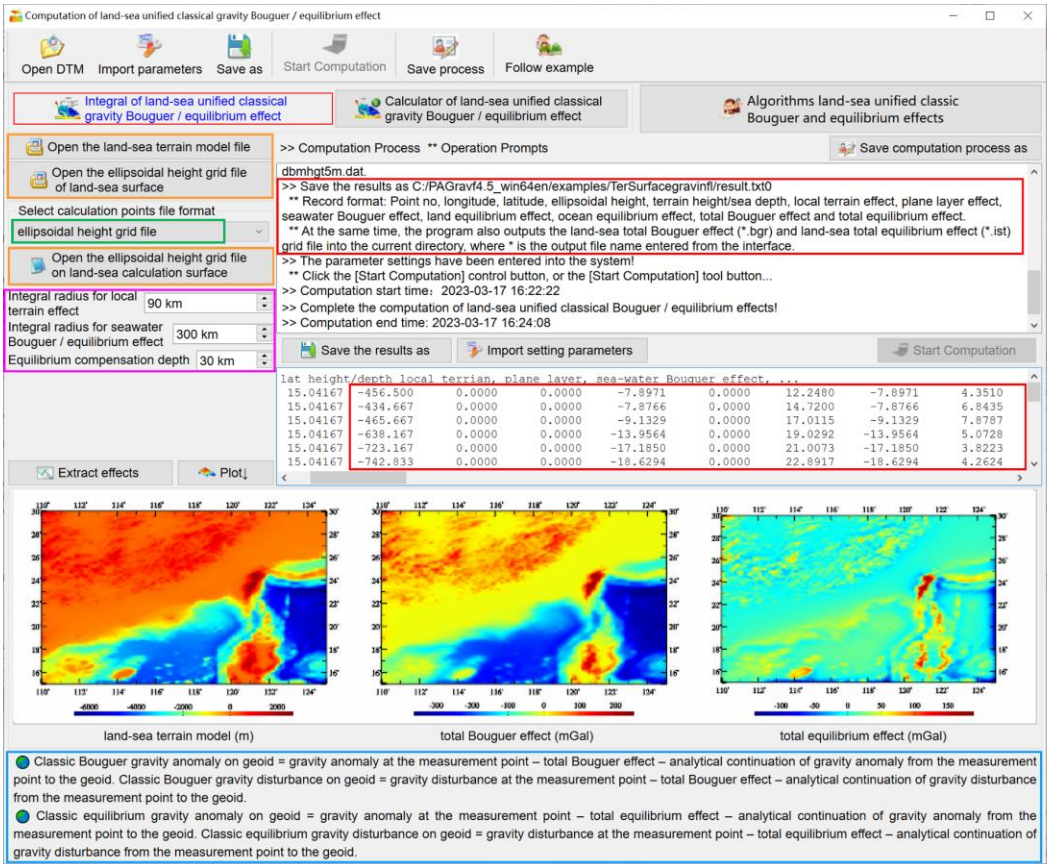
The seawater Bouguer and equilibrium effects include the medium and long wave effects of the terrain, and the integral radius should be much larger than that of local terrain effect.

[Output file] The gravity Bouguer / equilibrium effect file.

When the discrete calculation points file input, the output file record format: Behind the source calculation point file record, appends the local terrain effect, plane layer effect, seawater Bouguer effect, land equilibrium effect, ocean equilibrium effect, total Bouguer effect and total equilibrium effect, a total of 7 attribute values, keeps 4 significant figures.

When the ellipsoidal height grid file input, the output file record format: Point no, longitude, latitude, ellipsoidal height, local terrain effect, plane layer effect, seawater Bouguer effect, land equilibrium effect, ocean equilibrium effect, total Bouguer effect and total equilibrium effect.

At the same time, the program also outputs the land-sea total Bouguer effect (*.bgr) and land-sea total equilibrium effect (*.ist) grid files into the current directory, where * is the output file name entered from the interface.



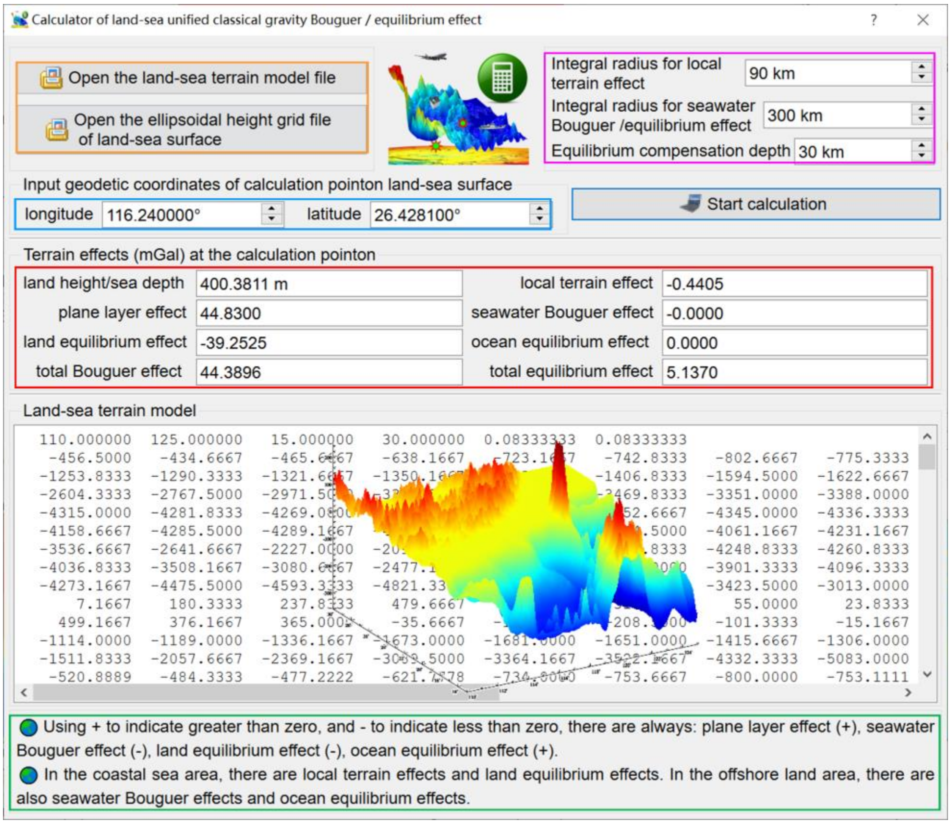
The program is suitable for the unified computation of the classical Bouguer / equilibrium effect on gravity in land, land-sea junction, and sea area. The analytical continuation need be computed by calling the programs in the subsystem [Data analysis and preprocessing calculation of the Earth gravity field].

In any case, the classical Bouguer/equilibrium gravity anomaly (disturbance) can be achieved with a simple two-step calculation. The first step is to obtain the gravity anomaly (disturbed gravity) on the geoid from terrestrial, marine and airborne observed gravity (or from a geopotential coefficient model), the second step is to call this program to obtain the total Bouguer/equilibrium effect, and the two-step results subtraction is the classical Bouguer /equilibrium gravity anomaly (disturbance).

3.5.2 Calculator of land-sea unified classical gravity Bouguer / equilibrium effect

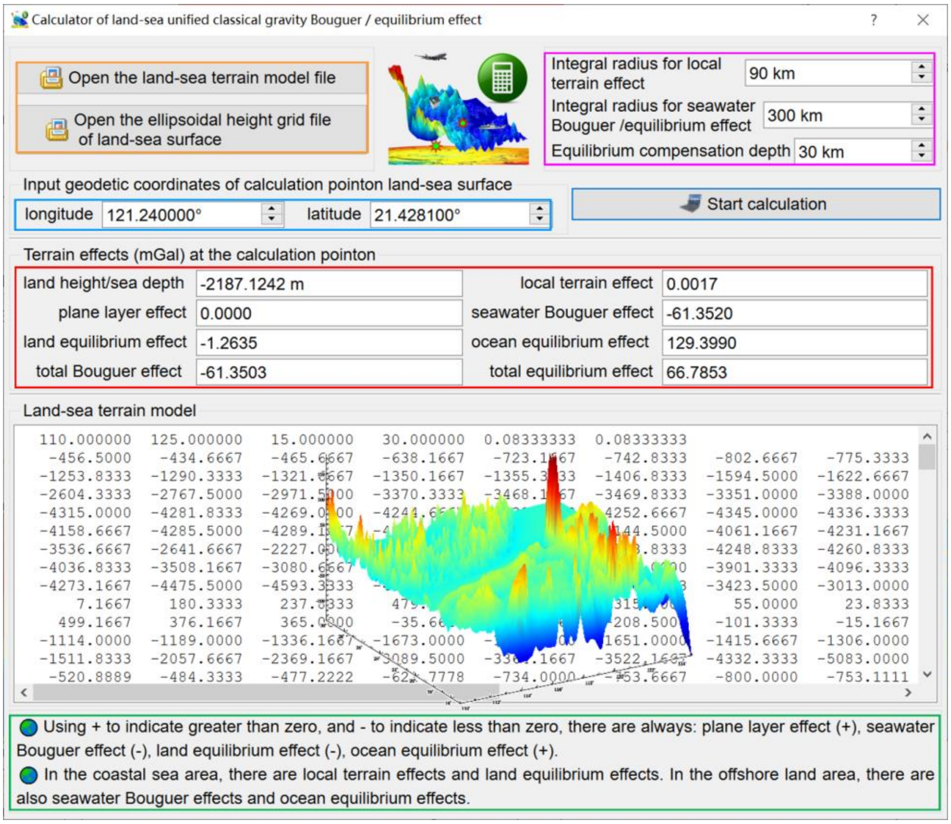
[Function] From the land-sea terrain model file and ellipsoidal height grid file of the land-sea surface with the same grid specifications, given the longitude and latitude of the calculation point on land-sea surface, calculate the land-sea unified classical Bouguer / equilibrium effect and on land-sea surface gravity (mGal).

Using + to indicate greater than zero, and - to indicate less than zero, there are always: plane layer effect (+), seawater Bouguer effect (-), land equilibrium effect (-), ocean equilibrium effect (+).



In the coastal sea area, there are local terrain effects and land equilibrium effects.

In the offshore land area, there are also seawater Bouguer effects and ocean equilibrium effects.



Classic Bouguer gravity anomaly on geoid = gravity anomaly at the measurement point – total Bouguer effect – analytical continuation of gravity anomaly from the measurement point to the geoid.

Classic Bouguer gravity disturbance on geoid = gravity disturbance at the measurement point – total Bouguer effect – analytical continuation of gravity disturbance from the measurement point to the geoid.

Classic equilibrium gravity anomaly on geoid = gravity anomaly at the measurement point – total equilibrium effect – analytical continuation of gravity anomaly from the measurement point to the geoid.

Classic equilibrium gravity disturbance on geoid = gravity disturbance at the measurement point – total equilibrium effect – analytical continuation of gravity disturbance from the measurement point to the geoid.

3.6 Ultrahigh degree spherical harmonic analysis on land-sea terrain and construction of model

[Purpose] Perform spherical harmonic analysis on global land-sea terrain (terrain height/sea depth), and then generate a normalized global land-sea terrain mass spherical harmonic coefficient model, which can be employed to calculate the land-sea unified complete Bouguer effects or residual terrain effects on various gravity field elements on the geoid or in its outer space.

3.6.1 Construction of global surface data grid in spherical coordinates

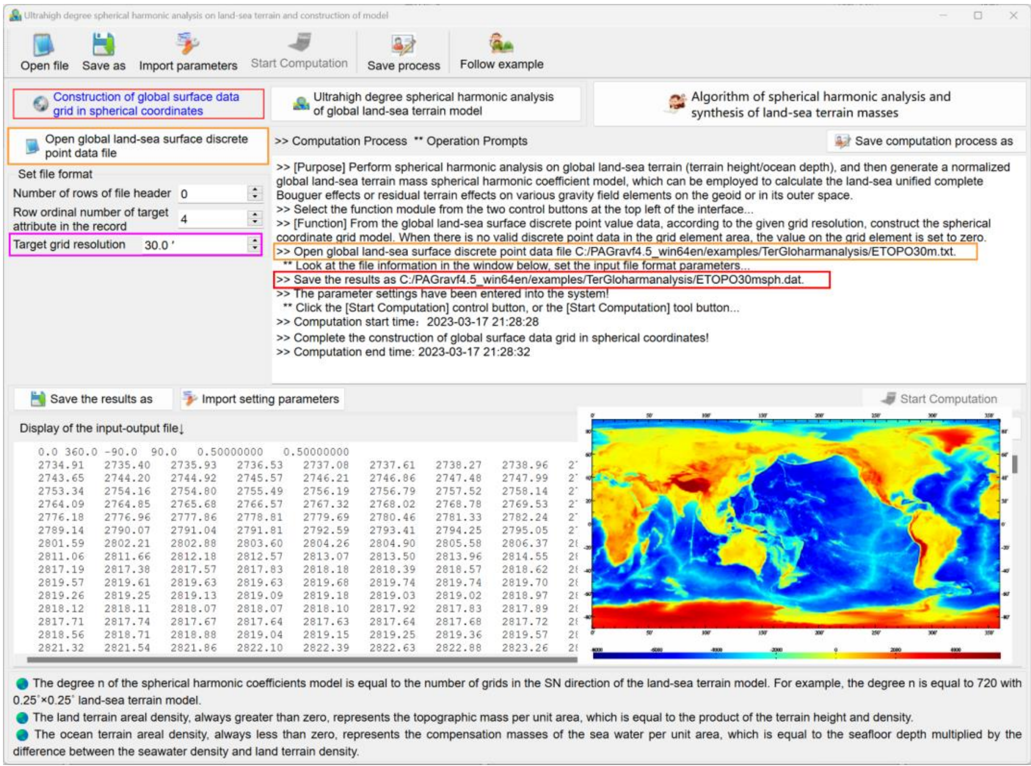
[Function] From the global land-sea surface discrete point value data, according to the given grid resolution, construct the spherical coordinate grid model. When there is no valid discrete point data in the grid element area, the value on the grid element is set to zero.

[Input file] A global land-sea surface discrete point file.

The file record format: ID (Point number / point name), longitude, latitude (decimal degrees), …, attribute to be grid, ….

[Parameter settings] Enter the number of rows of the input file header, row ordinal number of target attribute in the file record, and grid resolution.

[Output file] The spherical coordinate grid file.



3.6.2 Ultrahigh degree spherical harmonic analysis of global land-sea terrain model

[Function] From the global land-sea terrain model grid in the spherical coordinate system, calculate the land-sea terrain mass represented by areal density, perform spherical harmonic analysis, and then generate the global land-sea terrain mass normalized spherical harmonic coefficient model (kg/m²).

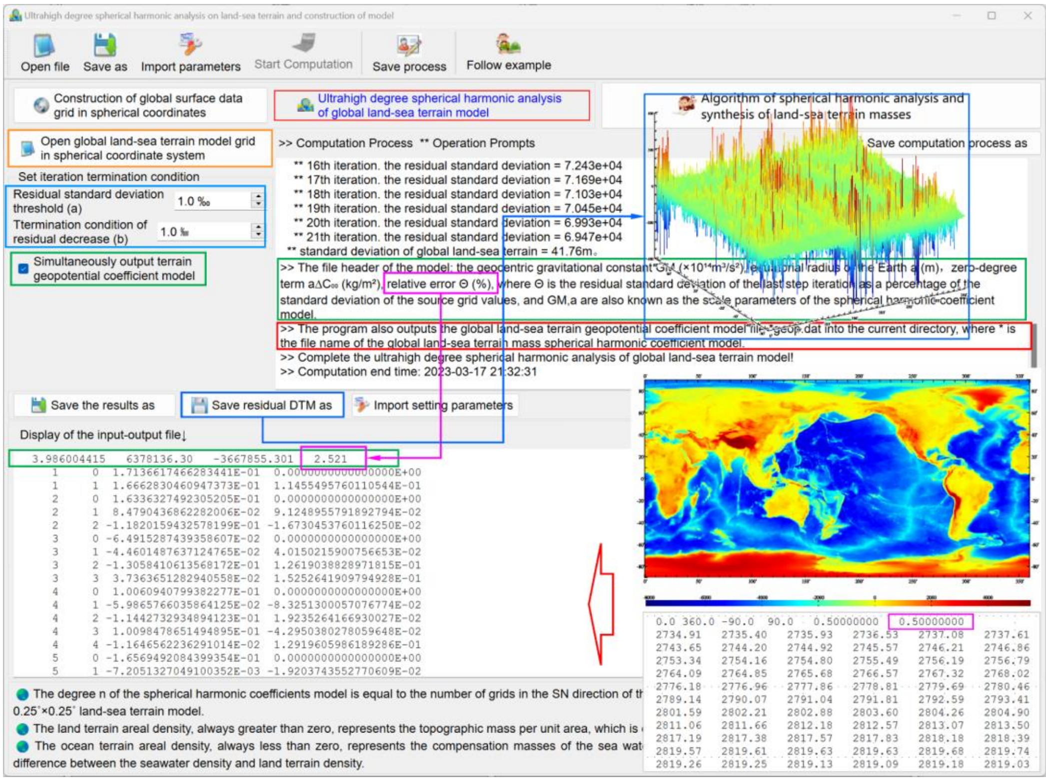
The land terrain areal density, always greater than zero, represents the topographic mass per unit area, which is equal to the product of the terrain height and density. The ocean terrain areal density, always less than zero, represents the compensation masses of the sea water per unit area, which is equal to the seafloor depth multiplied by the difference between the seawater density and land terrain density.

The degree n of the spherical harmonic coefficients model is equal to the number of grids in the SN direction of the land-sea terrain model. For example, the degree n is equal to 720 with 0.25˚×0.25˚ land-sea terrain model.

[Input file] The global land-sea terrain model grid file in the spherical coordinate system.

[Parameter settings] Enter the iteration condition parameters.

Iteration termination condition: The standard deviation of the residual terrain areal density is less than a% of the standard deviation of the source terrain areal density, or the difference of the residual standard deviation of the previous step iteration relative to the current step iteration is less than b‰ of the standard deviation of the source terrain areal density.



[Output files] The land-sea terrain mass spherical harmonic coefficient model file and the residual land-sea terrain model grid file.

The file header of the spherical harmonic coefficient model: the geocentric gravitational constant GM ($\times10^{14}m^3/s^2$), equatorial radius of the Earth a (m), zero-degree term $a\Delta C_{00}$ (kg/m^2), relative error Θ (%), where Θ is the residual standard deviation of the last step iteration as a percentage of the standard deviation of the source grid values, and GM,a are also known as the scale parameters of the spherical harmonic coefficient model.

The zero-degree term represents the variations of the total Earth mass caused by the variation of global terrain areal density, which is meaningless under the condition of Earth’s mass conservation. The three first degree spherical harmonic coefficients (ΔC_{10}, ΔC_{11}, ΔS_{11}) represent variations of the Earth's center of mass due to the variations of global terrain areal density.

The program also outputs the global land-sea terrain geopotential coefficient model file *geop.dat into the current directory, where * is the file name of the global land-sea terrain mass spherical harmonic coefficient model.

The program employs an iterative algorithm, and the calculation process need wait... During the period, you can open the file Harminf.txt in the current directory to look at the iterative process...

Harminf.txt record format: number of iterations, mean, standard deviation, minimum, maximum (of the residual terrain area density).

3.7 Spherical harmonic synthesis of complete Bouguer or residual terrain effects

[Purpose] From global land-sea terrain mass spherical harmonic coefficient model (kg/m²), calculate the model value of terrain height/sea depth, as well as the land-sea unified complete Bouguer or residual terrain effects on various gravity field elements on the geoid or in its outer space, and analyzes the spectral and spatial domain properties of the global land-sea terrain effect.

Since the normal gravity field keeps unchanged, the terrain effect on the gravity disturbance and gravity anomaly is always equal to that on gravity, and the terrain effect on disturbing geopotential is always equal to that on geopotential.

The program is suitable for the unified computation of the complete Bouguer and residual terrain effects on various gravity field elements in land, land-sea junction, and sea area. The calculation point may be on the geoid and its outer Earth space.

Using the equal minimum and maximum degree n, the program can calculate the contribution of the degree n land-sea terrain coefficients to various gravity field element. From the degree n, cumulative n degrees or n_1~n_2-degrees terrain effects calculated from a global land-sea terrain model, the spectral domain and spatial domain performance of the model can be observed and evaluated.

3.7.1 Calculation of model value for complete Bouguer or residual terrain effects

[Function] From global land-sea terrain mass spherical harmonic coefficient model (kg/m²), calculate the model value of terrain height/sea depth, as well as the land-sea unified complete Bouguer or residual terrain effects on the height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection vector (", south, west), disturbing gravity gradient (E, radial), tangential gravity gradient vector (E, north, west), or disturbing geopotential (m²/s²) on the geoid or in its outer space.

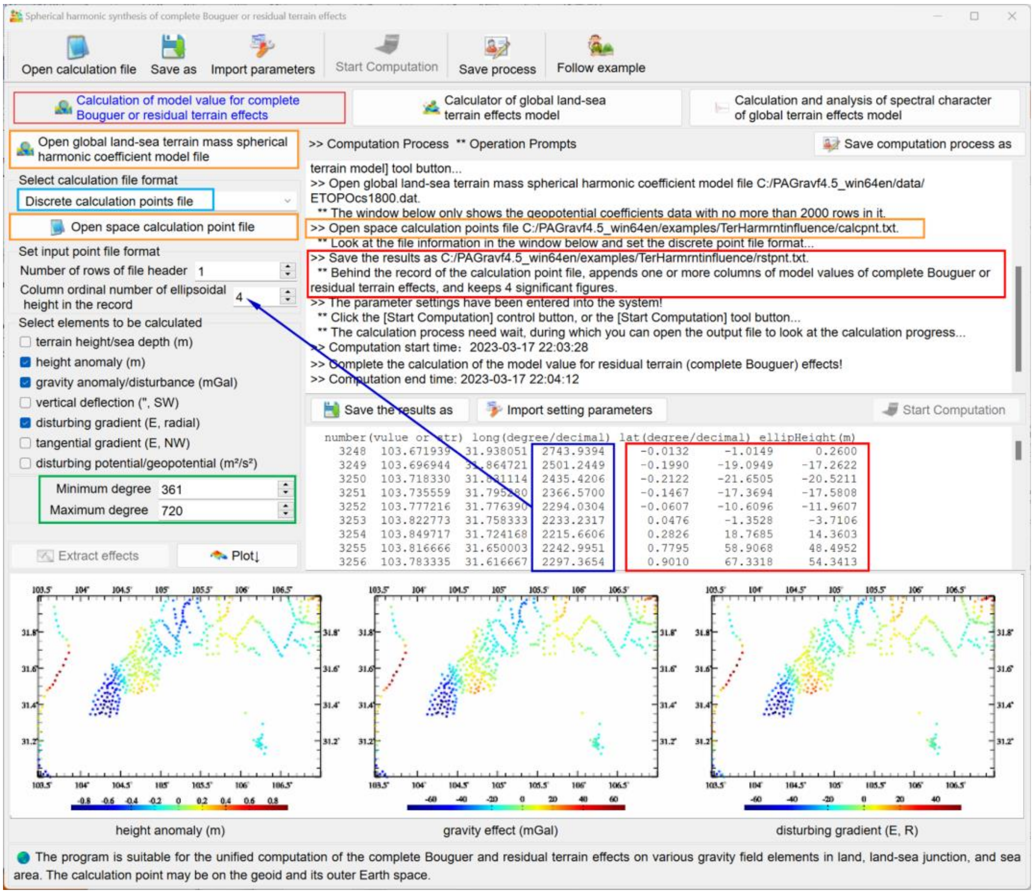
The global land-sea terrain mass spherical harmonic coefficient model can be constructed by the function [Ultrahigh degree land-sea terrain spherical harmonic analysis and model construction].

[Input files] The global land-sea terrain mass spherical harmonic coefficient model file, and the space calculation point location file.

The model file header: the geocentric gravitational constant GM ($\times10^{14}$m³/s²), equatorial radius of the Earth a (m), zero-degree term $a\Delta C_{00}$ (kg/m²), relative error Θ (%). (GM,a) are also known as the scale parameters of the spherical harmonic coefficient model.

The space calculation point location file may be a discrete calculation points file or an ellipsoidal height grid file of the calculation surface.

The record format of the calculation points file: point no / point name, longitude (decimal degrees), latitude (decimal degrees), ellipsoidal height (m)......

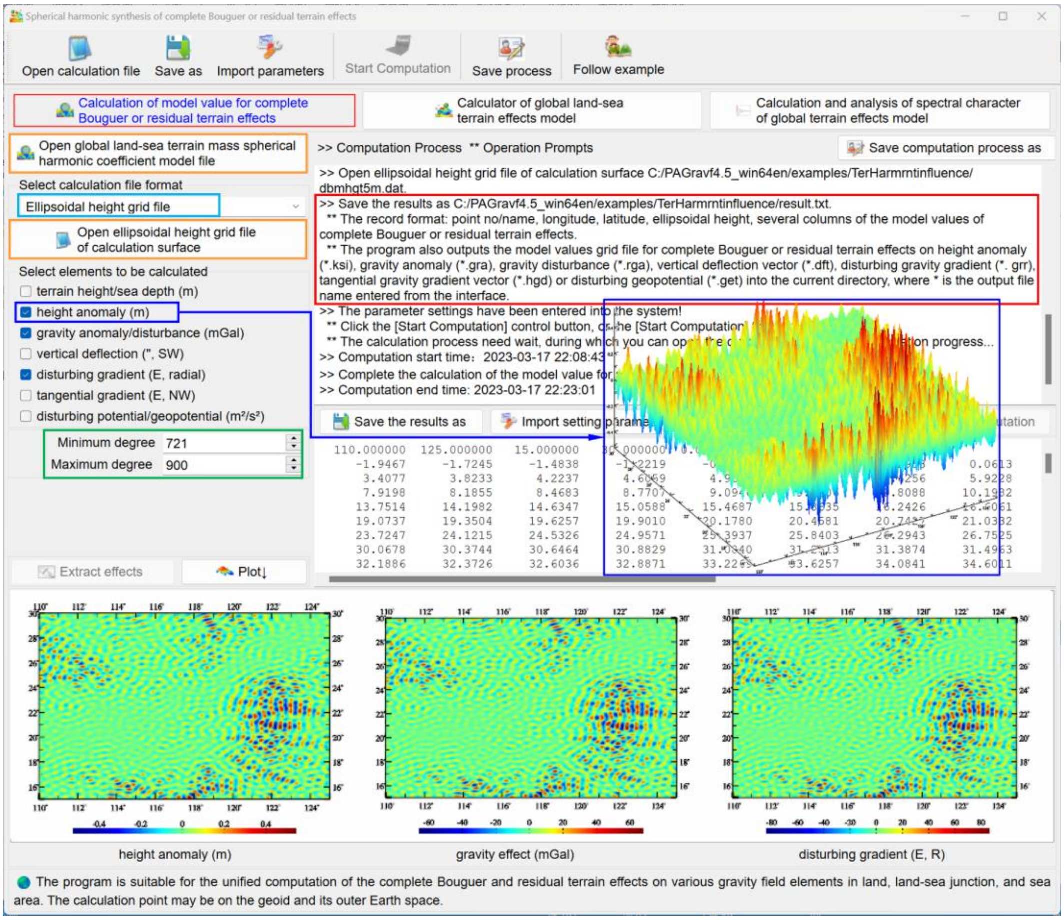


[Parameter settings] Set the calculation point file format parameters, input the minimum and the maximum calculation degree of the land-sea terrain geopotential model, and select the type of model residual terrain (complete Bouguer) effects to be calculated.

When the minimum calculation degree is less to 3, the program calculates the model value of land sea complete Bouguer effects. And when the minimum calculation degree is greater than 2, the program calculates the model value of land sea residual terrain effects.

The program selects the minimum of the maximum degree of the model and the input maximum degree as the calculation degree.

The calculation process need wait, during which you can open the output file to look at the calculation progress...



[Output file] The model value file of residual terrain (complete Bouguer) effects.

When the discrete calculation points file input, the output file record format: Behind the record of the calculation points file, appends one or more columns of the model values of residual terrain (complete Bouguer) effects selected, and keeps 4 significant figures.

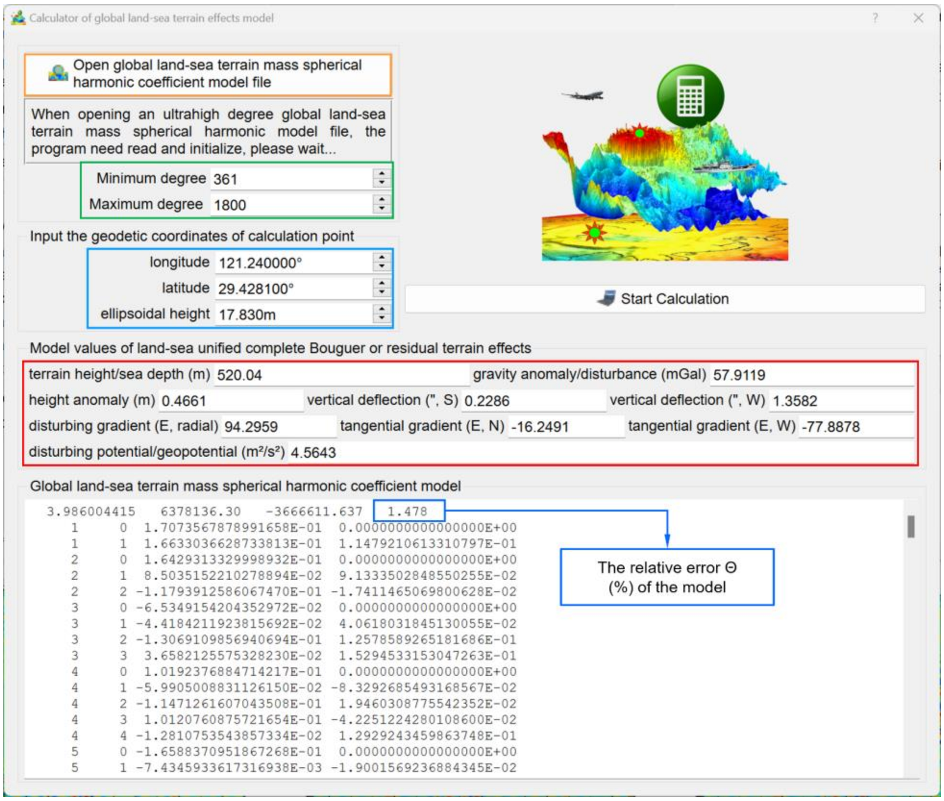
When the ellipsoidal height grid file input, the output file record format: point no/name, longitude, latitude, ellipsoidal height, several columns of the model values of residual terrain (complete Bouguer) effects selected.

The program also outputs the model value grid file for the terrain height/sea depth (m), complete Bouguer or residual terrain effects on height anomaly (*.ksi), gravity anomaly (*.gra), gravity disturbance (*.rga), vertical deflection vector (*.dft), disturbing gravity gradient (*. grr), tangential gravity gradient vector (*.hgd) or disturbing geopotential (*.get) into the current directory. Where * is the output file name entered from the interface.

When calculating the model value of terrain height/sea depth, the program ignores the ellipsoidal height of the calculation point.

3.7.2 Calculator of global land-sea terrain effects model

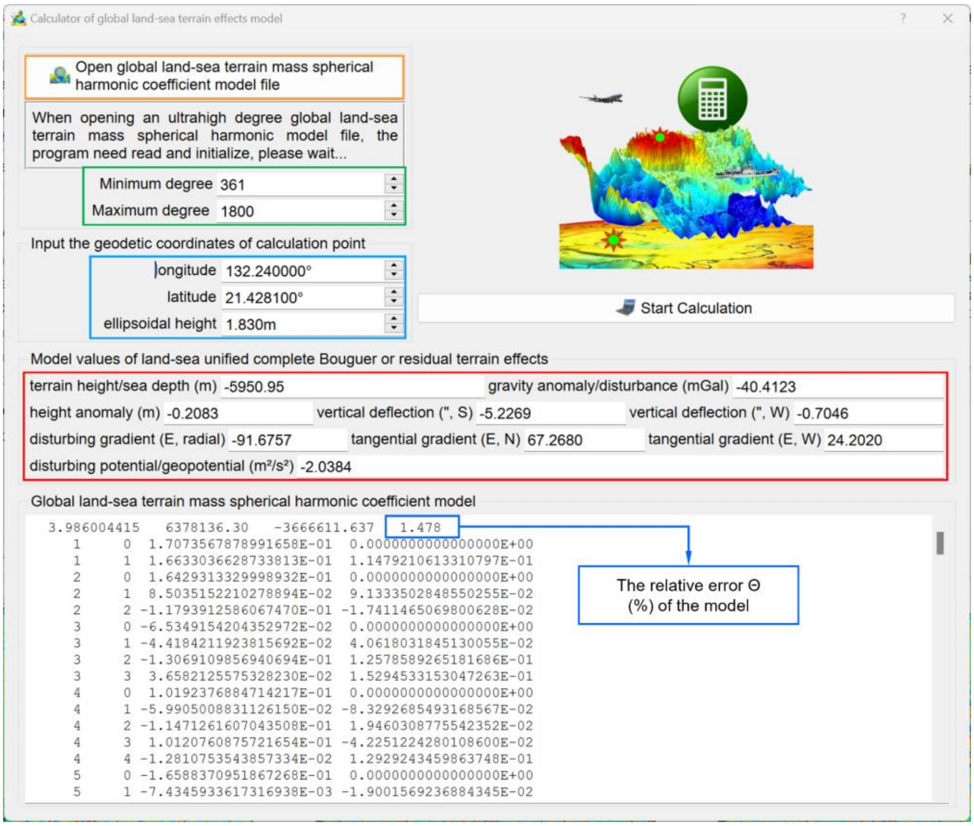
[Function] From the global land-sea terrain mass spherical harmonic coefficient model (kg/m²) file, given the geodetic coordinates of space calculation point, calculate the model values of terrain height/sea depth and the complete Bouguer or residual terrain effects on the height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection vector (", south, west), disturbing gravity gradient (E, radial), tangential gravity gradient vector (E, north, west) and disturbing geopotential (m²/s²). This function is suitable for classroom demonstrations.

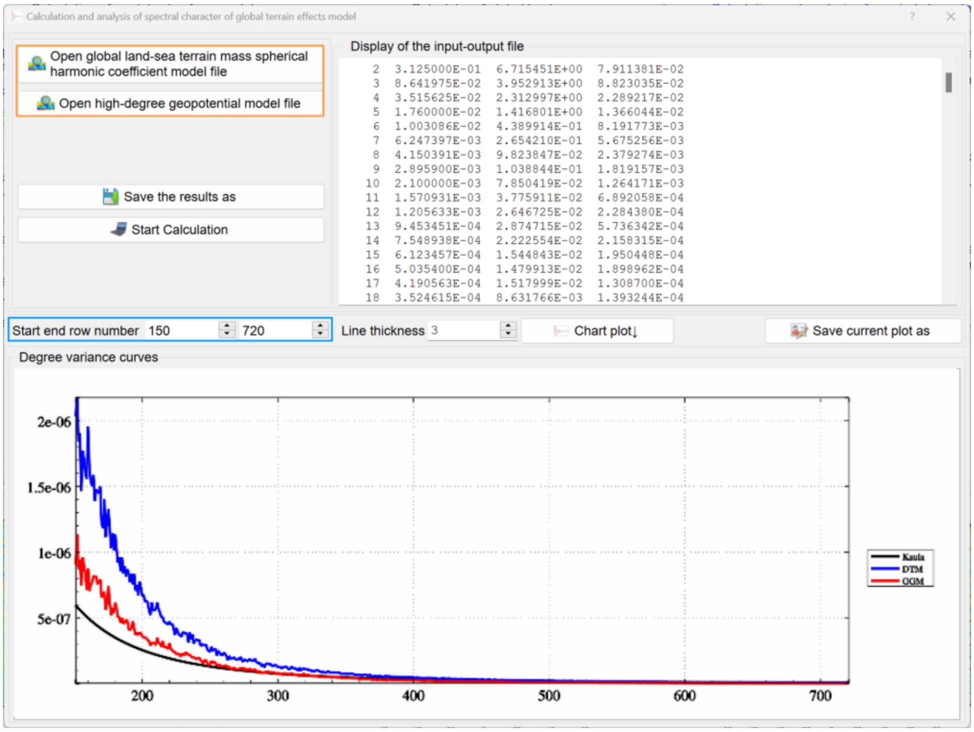


When an ultrahigh degree global land-sea terrain mass spherical harmonic model file opened, the program need read and initialize, please wait...

3.7.3 Calculation and analysis of spectral character of global terrain effects model

[Function] Calculate the degree variance of the land-sea terrain geopotential coefficient model and global geopotential model, and then analyze the spectral or spatial domain properties of the land-sea complete Bouguer and residual terrain effects from the degree variance curves.



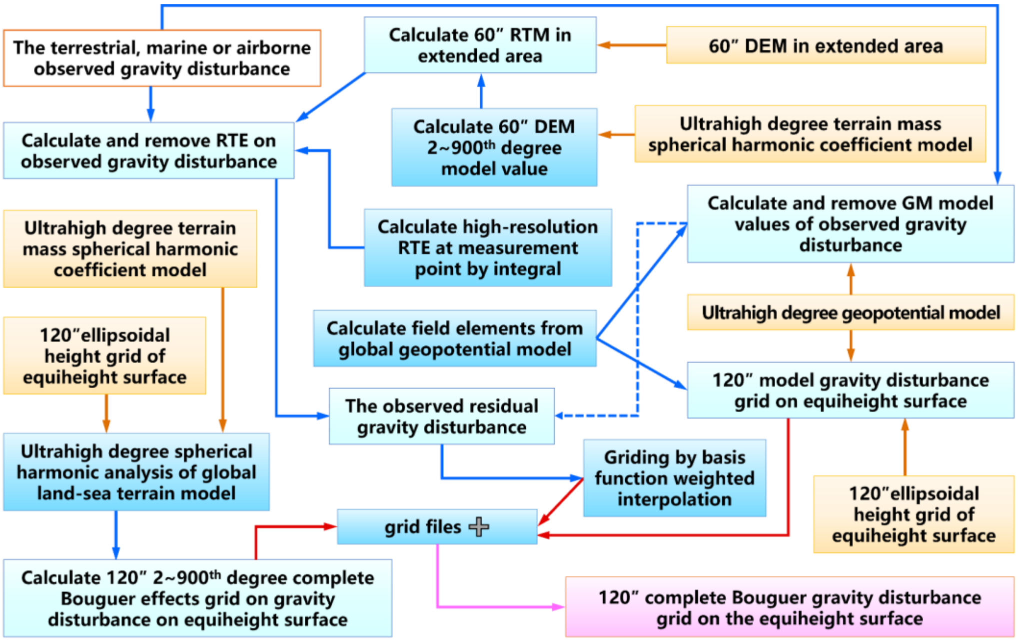


3.8 Computation process demo of various terrain effects outside geoid

3.8.1 Computation process demo of complete Bouguer anomaly on terrain equiheight surface

From the ground digital elevation model and discrete observed gravity disturbance calculated from EGM2008 geopotential model, a remove-restore scheme with the residual terrain effects employed, calculate the complete Bouguer gravity disturbance grid on an equipotential surface which is also the observation reduction surface, to show the basic computation scheme and process of the land-sea unified complete Bouguer effects on various gravity field elements near-Earth space.

The complete Bouguer effect is defined as the variation of Earth gravity field because of the terrain mass above the geoid removed and the seawater density compensated to the terrain density.



Computation process demo of complete Bouguer anomaly outside geoid

● **Input and output data and related terrain models**

Let terrain data range (extended area, E94.5~99.5°, N30.5~34.5°) ⊃ result range (measurement points distribution range / observation reduction surface range, E95.0 ~ 99.0°, N31.0 ~ 34.0°) to suppress the edge effect of integral.

(1) The observed gravity disturbance and disturbing gravity gradient file Obsgrav.txt.

The gravity disturbances are simulated from the 2~1800th degree EGM2008 model. PAGravf4.5 employs the exact same algorithms to process various terrestrial, marine, and airborne gravity data in a unified way, and there is no need to distinguish whether the observed point is on the ground, at the air altitude or in the sea area.

The format of the file record: ID, longitude (°), latitude (°), ellipsoidal height (m), gravity disturbance (mGal). The distribution of the observed points is shown in Fig.

(2) The 1800-degree terrain mass spherical harmonic coefficient model file ETOPOcs1800.dat and the 2190-degree EGM2008 geopotential coefficient model file EGM2008.gfc.

The two model files are stored in the directory C:\PAGravf4.5_win64en\data. The 1800-degree global land-sea terrain mass spherical harmonic coefficient model ETOPOcs1800.dat is generated by the PAGravf4.5 function [Ultrahigh degree spherical harmonic analysis of global land-sea terrain model] from the global 2′×2′ land-sea terrain model ETOPO2v2g.

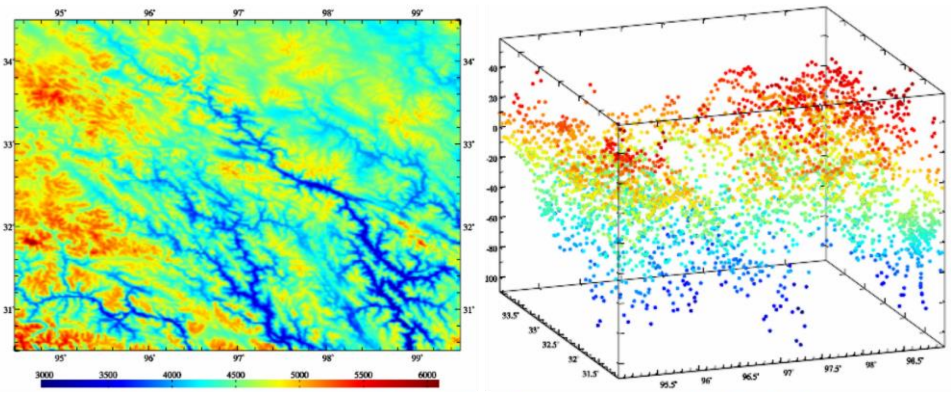
(3) The ground digital elevation model (DEM)

It is required that the DEM grid range (extended area) be larger than the calculation area to eliminate the integral edge effect.

Two kinds of DEM resolutions are required. The high-resolution is employed for the observed data reduction, that is, to calculate and remove the residual terrain effects on the observations. The other resolution is consistent with the calculation result resolution and is employed to restore the residual terrain effects on the result. In this example, they are 60" and 120" respectively, and the corresponding files are extdtm60s.dat and extdtm120s.dat. 339.5

(4) 60″ ground ellipsoidal height grid file surfhgt60s.dat

The ground ellipsoidal height grid is employed to give the space location of the residual terrain mass (the integral move cell) which is indispensable for high-precision calculations. In this example, the 60" ground ellipsoidal height grid file surfhgt60s.dat is for the 60" residual terrain model.



Ground digital elevation model (m) and gravity measurement point distribution

(5) 120″ ellipsoidal height grid file equihgt120s.dat of terrain equiheight surface.

The terrain equiheight surface is the reduction surface of the ground observations and calculation surface of the result grid, which is regarded as an equipotential surface. The ellipsoidal height of the terrain equiheight surface is equal to the sum of the 2~180th degree EGM2008 model geoidal height and mean of DEM.

The griding operation is not analytical, which is easy to weaken the analytical nature of the gravity field. Non-analytical operations are required to be performed on some an equipotential surface to minimize the negative effects on gravity field. In this example, the terrain equiheight surface is regarded as an equipotential surface.

When the normal (orthometric) heights of the surface are zero, namely whose ellipsoidal height are the geoidal height, the reduction surface and calculation surface are the geoid in the traditional sense.

(6) The result products: 120″×120″ complete Bouguer gravity disturbance grid file on the terrain equiheight surface.

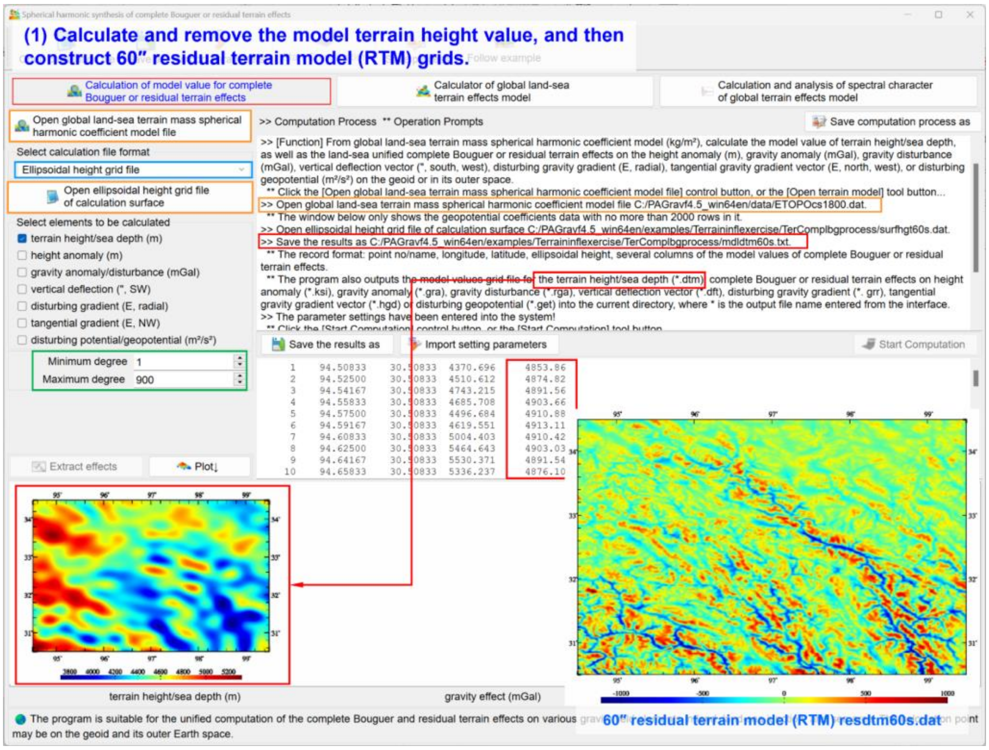
● **Programs call and input-output data flow**

(1) Calculate and remove the model terrain height value, and then construct 60″ residual terrain model (RTM) grid.

Call the function [Calculation of model value for complete Bouguer or residual terrain effects] with the minimum degree 1 and the maximum degree 900, select the calculation type ‘terrain height/sea depth (m)’, input the land-sea terrain mass spherical harmonic coefficient model file ETOPOcs1800.dat and ground ellipsoidal height grid file surfhgt60s.dat, and generate 60″ model terrain height grid files mdldtm60s.dtm.

Let extdtm60s.dat minus mdldtm60s.dat, the residual terrain models (RTM) resdtm60s.dat in the extended area are obtained, as shown in the figure.

	mean	standard deviation	minimum	maximum
30″ RTM (m)	0.0053	175.5869	-959.5450	59.0160
60″ RTM (m)	0.0053	175.5869	-959.5450	886.2500



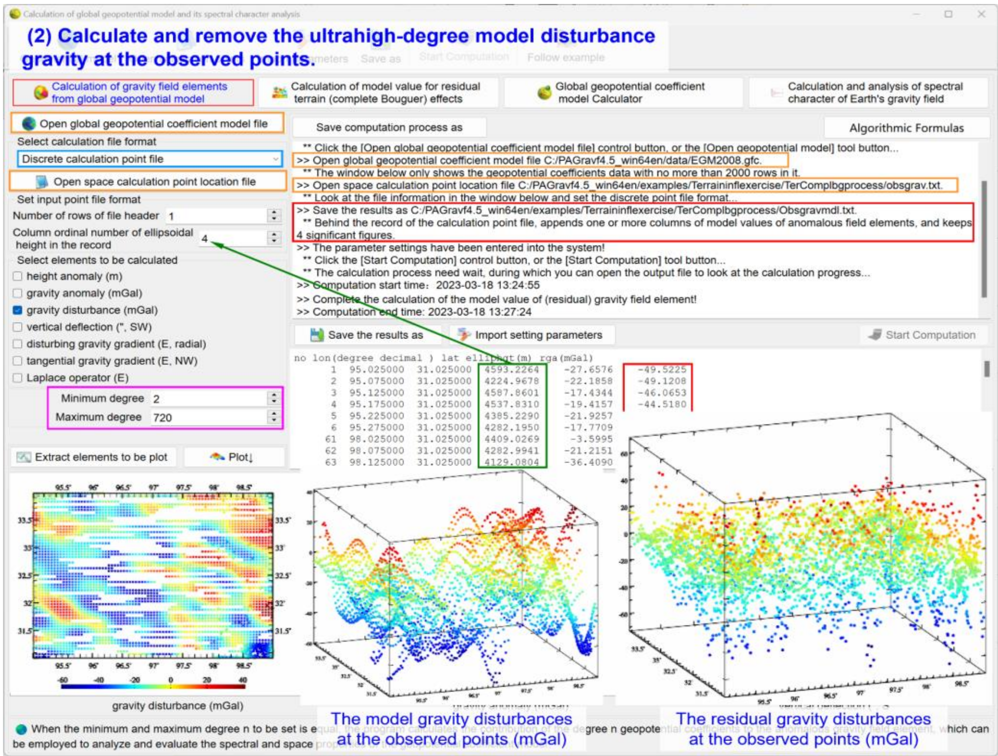
(2) Calculate and remove the ultrahigh-degree model gravity disturbances at the observed points.

Call the function [Calculation of gravity field elements from global geopotential model] with the minimum degree 2 and the maximum degree 720, input the file EGM2008.gfc and the observation file Obsgrav.txt, select the type ‘gravity disturbance’, and generate the model gravity disturbance file Obsgravmdl.txt (columns 6) at the observed points.

Subtract the observed gravity disturbance (column 5) and model gravity disturbance (column 6) to generate the model residual gravity disturbance (column 7) file Obsgravmdlresd.txt.

Table 2 shows the statistical results on the gravity disturbances after the 2~720th degree model values removed.

Measurement points	mean	standard deviation	minimum	maximum
Observed gravity disturbance mGal	-15.6106	25.5080	-110.7251	59.0160
Residual of gravity disturbance	-0.4881	17.4588	-74.6129	71.5003



(3) Calculate and remove the residual terrain effects on the gravity disturbance at the observed points.

Call the function [Numerical integral of land-sea residual terrain effects on various gravity field elements], input the file Obsgravmdlresd.txt, the high-resolution DEM extdtm60s.dat, low-pass DEM mdldtm60s.dtm and the ground ellipsoidal height grid file surfhgt60s.dat, set the integral radius 90km, and generate the residual terrain effects

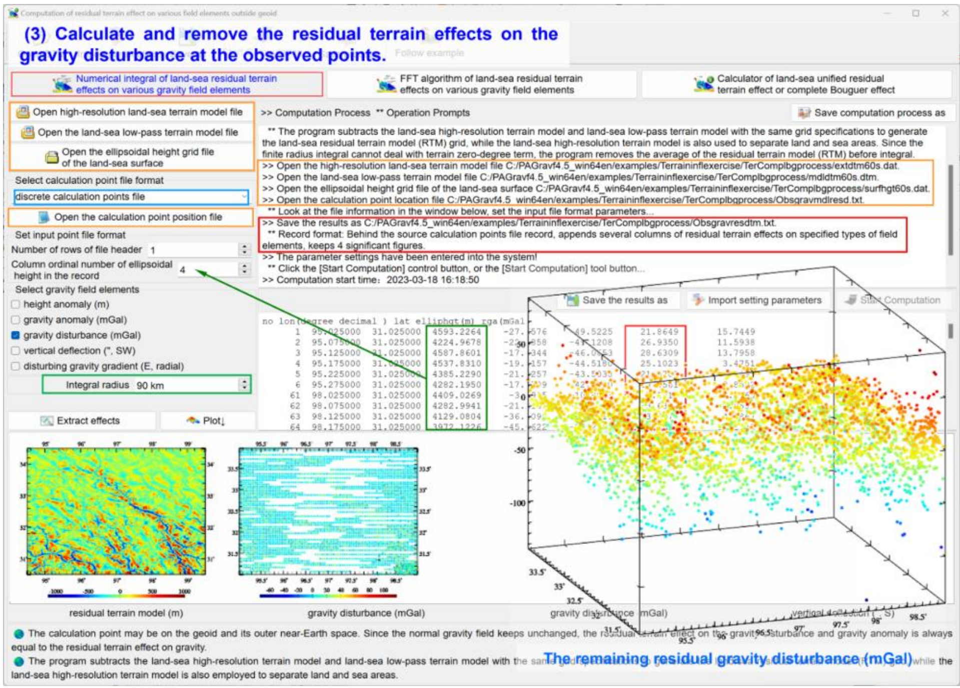
(RTE) file on the gravity disturbance, Obsgravresdtm.txt (columns 8).

Subtract the residual gravity disturbance (column 7) and its residual terrain effect (column 8), to generate the remaining residual gravity disturbance (column 9) file Obsgravresidual.txt.

After the residual terrain effects removed, the statistical results on the residual gravity disturbances are shown in Table 3.

Measurement points	mean	standard deviation	minimum	maximum
RTE on gravity disturbance mGal	4.8843	7.2038	-73.7901	118.6158
Remaining residual gravity disturbance mGal	-5.3034	19.7638	-144.5444	92.4782

In this example, the continuation correction of residual radial gradient (within a height difference of 1000m, it is small) is ignore. In this case, the remaining residual gravity disturbance at the observed point is equal to that on the equipotential surface.



The basic purpose of the statistics in Tables 1 to 3 is to improve the residual terrain effect algorithm and relative parameters according to the griding optimization criteria. Since the simulated data lack sufficient ultrashort wave information of the real gravity field, the optimization criterion analysis process is omitted in this example.

So far, the reduction processing of the gravity disturbances from the observed points to the terrain equiheight surface has been completed.

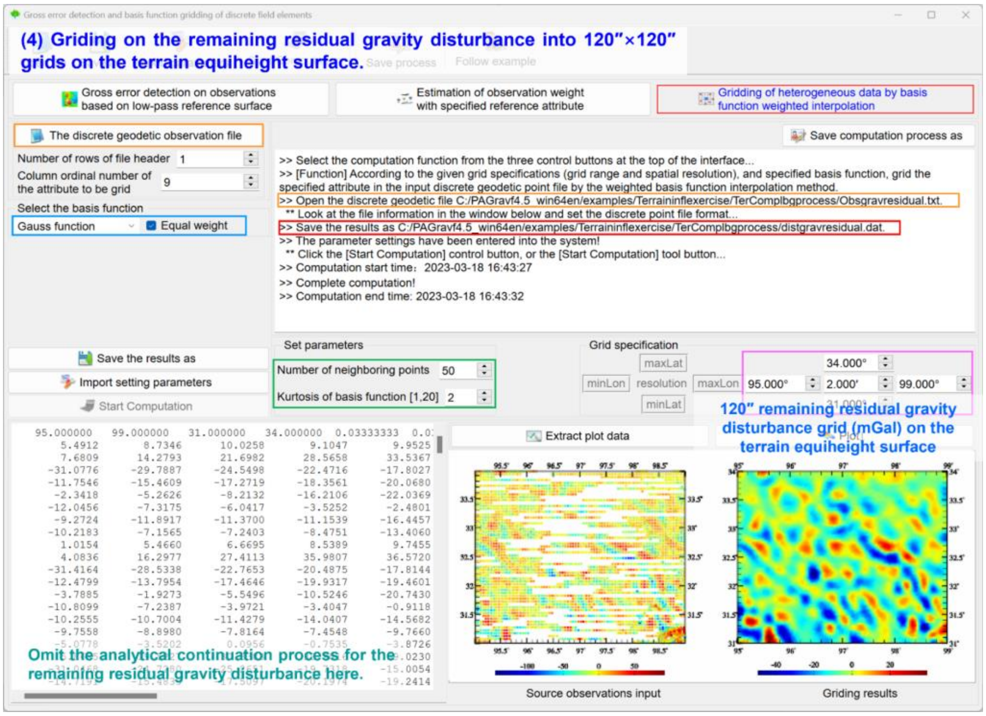
(4) Griding on the remaining residual gravity disturbance into 120″×120″ grids on the terrain equiheight surface.

Call the function [Gridding of heterogeneous data by basis function weighted interpolation], select ‘equal weights of observations’ (the weights can be estimated with the residual terrain effects as the reference attribute in advance), and grid on the 9th column of attributes (from the file Obsgravresidual.txt), to generate 120″ remaining residual gravity disturbance file distgravresidual.dat on the terrain equiheight surface.

The range and resolution of the grid is the same as the result grid.

(5) Calculate the 120″ EGM2008 720-degree model value grid of the gravity disturbance on the terrain equiheight surface.

Call the function [Calculation of gravity field elements from global geopotential model] with the minimum degree 2 and the maximum degree 720, input the file EGM2008.gfc and the ellipsoidal height grid file equihgt120s1.dat (from equihgt120s.dat with grid edge removed) of the terrain equiheight surface, and select the calculation type ‘gravity disturbance’, to generate 120″ model gravity disturbance grid file distgravmdl.rga on the terrain equiheight surface.

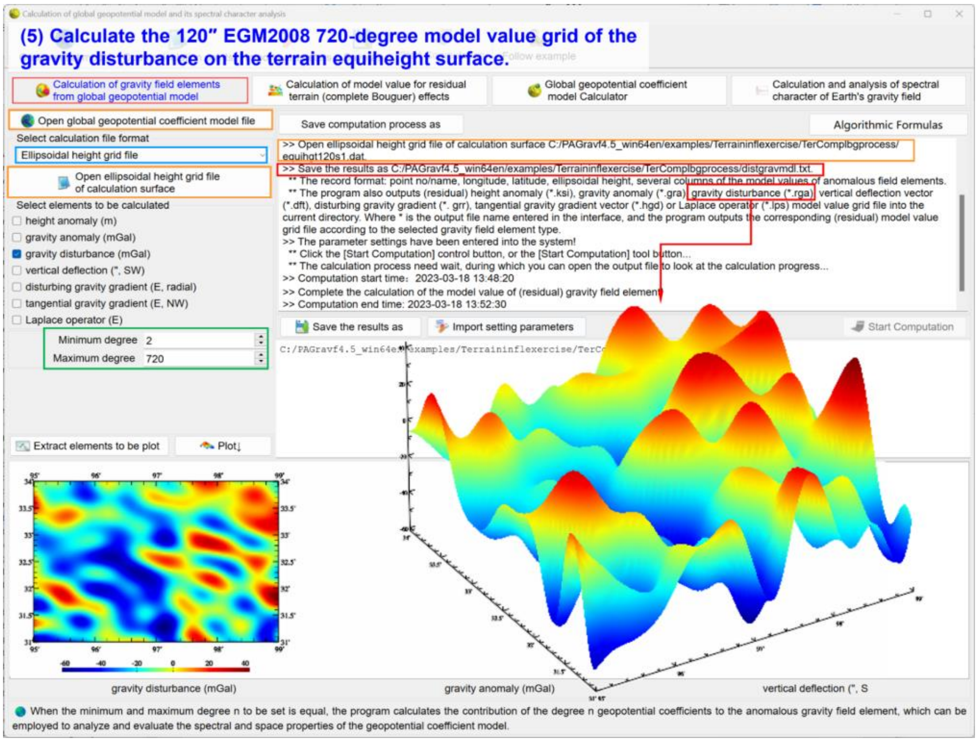


Here, the geopotential model, the minimum and maximum degree are required to be the same as in step (2).

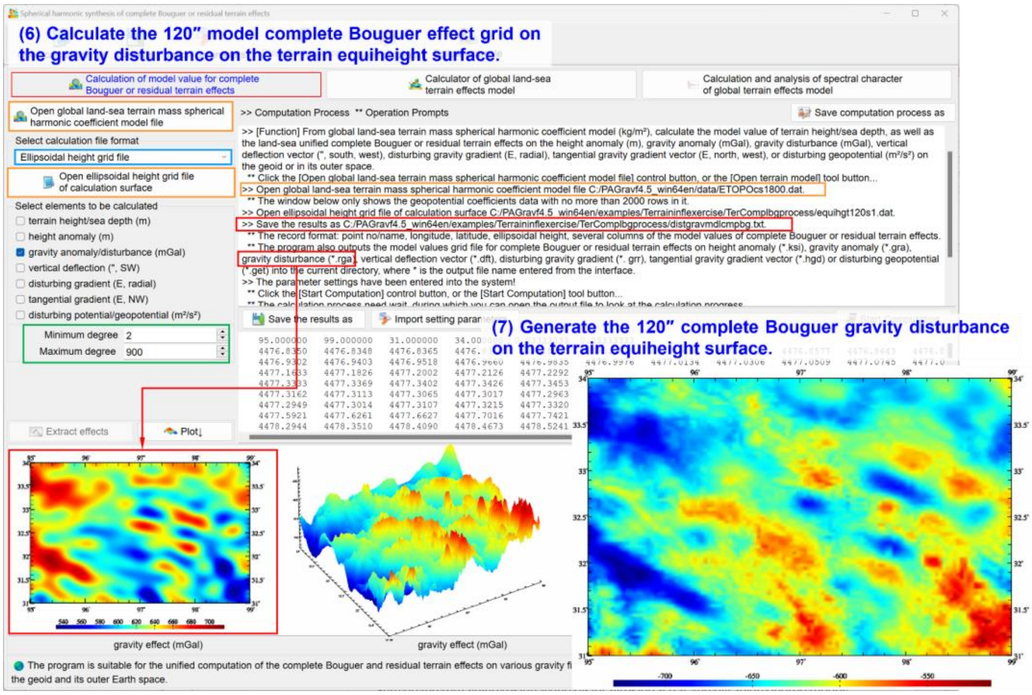
(6) Calculate the 120″ model complete Bouguer effect grid on the gravity disturbance on the terrain equiheight surface.

Call the function [Calculation of model value for complete Bouguer or residual terrain effects] with the minimum degree 2 and the maximum degree 900, select the type ‘gravity disturbance’ input the land-sea terrain mass spherical harmonic coefficient model file ETOPOcs1800.dat and 120″ ground ellipsoidal height grid file surfhgt120s1.dat of the terrain equiheight surface and, and then generate the 120″ model complete Bouguer effect grid file distgravmdlcmpbg.gra on the gravity disturbance on the terrain equiheight surface.

Here, the terrain mass spherical harmonic coefficient model and the maximum degree are required to be the same as in step (1).



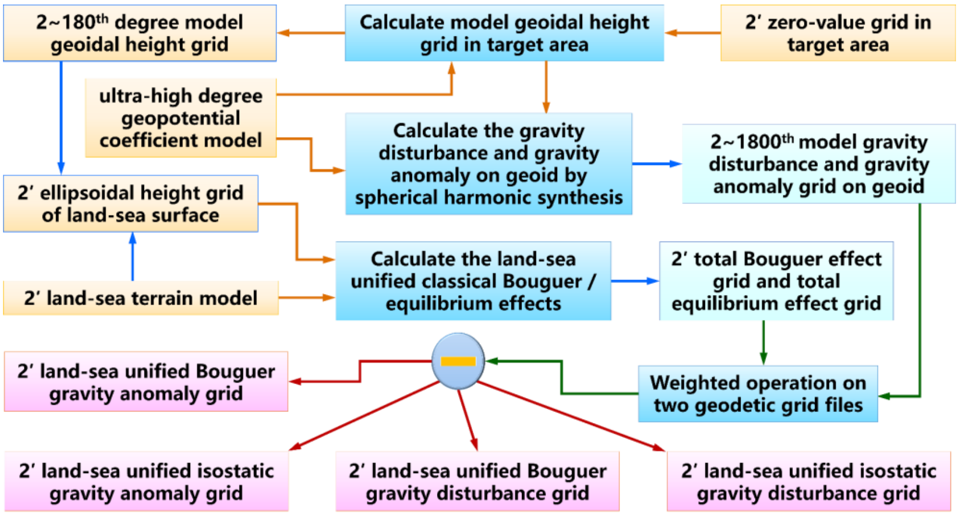
(7) Generate the 120″ complete Bouguer gravity disturbance on the terrain equiheight surface.



Sum up the remaining residual grid distgravresidual.dat, the ultrahigh degree model value grid distgravmdl.dat, the residual terrain effect grid distgravresidtm.dat and the model complete Bouguer effect grid distgravmdlcmpbg.gra four grid files of the gravity disturbance with the same grid specifications, to generate the 120″ complete Bouguer gravity disturbance grid distgravcmpbg.dat on the terrain equiheight surface.

3.8.2 Computation process demo of land-sea Bouguer / equilibrium anomaly from geopotential model

From the Earth geopotential coefficient model and land-sea topographic relief model, the classical Bouguer gravity anomaly (disturbance) and isostatic gravity anomaly (disturbance) are calculated synchronously in four steps in any region of the world to demonstrate the fast and convenient computation process of the land-sea unified classical Bouguer / isostatic anomaly.



Computation process demo of land-sea Bouguer/equilibrium anomaly from geopotential model

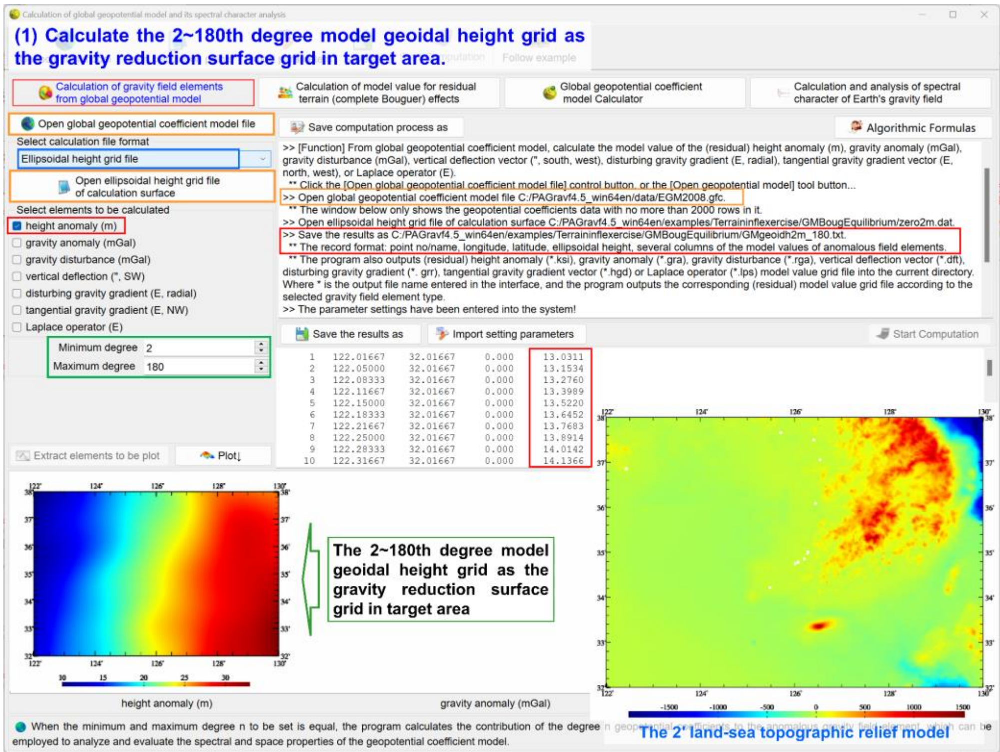
(1) Calculate the 2~180th degree model geoidal height grid as the gravity reduction surface grid in target area.

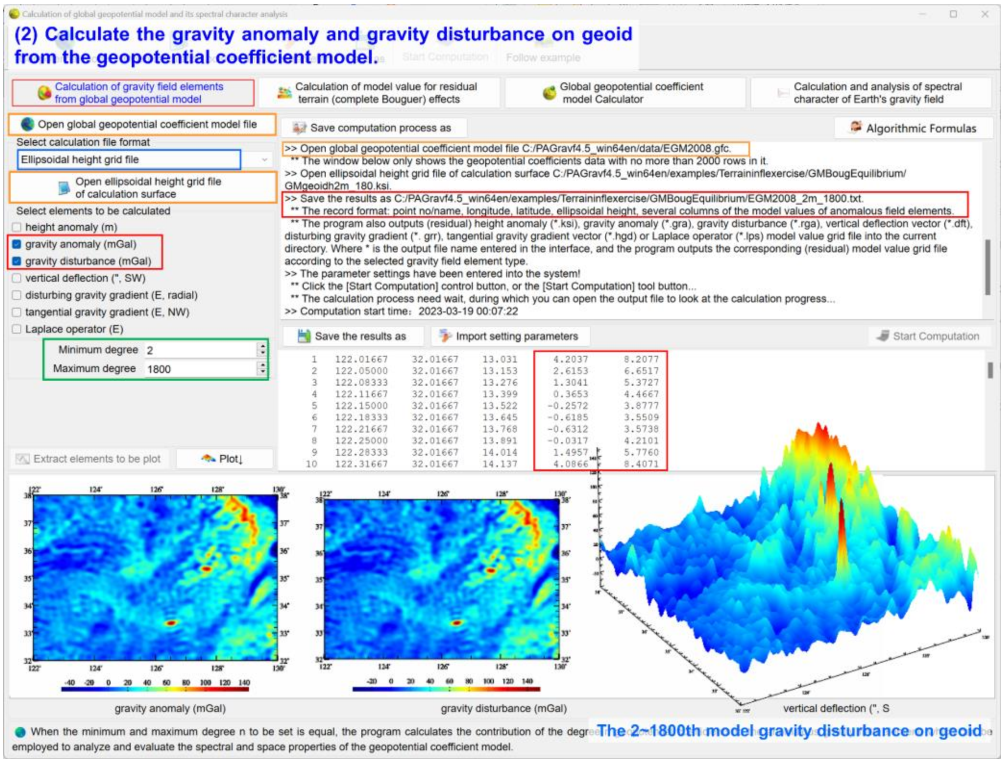
Call the function [Calculation of gravity field elements from global geopotential model] with the minimum degree 2 and the maximum degree 180, input the file EGM2008.gfc and the zero-value grid file zero2m.dat of the target area, and select the calculation type ‘height anomaly’, to generate 2′×2′ model geoidal height grid file GMgeoidh2m_180.ksi.

The 2~180th degree model geoidal height grid here is employed as the reduction surface and location for the classical Bouguer / equilibrium anomaly.

(2) Calculate the gravity anomaly and gravity disturbance on geoid from the geopotential coefficient model.

Call the function [Calculation of gravity field elements from global geopotential model] with the minimum degree 2 and the maximum degree 1800, input the file EGM2008.gfc and model geoidal height grid file GMgeoidh2m_180.ksi, and select the calculation type ‘gravity anomaly’ and ‘gravity disturbance’, to generate the 2′×2′ gravity anomaly grid file EGM2008_2m_1800.gra and gravity disturbance grid file EGM2008_2m_1800.rga in the target area.

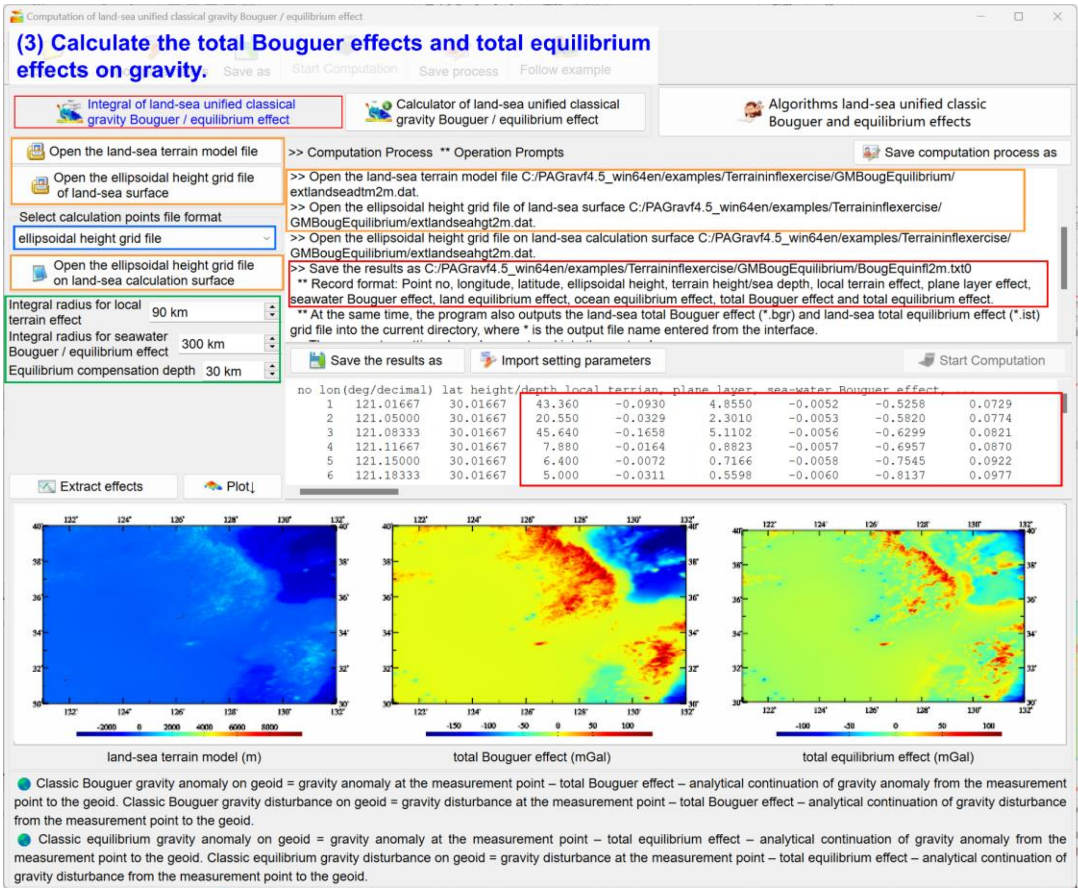




(3) Calculate the total Bouguer effects and total equilibrium effects on gravity.

Call the function [Computation of land-sea unified classical gravity Bouguer / equilibrium effect], input the land-sea topographic relief model file extlandseadtm2m.dat and land-sea surface ellipsoidal height grid file extlandseahgt2m.dat, and set the land integral radius 90km, sea integral radius 200km and equilibrium compensation depth 30km. to generate the 2′×2′ total Bouguer effect grid file BougEquinfl2m.bgr and total equilibrium effect grid file BougEquinfl2m.ist.

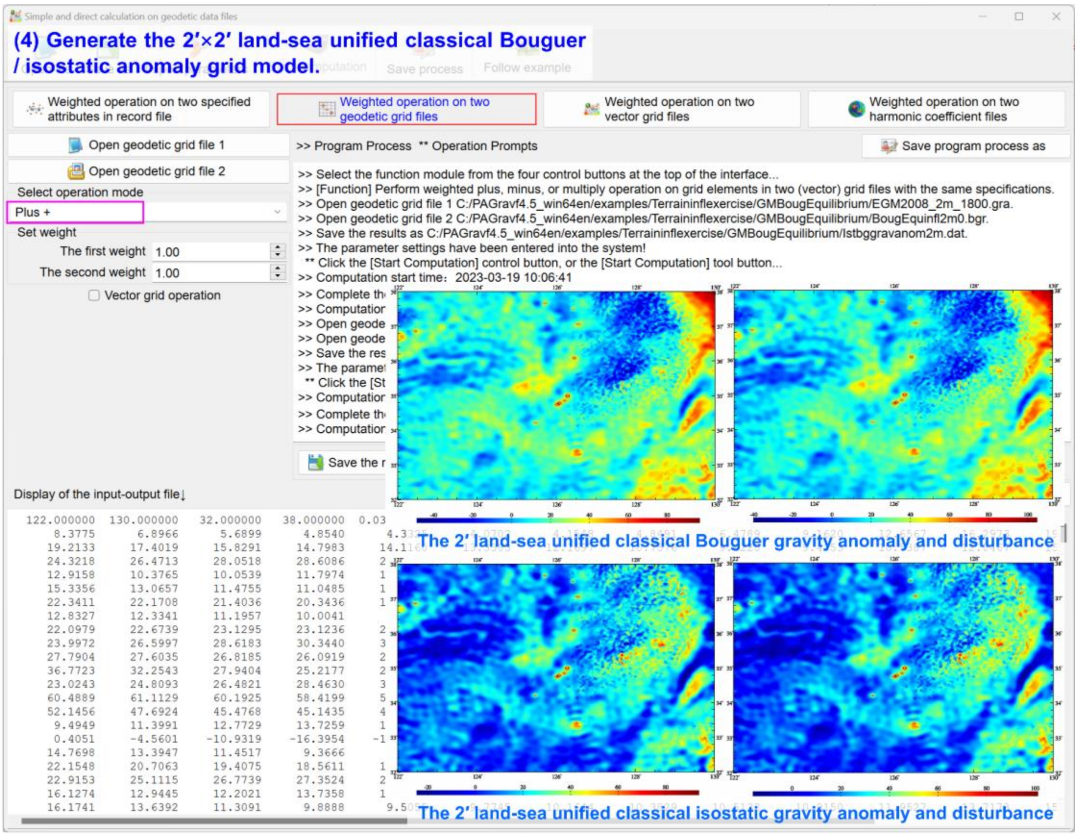
Because the normal gravity field has nothing to do with the terrain effect, the Bouguer / equilibrium effect on the gravity anomaly, gravity disturbance and gravity is equal everywhere and does not need to be distinguished.



(4) Generate the 2′×2′ land-sea unified classical Bouguer / isostatic anomaly grid model.

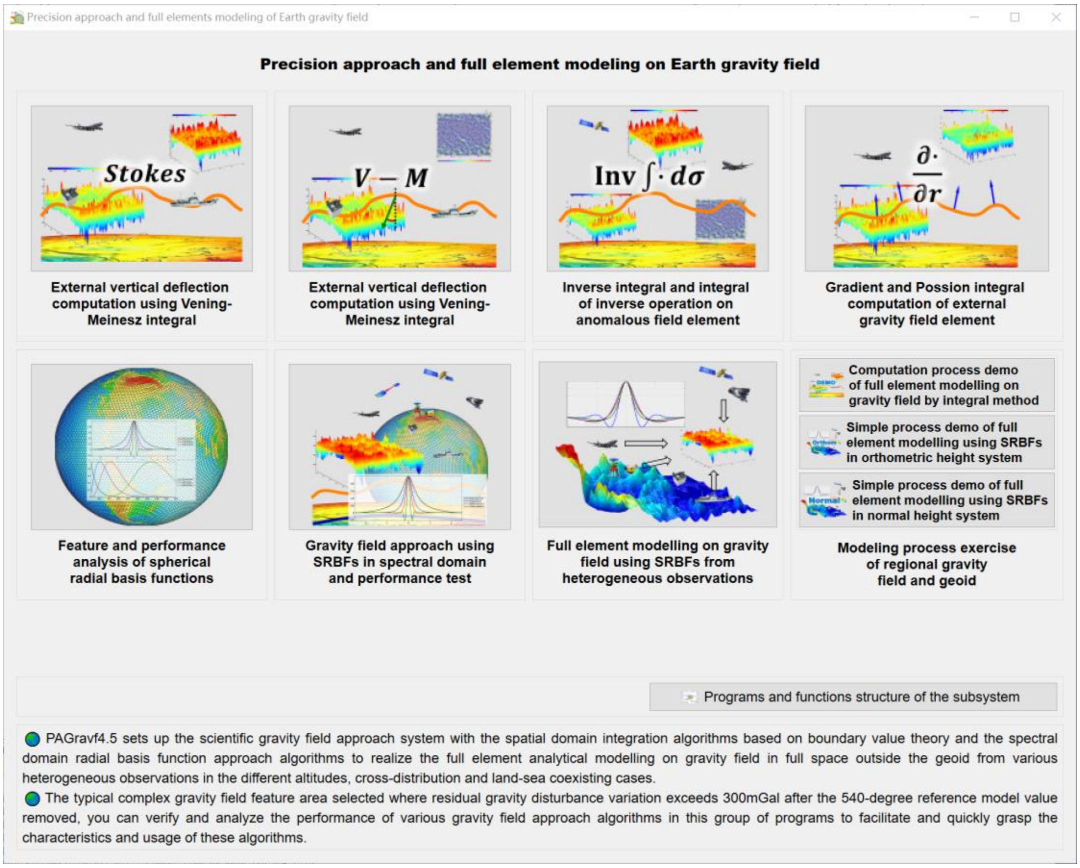
Subtract the gravity anomaly grid EGM2008_2m_1800.gra and gravity disturbance grid EGM2008_2m_1800.rga on geoid from the total Bouguer effect grid (the grid edge removed) BougEquinfl2m0.bgr respectively to get the classical Bouguer gravity anomaly grid model Clsbggravanom2m.dat and classical Bouguer gravity disturbance grid model Clsbgdistgrav2m.dat.

Subtract the gravity anomaly grid EGM2008_2m_1800.gra and gravity disturbance grid EGM2008_2m_1800.rga on geoid from the total isostatic effect grid (the grid edge removed) BougEquinfl2m0.ist respectively to get the classical isostatic gravity anomaly grid model Istbggravanom2m.dat and classical isostatic gravity disturbance grid model Istbgdistgrav2m.dat.



4 Precision approach and full element modelling on Earth gravity field

PAGravf4.5 sets up the scientific gravity field approach system with the spatial domain integration algorithms based on boundary value theory and spectral domain radial basis function approach algorithms to realize the full element analytical modelling on gravity field in full space outside the geoid from various heterogeneous observations in the different altitudes, cross-distribution and land-sea coexisting cases.



The typical complex gravity field feature area selected where residual gravity disturbance variation exceeds 300mGal after the 540-degree reference model value removed, you can verify and analyze the performance of various gravity field approach algorithms in this group of programs to facilitate and quickly grasp the characteristics and usage of these algorithms.

4.1 External height anomaly computation using Stokes/Hotine integral

[Purpose] Using the generalized Stokes/Hotine rigorous numerical integral or FFT algorithm, from the ellipsoidal height grid of the equipotential surface and gravity anomaly or disturbance (mGal) grid on the surface, compute the height anomaly (m) on the geoid or in its outer space.

Height anomaly on the geoid is equal to the geoid undulation, that is, the geoidal (ellipsoidal) height.

The Stokes boundary value theory requires that the boundary surface should be an equipotential surface, that is, the gravity anomaly/disturbance should be on the equipotential surface.

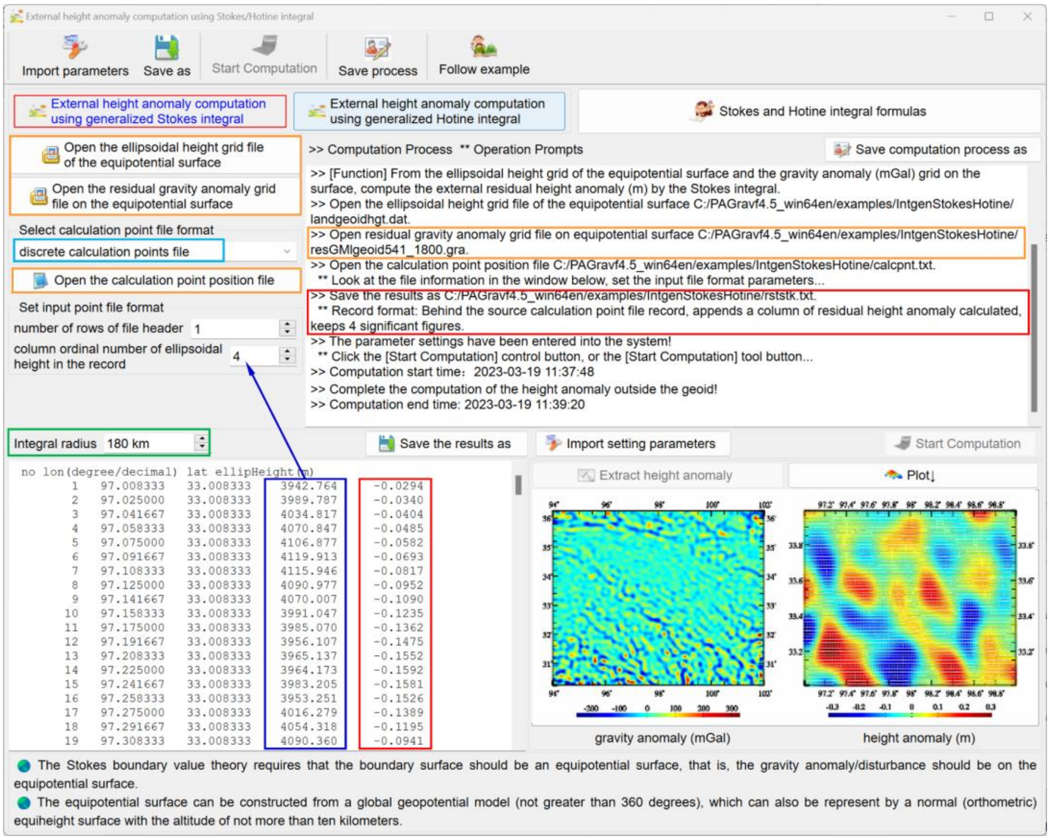
It is usually necessary to employ the remove-restore scheme with a reference geopotential model to use the finite radius for gravity field integral. Firstly, remove model gravity anomaly/disturbance on the boundary surface, then integrate to obtain the residual height anomaly at the calculation point, and finally restore the model height anomaly at the calculation point.

The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represent by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

4.1.1 External height anomaly computation using generalized Stokes integral

[Function] From the ellipsoidal height grid of the equipotential surface and gravity anomaly (mGal) grid on the surface, compute the external residual height anomaly (m) by the Stokes integral.

[Input files] The ellipsoidal height grid file of the equipotential surface and the residual gravity anomaly grid file on the surface with the same grid specifications, and the calculation point position file or the ellipsoidal height grid file of the calculation surface.



The ellipsoidal height grid of the equipotential surface stands for the space position of the equipotential surface, employed to calculate the integral distance.

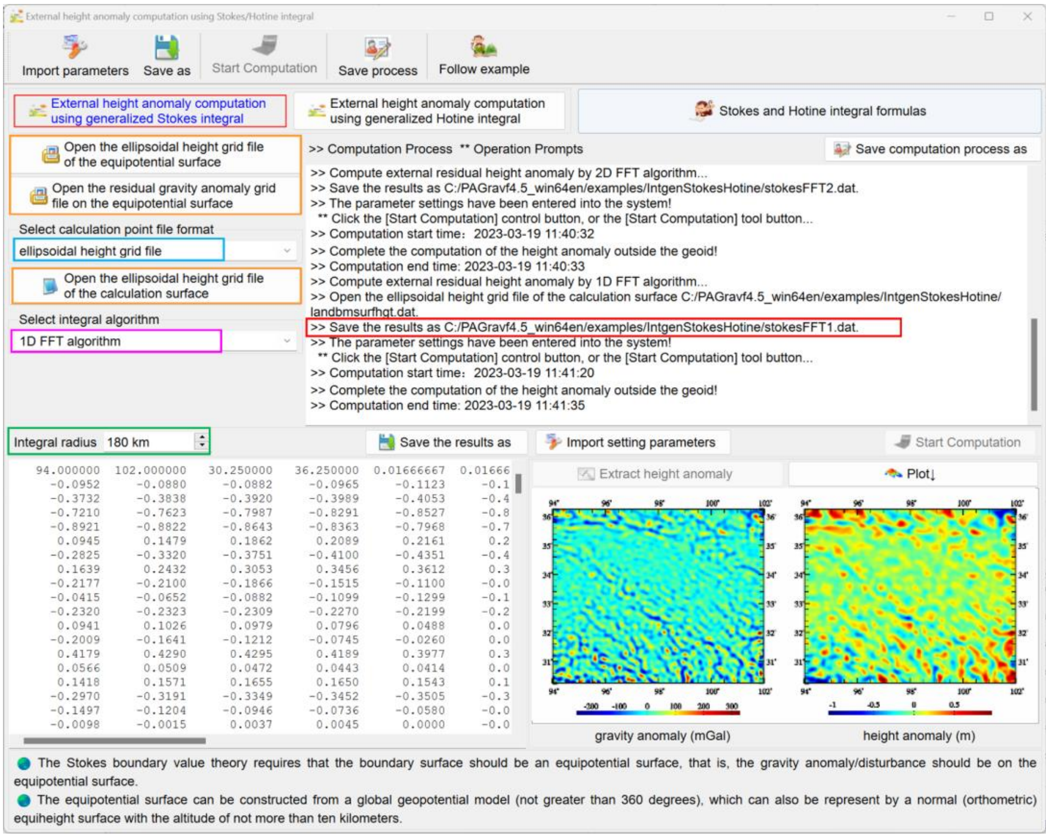
The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude, ellipsoidal height (m), ...

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

[Output file] The residual height anomaly result file.

When the discrete calculation points file input, the output file record format: Behind the source calculation point file record, appends a column of residual height anomaly calculated, keeps 4 significant figures.

When the ellipsoidal height grid file of the calculation surface input, the program outputs the residual height anomaly grid file with the same grid specifications as the input grid file.



In this example, the residual ground height anomaly is calculated by the Stokes integral from the residual gravity anomaly on the geoid, and the integral radius is 180km. After the 2° area of the grid margin with integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual height anomalies (regarded as the true reference value), and the difference between the results of the Stokes integral and model reference value, which can be employed to examine the performance of the algorithm.

The true reference model value (m)	mean	standard deviation	minimum	maximum
541~1800th degree ground height anomalies	0.0010	0.1182	-0.6745	0.4760

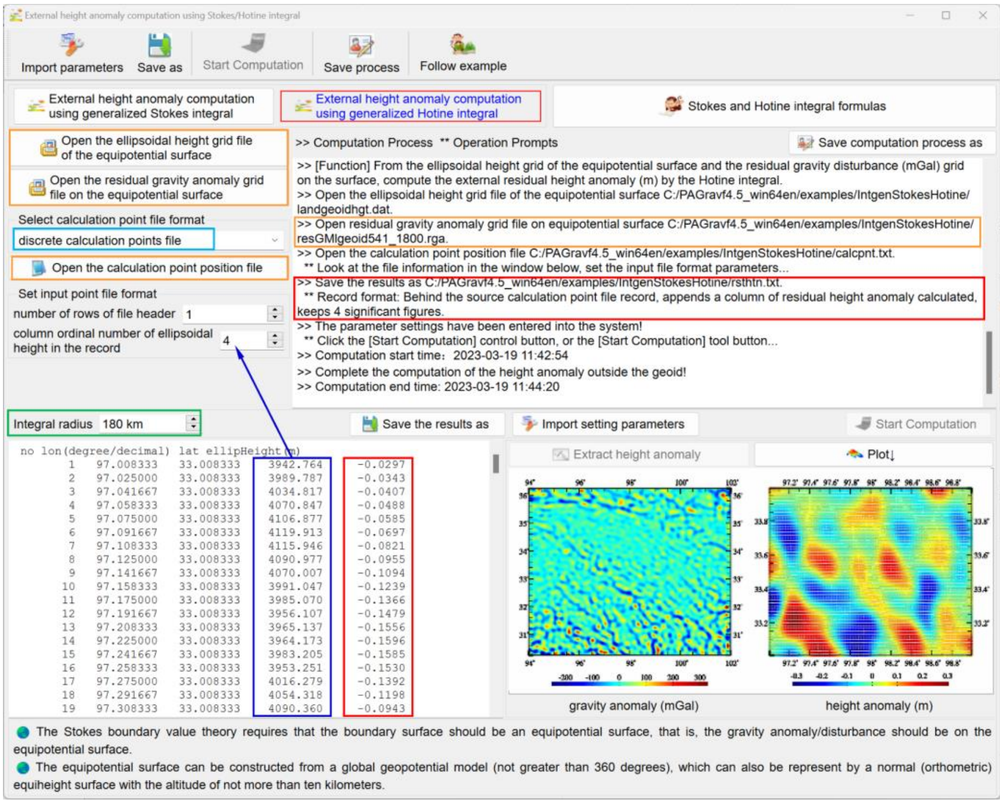
Differences between Integral and reference value(m)	mean	standard deviation	minimum	maximum
numerical integral	-0.0002	0.0324	-0.1159	0.1211
FFT2	0.0018	0.0326	-0.1150	0.1280
FFT1	0.0018	0.0327	-0.1178	0.1251

Furtherly, statistically analyze the differences between the results of Stokes numerical integral and fast FFT algorithm.

Differences between algorithms (m)	mean	standard deviation	minimum	maximum
FFT1- numerical integral	0.0021	0.0059	-0.0150	0.0683
FFT2- FFT1	0.0021	0.0059	-0.0150	0.0683

4.1.2 External height anomaly computation using generalized Hotine integral

[Function] From the ellipsoidal height grid of the equipotential surface and residual gravity disturbance (mGal) grid on the surface, compute the external residual height anomaly (m) by the Hotine integral.



[Input files] The ellipsoidal height grid file of the equipotential surface and the residual gravity disturbance grid file on the surface with the same grid specifications, and the calculation point position file or the ellipsoidal height grid file of the calculation surface.

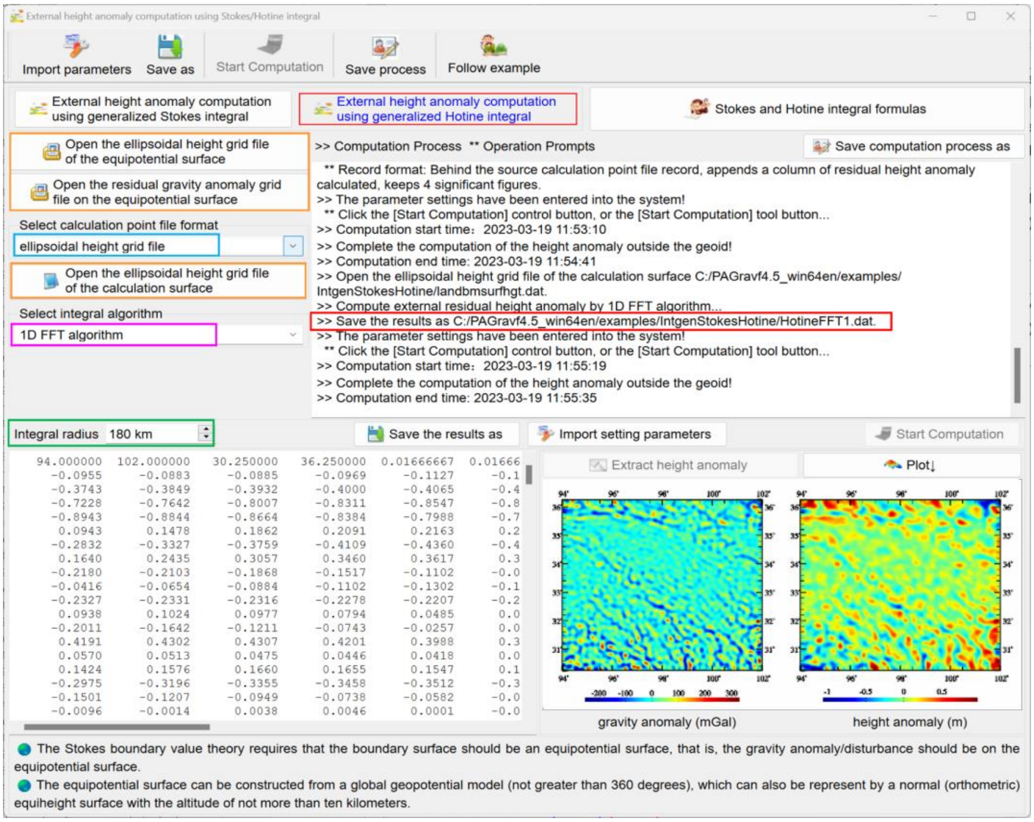
The ellipsoidal height grid of the equipotential surface stands for the space position of the equipotential surface, employed to calculate the integral distance.

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

[Output file] The residual height anomaly result file.

When the discrete calculation points file input, the output file record format: Behind the source calculation point file record, appends a column of residual height anomaly calculated, keeps 4 significant figures.

When the ellipsoidal height grid file of the calculation surface input, the program outputs the residual height anomaly grid file with the same grid specifications as the input grid file.



As the Stokes integral algorithm analysis example, statistically analyze the difference between the results of the Hotine integral and model reference value, which can be employed to examine the performance of the algorithm.

Differences between Integral and reference value(m)	mean	standard deviation	minimum	maximum
numerical integral	-0.0001	0.0256	-0.0915	0.0957

FFT2	0.0019	0.0259	-0.0911	0.1065
FFT1	0.0018	0.0261	-0.0934	0.1036

Furtherly, statistically analyze the differences between the results of the Stokes and Hotine integral.

Stokes-Hotine(m)	mean	standard deviation	minimum	maximum
numerical integral	0.0020	0.0090	-0.0264	0.0658
FFT2	-0.0000	0.0067	-0.0239	0.0258
FFT1	-0.0000	0.0068	-0.0245	0.0250

This example shows that the performance of the Hotine integral is slightly better than that of the Stokes integral.

4.2 External vertical deflection computation using Vening-Meinesz integral

[Purpose] Using the generalized Vening-Meinesz rigorous numerical integral or FFT algorithm, compute the vertical deflection (″, SW, to south, to west) on the geoid or its outer space from the ellipsoidal height grid of the equipotential surface and its gravity anomaly or disturbance (mGal) grid.

The generalized Vening-Meinesz formula is derived from the generalized Stokes/Hotine formula and belongs to the solution of the Stokes boundary value problem. Which requires the integrand gravity anomaly/disturbance to be on the equipotential surface.

4.2.1 Computation of external vertical deflection from gravity anomaly

[Function] From the ellipsoidal height grid of the equipotential surface and residual gravity anomaly (mGal) grid on the surface, compute the external residual vertical deflection (″, SW, to south, to west) by the generalized Vening-Meinesz integral.

[Input files] The ellipsoidal height grid file of the equipotential surface and the residual gravity anomaly grid file on the surface with the same grid specifications, and the calculation point position file or the ellipsoidal height grid file of the calculation surface.

The ellipsoidal height grid of the equipotential surface stands for the space position of the equipotential surface, employed to calculate the integral distance.

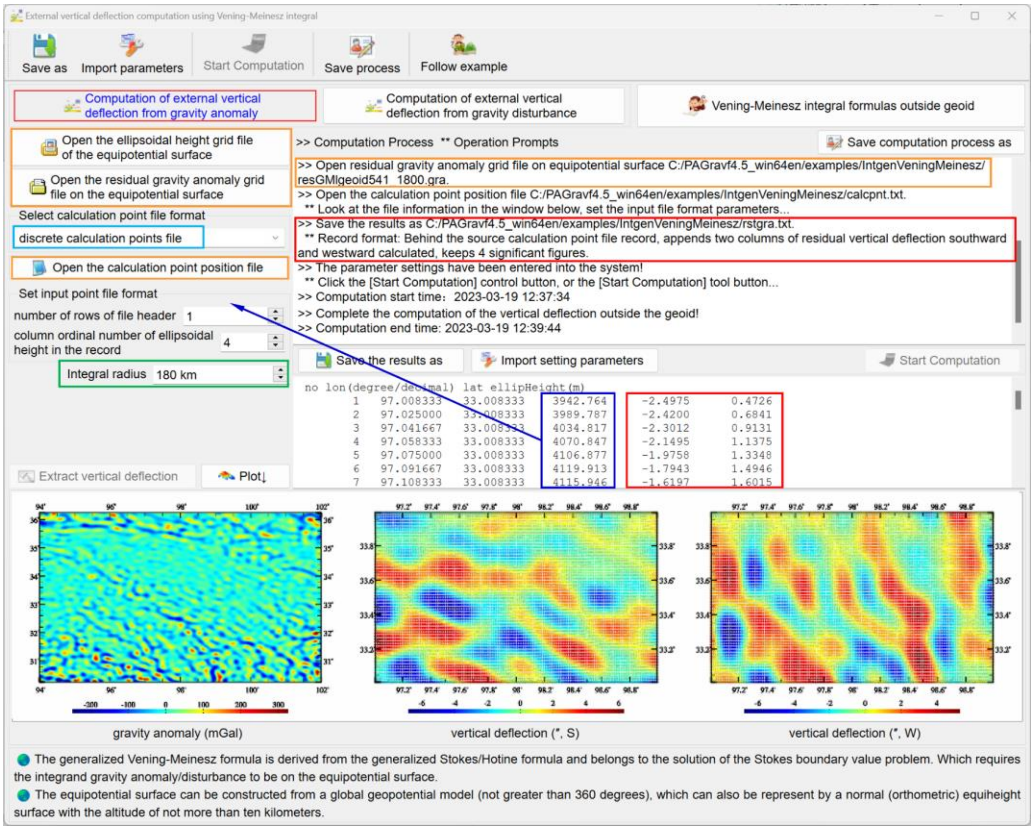
The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude, ellipsoidal height (m), ...

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

[Output file] The residual vertical deflection result file.

When the discrete calculation points file input, the output file record format Behind the source calculation point file record, appends two columns of residual vertical deflection southward and westward calculated, keeps 4 significant figures.

When the ellipsoidal height grid file of the calculation surface input, the program outputs the residual vertical deflection vector grid file with the same grid specifications as the input grid file.

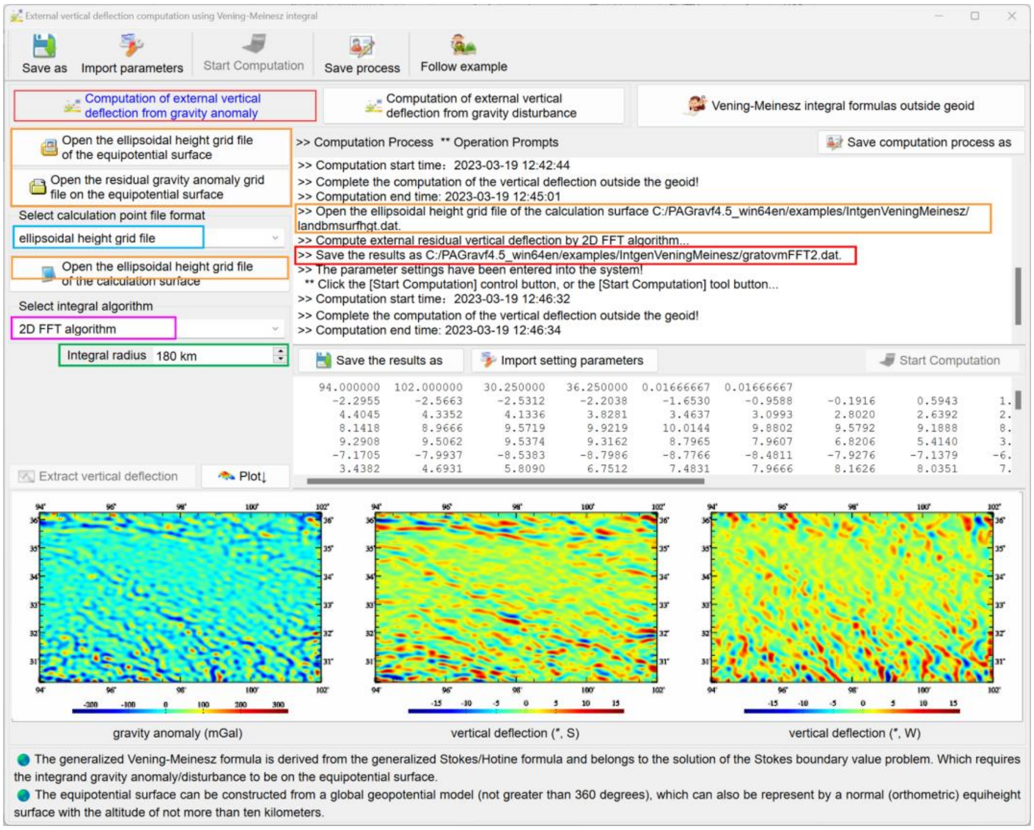


In this example, the residual ground vertical deflection is calculated by the generalized Vening-Meinesz integral from the residual gravity anomaly on the geoid, and the integral radius is 180km. After the 2° area of the grid margin with integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual vertical deflections (regarded as the true reference value), and the difference between the results of the Vening-Meinesz integral and model reference value, which can be employed to examine the performance of the algorithm.

The true reference model value (″)	mean	standard deviation	minimum	maximum
Ground vertical deflection (S)	0.0014	2.4951	-12.7789	14.2346
Ground vertical deflection (W)	0.0097	2.1772	-9.1577	10.4499

Differences between Integral and reference value(m)		mean	standard deviation	minimum	maximum
Numerical integral	S″	0.0003	0.0380	-0.1061	0.1387
	W″	0.0011	0.0289	-0.0830	0.1091
FFT2	S″	0.0002	0.1107	-0.8974	0.7395

	W″	0.0011	0.1003	-0.6887	1.0743
FFT1	S″	0.0007	0.1090	-0.8189	0.6581
	W″	0.0011	0.0984	-0.6078	1.0866



4.2.2 Computation of external vertical deflection from gravity disturbance

[Function] From the ellipsoidal height grid of the equipotential surface and residual gravity disturbance (mGal) grid on the surface, compute the external residual vertical deflection (″, SW, to south, to west) by the generalized Vening-Meinesz integral.

[Input files] The ellipsoidal height grid file of the equipotential surface and the residual gravity disturbance grid file on the surface with the same grid specifications, and the calculation point position file or the ellipsoidal height grid file of the calculation surface.

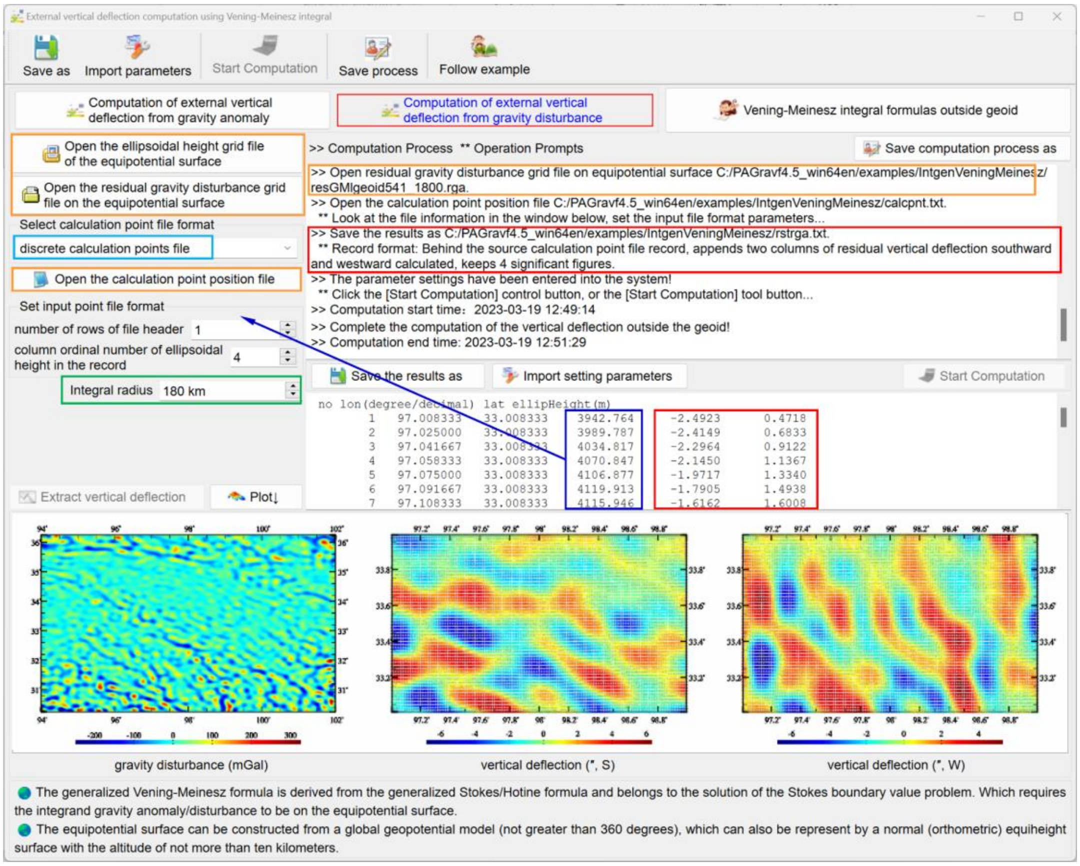
The ellipsoidal height grid of the equipotential surface stands for the space position of the equipotential surface, employed to calculate the integral distance.

The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude, ellipsoidal height (m), ...

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

[Output file] The residual vertical deflection result file.

When the discrete calculation points file input, the output file record format Behind the source calculation point file record, appends two columns of residual vertical deflection southward and westward calculated, keeps 4 significant figures.

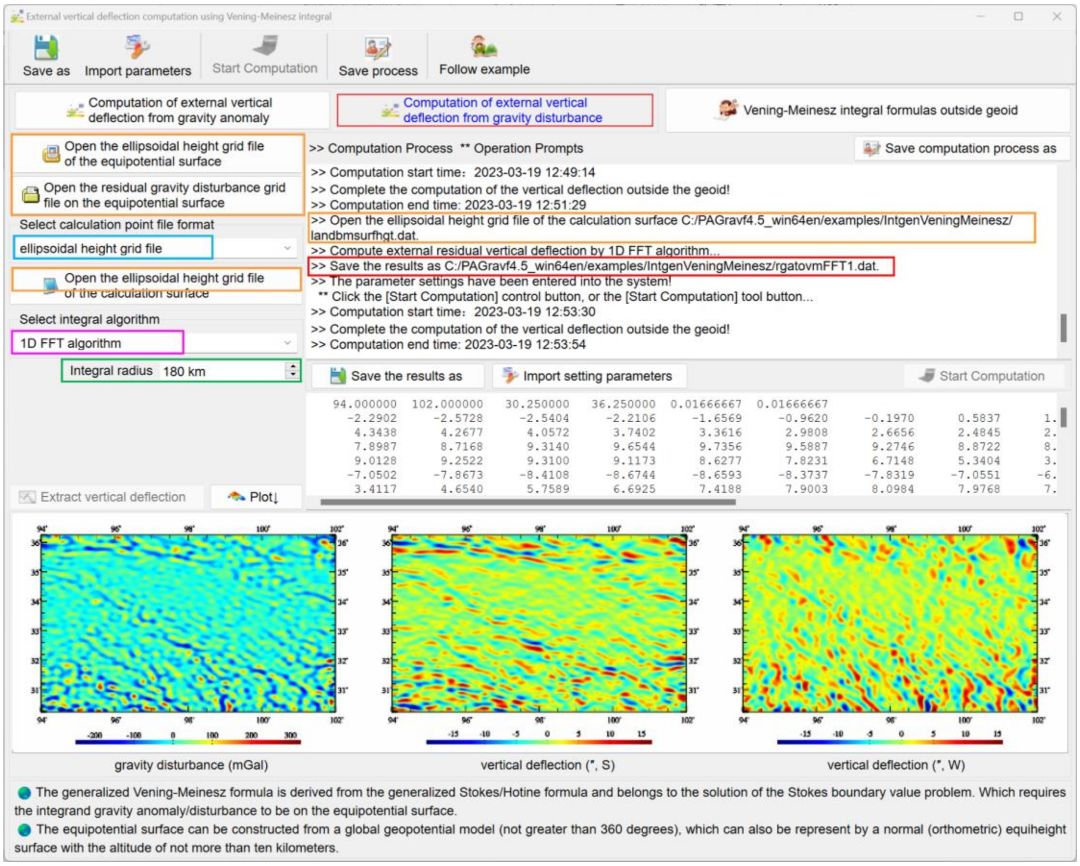


When the ellipsoidal height grid file of the calculation surface input, the program outputs the residual vertical deflection vector grid file with the same grid specifications as the input grid file.

Similarly, statistically analyze the difference between the results of integral and model reference value, which can be employed to examine the performance of the algorithm.

Differences between Integral and reference value		mean	standard deviation	minimum	maximum
Numerical integral	S″	-0.0003	0.0274	-0.0872	0.0994
	W″	0.0005	0.0200	-0.0646	0.0836
FFT2	S″	0.0002	0.1072	-0.8692	0.7094
	W″	0.0010	0.0959	-0.6204	1.0981
FFT1	S″	0.0007	0.1058	-0.7910	0.6481
	W″	0.0010	0.0959	-0.6204	1.0981

Furtherly, statistically analyze the differences between the results of Vening-Meinesz integral from gravity anomaly and gravity disturbance.



Differences from gravity anomaly and disturbance		mean	standard deviation	minimum	maximum
Numerical integral	S″	-0.0000	0.0062	-0.0286	0.0329
	W″	0.0003	0.0059	-0.0206	0.0261
FFT2	S″	-0.0000	0.0062	-0.0286	0.0329
	W″	0.0001	0.0050	-0.0189	0.0244
FFT1	S″	-0.0000	0.0062	-0.0284	0.0325
	W″	0.0001	0.0050	-0.0189	0.0241

4.3 Inverse integral and integral of inverse operation on anomalous field element

[Purpose] Using the inverse integral or integral of inverse operation method, compute the anomalous gravity field element on the geoid or in its outer space from the ellipsoidal height grid of the equipotential surface and height anomaly (m) or vertical deflection vector (″) grid on the surface.

The integral of inverse operation formula belongs to the solution of the Stokes boundary value problem, which requires the integrand height anomaly or vertical deflection to be on the equipotential surface.

The inverse integral adopts the combination algorithm with Poisson integral and differentiation of anomalous field element, which does not require that the boundary surface should be a gravity equipotential surface.

It is usually necessary to employ the remove-restore scheme with a reference geopotential model to use the finite radius for gravity field integral. Firstly, remove the model values of source field element on the boundary surface, then compute the residual values of target field element at the calculation point by inverse integral and integral of inverse operation, and finally restore the model values of target field element at the calculation point.

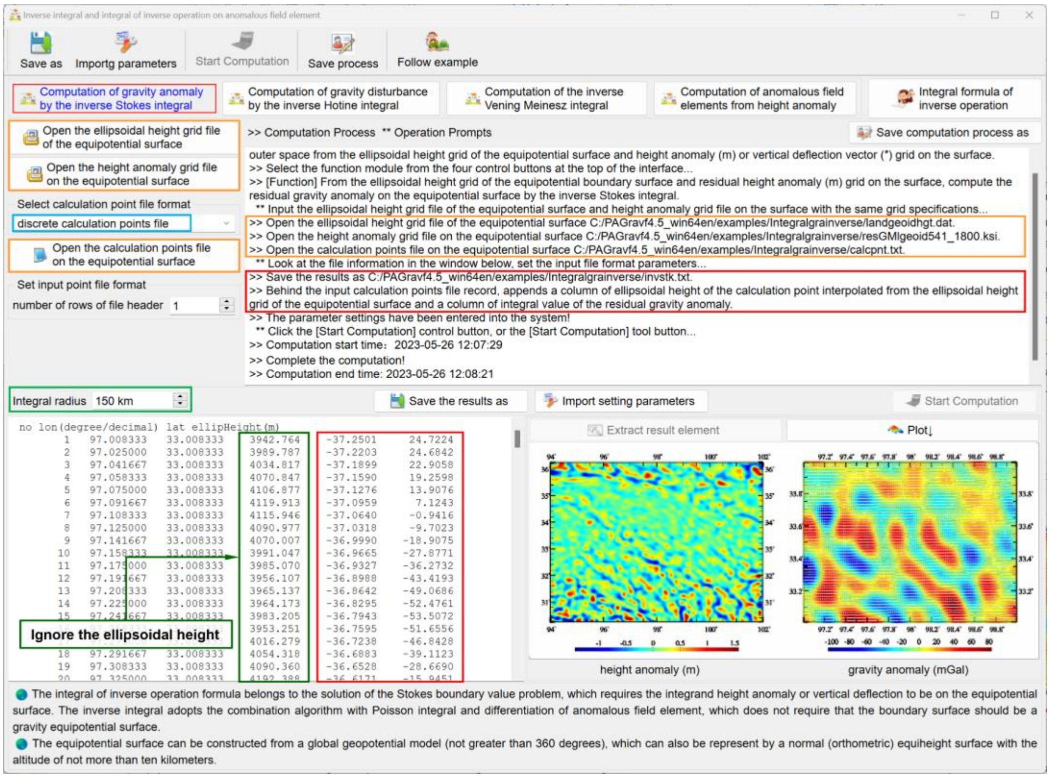
4.3.1 Computation of gravity anomaly by the inverse Stokes integral

[Function] From the ellipsoidal height grid of the equipotential boundary surface and residual height anomaly (m) grid on the surface, compute the residual gravity anomaly (mGal) on the equipotential surface by the inverse Stokes integral.

[Input files] The ellipsoidal height grid file of the equipotential surface and height anomaly grid file on the surface with the same grid specifications, and the calculation point position file on the surface.

The ellipsoidal height grid of the equipotential surface stands for the space position of the equipotential surface, which is employed to calculate the integral distance.

The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude, ...



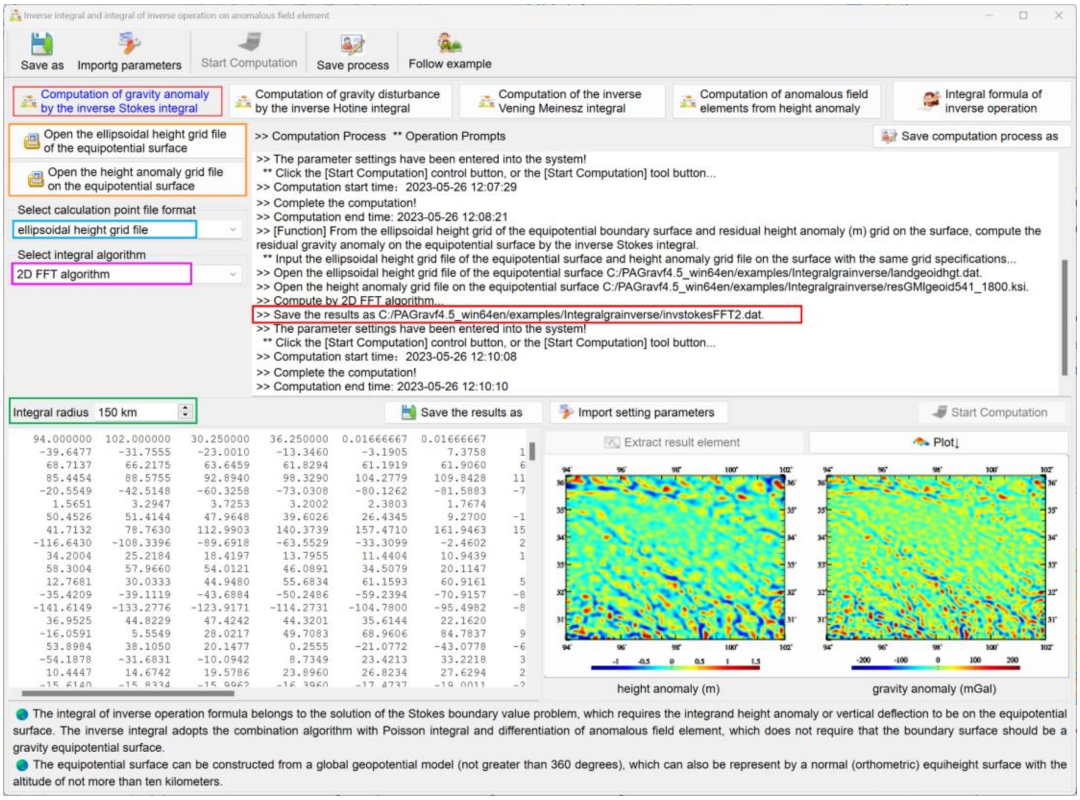
[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

When the ellipsoidal height grid file selected, the program let the ellipsoidal height grid of the equipotential surface as the calculation surface.

[Output file] The residual gravity anomaly result file.

When the discrete calculation points file input, the output file record format: Behind the input calculation points file record, appends a column of ellipsoidal height of the calculation point interpolated from the ellipsoidal height grid of the equipotential surface and a column of integral value of the residual gravity anomaly.

When calculated on the calculation surface, the program outputs the residual vertical deflection vector grid file with the same grid specifications as the input grid file.



In this example, the residual gravity anomaly on the geoid is calculated by the inverse Stokes integral from the residual height anomaly on the geoid, and the integral radius is 150km. After the 2° area of the grid margin with integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual gravity anomalies (regarded as the true reference value) on the geoid, and the differences between the results of the integral and model reference values, which can be used to examine the performance of the algorithm.

The true reference model value (mGal)	mean	standard deviation	minimum	maximum
541~1800th degree residual gravity anomalies	0.4334	34.0852	-170.9556	177.8780

Differences between Integral and reference value (mGal)	mean	standard deviation	minimum	maximum
Numerical integral	-0.0415	2.9285	-15.2734	14.6970
FFT2	-0.0433	2.8208	-11.9201	13.3607
FFT1	-0.0444	3.0509	-15.7950	15.5242

4.3.2 Computation of gravity disturbance by the inverse Hotine integral

[Function] From the ellipsoidal height grid of the equipotential boundary surface and residual height anomaly (m) grid on the surface, compute the residual gravity disturbance on the surface by the Hotine integral of inverse operation.

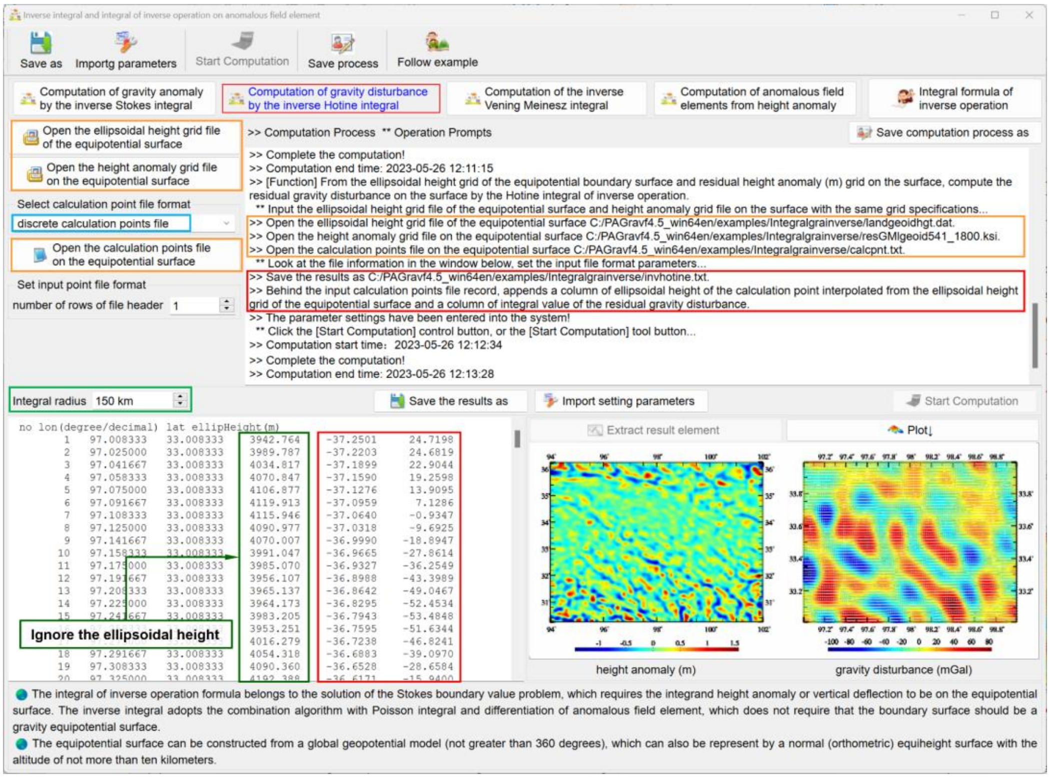
[Input files] The ellipsoidal height grid file of the equipotential surface and height anomaly grid file on the surface with the same grid specifications, and the calculation point position file on the surface.

The ellipsoidal height grid of the equipotential surface stands for the space position of the equipotential surface, which is employed to calculate the integral distance.

The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude, ...

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

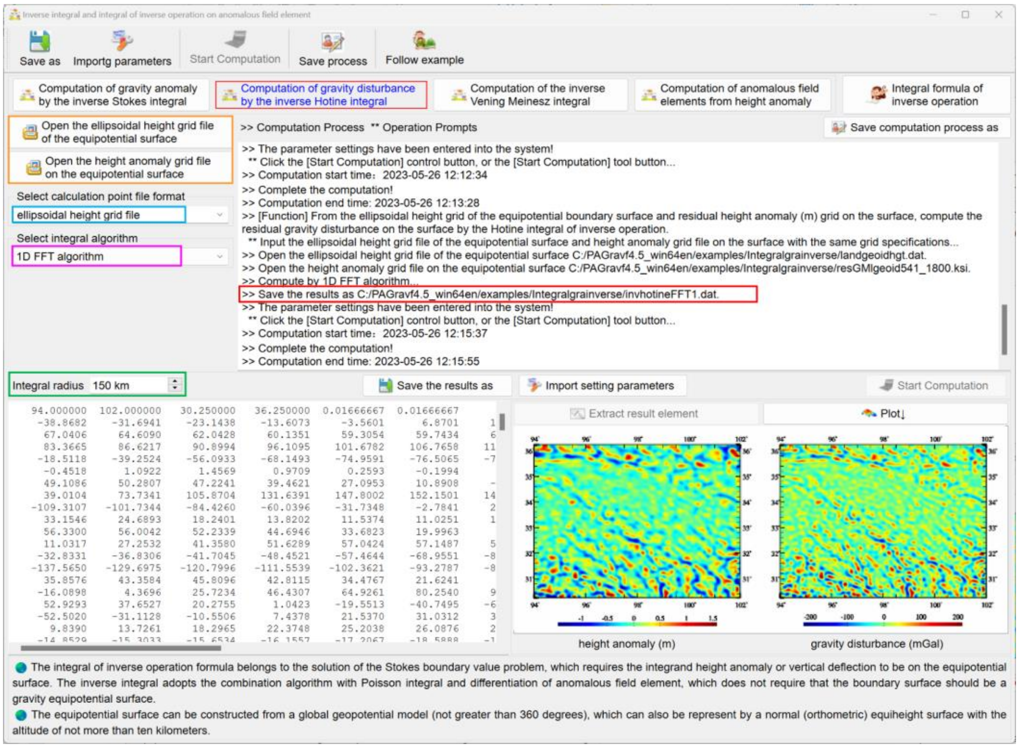
When the ellipsoidal height grid file selected, the program let the ellipsoidal height grid of the equipotential surface as the calculation surface.



[Output file] The residual gravity disturbance result file.

When the discrete calculation points file input, the output file record format: Behind the input calculation points file record, appends a column of ellipsoidal height of the calculation point interpolated from the ellipsoidal height grid of the equipotential surface and a column of integral value of the residual gravity disturbance.

When calculated on the calculation surface, the program outputs the residual vertical deflection vector grid file with the same grid specifications as the input grid file.



In this example, the residual gravity disturbance on the geoid is calculated by Hotine integral of inverse operation from the residual height anomaly on the geoid, and the integral radius is 150km. After the 2° area of the grid margin with integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual gravity disturbances (regarded as the true reference value) on the geoid, and the differences between the results of the integral and model reference values, which can be used to examine the performance of the algorithm.

The true reference model value (mGal)	mean	standard deviation	minimum	maximum
541~1800th degree residual gravity disturbances	0.4348	34.1479	-171.3088	178.1561

Differences between Integral and reference value (mGal)	mean	standard deviation	minimum	maximum
Numerical integral	0.1277	2.9277	-9.6849	9.6267
FFT2	-0.0451	2.8982	-12.2656	13.7990
FFT1	0.1326	2.9671	-9.8383	9.7441

4.3.3 Computation of the inverse Vening Meinesz integral

[Function] From the ellipsoidal height grid of the equipotential boundary surface and residual vertical deflection vector (″, SW) grid on the surface, compute the residual height anomaly, gravity anomaly and gravity disturbance on the surface by the inverse Vening Meinesz integral.

[Input files] The ellipsoidal height grid file of the equipotential surface and vertical deflection vector grid file on the surface with the same grid specifications, and the calculation point position file on the surface.

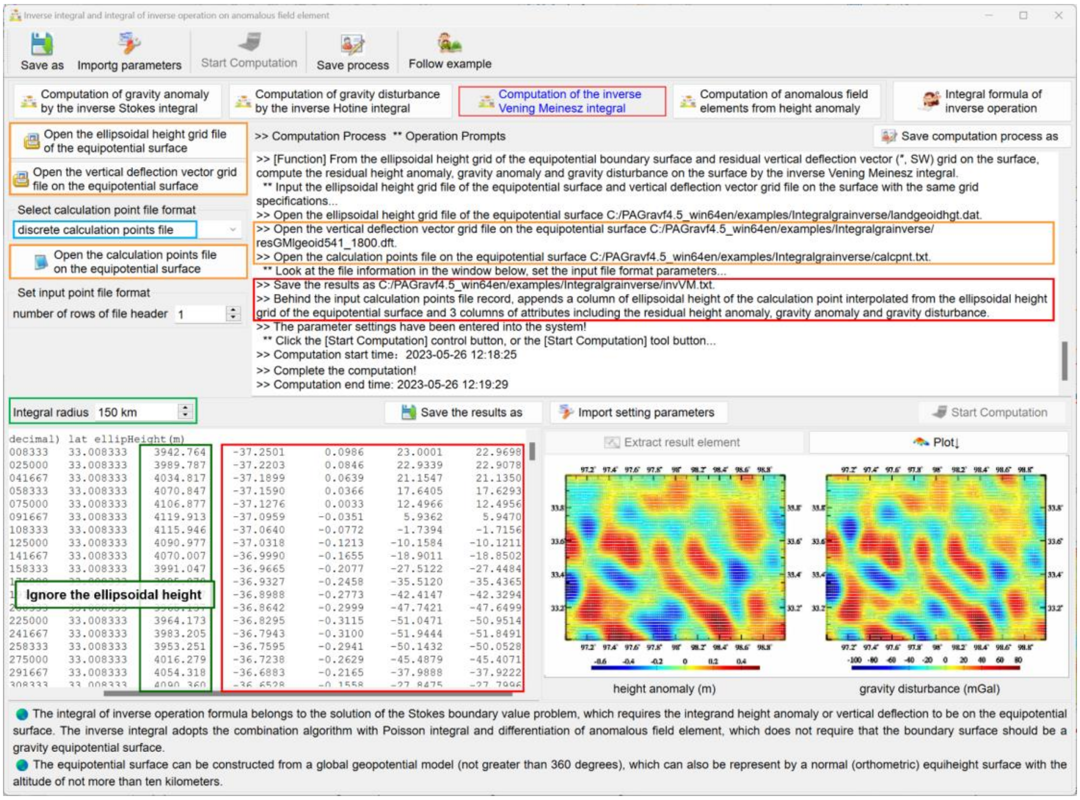
The ellipsoidal height grid of the equipotential surface stands for the space position of the equipotential surface, which is employed to calculate the integral distance.

The record format of the discrete calculation point file: point no / point name, longitude (decimal degrees), latitude, ...

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

When the ellipsoidal height grid file selected, the program let the ellipsoidal height grid of the equipotential surface as the calculation surface.

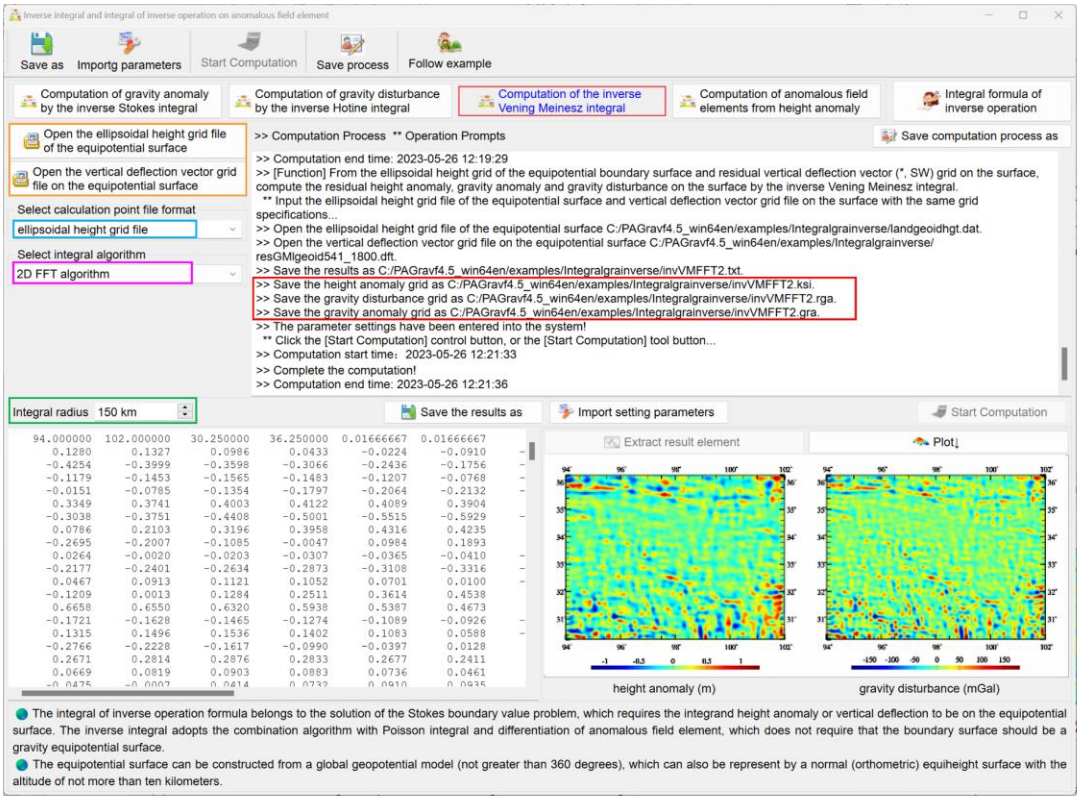
[Output file] The inverse Vening Meinesz integral result file.



When the discrete calculation point file input, the output file record format: Behind the input calculation points file record, appends a column of ellipsoidal height of the calculation point interpolated from the ellipsoidal height grid of the equipotential surface and 3 columns of attributes including the residual height anomaly, gravity anomaly and gravity disturbance.

When calculated on the calculation surface, the output file record format: point no/name, longitude, latitude, ellipsoidal height, residual height anomaly, gravity anomaly and gravity disturbance.

The program also outputs (residual) height anomaly (*.ksi), gravity disturbance (*.rga) and gravity anomaly (*.gra) grid file into the current directory, where * is the output file name entered from the interface.



In this example, the residual geoidal height, gravity anomaly and gravity disturbance on the geoid are calculated by the inverse Vening Meinesz integral from the residual vertical deflection vector on the geoid, and the integral radius is 150km. After the 2° area of the grid margin with integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual geoidal heights, gravity anomalies and gravity disturbances (regarded as the true reference value) on the geoid, and the differences between the results of the integral and model reference values, which can be used to examine the performance of the algorithm.

The true reference model value	mean	standard deviation	minimum	maximum
residual geoidal heights (m)	0.0045	0.2172	-1.1490	0.9110
residual gravity disturbances (mGal)	0.4348	34.1479	-171.3088	178.1561
residual gravity anomalies (mGal)	0.4334	34.0852	-170.9556	177.8780

Differences between Integral and reference value		mean	standard deviation	minimum	maximum
Numerical integral	residual geoidal height (m)	-0.0002	0.0331	-0.1066	0.1266
	residual gravity disturbances (mGal)	-0.0281	3.5304	-19.8306	15.5799
	residual gravity anomalies (mGal)	-0.0280	3.5293	-19.8233	15.5701
FFT2	residual geoidal height (m)	0.0001	0.0337	-0.1057	0.1271
	residual gravity disturbances (mGal)	-0.0131	2.4732	-12.3026	9.8919
	residual gravity anomalies (mGal)	-0.0131	2.4729	-12.2973	9.8831
FFT1	residual geoidal height (m)	-0.0001	0.0332	-0.1068	0.1263
	residual gravity disturbances (mGal)	-0.0283	3.5527	-19.9907	15.7164
	residual gravity anomalies (mGal)	-0.0283	3.5516	-19.9830	15.7072

4.3.4 Computation of external anomalous gravity field elements from height anomaly

[Function] From the ellipsoidal height grid of the boundary surface and residual height anomaly grid (m) on the surface, compute the residual gravity anomaly (mGal), gravity disturbance (mGal) and vertical deflection vector (″, SW) on the geoid or in its outer space. The inverse operation of height anomaly adopts the combination algorithm with Poisson integral and differentiation, which does not require that the boundary surface should be a gravity equipotential surface.

[Input files] The ellipsoidal height grid file of the equipotential surface and height anomaly grid file on the surface with the same grid specifications, and the calculation point position file or ellipsoidal height grid file of the calculation surface.

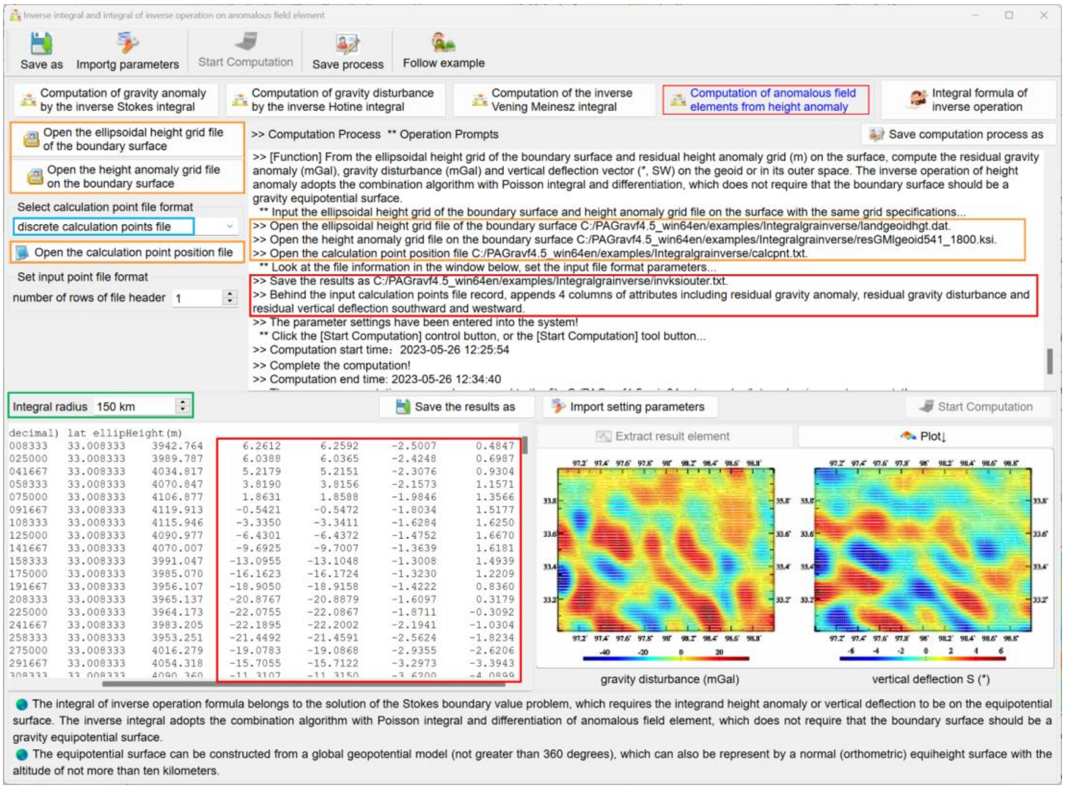
The ellipsoidal height grid of the equipotential surface stands for the space position of the equipotential surface, which employed to calculate the integral distance.

The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude, ellipsoidal height (m), ...

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

[Output file] The external anomalous field elements result file.

When the discrete calculation points file input, the output file record format: Behind the input calculation points file record, appends 4 columns of attributes including residual gravity anomaly, residual gravity disturbance and residual vertical deflection southward and westward.



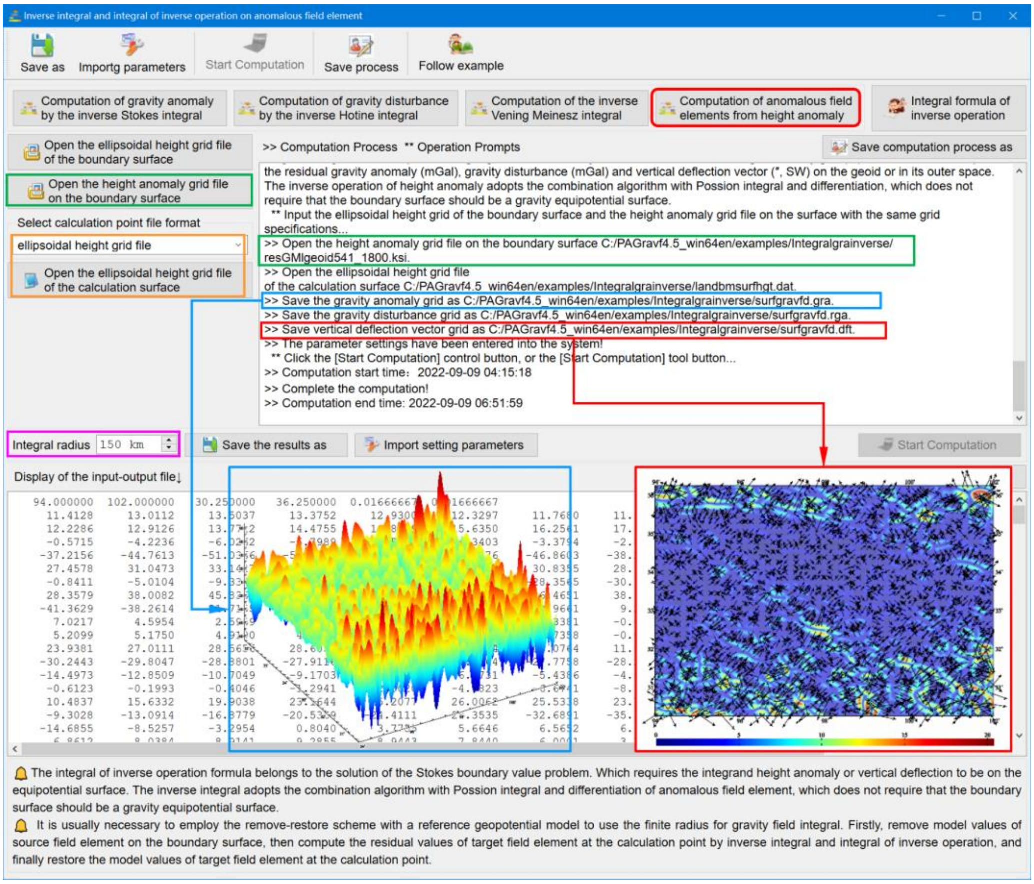
When the ellipsoidal height grid file input, the output file record format: point no/name, longitude, latitude, ellipsoidal height residual gravity anomaly, residual gravity disturbance and residual vertical deflection southward and westward.

The program also outputs (residual) gravity anomaly (*.gra), gravity disturbance (*.rga) and vertical deflection vector(*.dft) grid file into the current directory, where * is the output file name entered from the interface.

In this example, the ground residual gravity anomaly, gravity disturbance and vertical deflection are computed from the residual geoidal height, and the integral radius is 150km. After the 2° area of the grid margin with integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual gravity anomalies, residual gravity disturbances and residual vertical deflections (regarded as the true reference value) on the ground, and the differences between the results of the integral and model reference values, which can be employed to examine the performance of the algorithm.

The 541 to 1800th degree true reference model value	mean	standard deviation	minimum	maximum
Ground residual gravity anomalies (mGal)	-0.0349	15.7184	-93.7784	66.5507
Ground residual gravity disturbances (mGal)	-0.0346	15.7527	-93.9854	66.6638
Ground vertical deflection (″, S)	0.0014	2.4951	-12.7789	14.2346
Ground vertical deflection (″, W)	0.0097	2.1772	-9.1577	10.4499

Differences between Integral and reference value	mean	standard deviation	minimum	maximum
Ground residual gravity anomalies (mGal)	-0.0104	2.0577	-8.2097	10.7064
Ground residual gravity disturbances (mGal)	-0.0100	2.0929	-8.3217	10.8982
Ground vertical deflection (″, S)	0.0004	0.0075	-0.0357	0.0388
Ground vertical deflection (″, W)	0.0003	0.0076	-0.0362	0.0358



4.4 Gradient and Poisson integral computation of external gravity field element

[Purpose] Using rigorous numerical integral method, carry out the radial gradient integral, integral inverse, inverse operation integral and Poisson integral operation on the anomalous gravity field element.

The radial gradient integral algorithm of the anomalous field element is derived from the solution of the Stokes boundary value problem, which requires the integrand field elements to be on the equipotential surface. The Poisson integral is the solution of the first boundary value problem in the mathematical sense, and the boundary surface is not required to be an equipotential surface.

4.4.1 Operation of radial gradient integral on anomalous gravity field element

[Function] From the ellipsoidal height grid of the equipotential boundary surface and anomalous gravity field element grid on the surface, compute the radial gradient (/km) of the field element on the surface by the numerical integral.

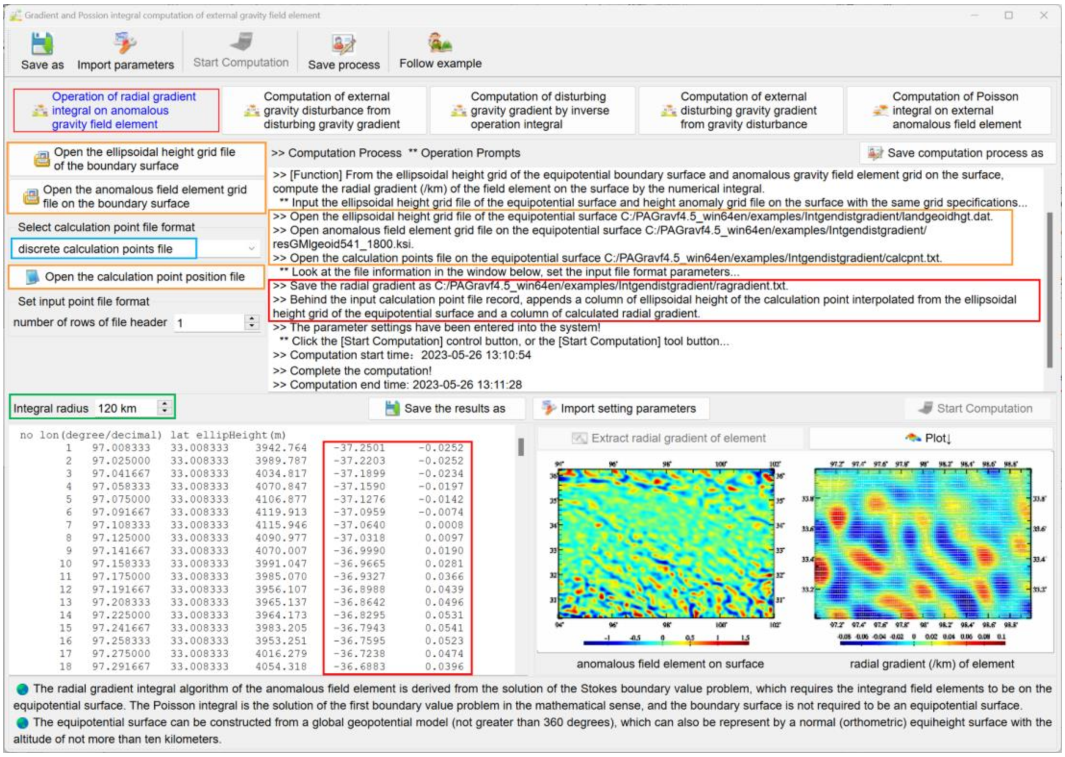
[Input files] The ellipsoidal height grid file of the equipotential surface and anomalous field element grid file on the surface with the same grid specifications, and the calculation point position file on the surface.

The ellipsoidal height grid of the equipotential surface stands for the space position of the equipotential surface, employed to calculate the integral distance.

The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude, ...

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

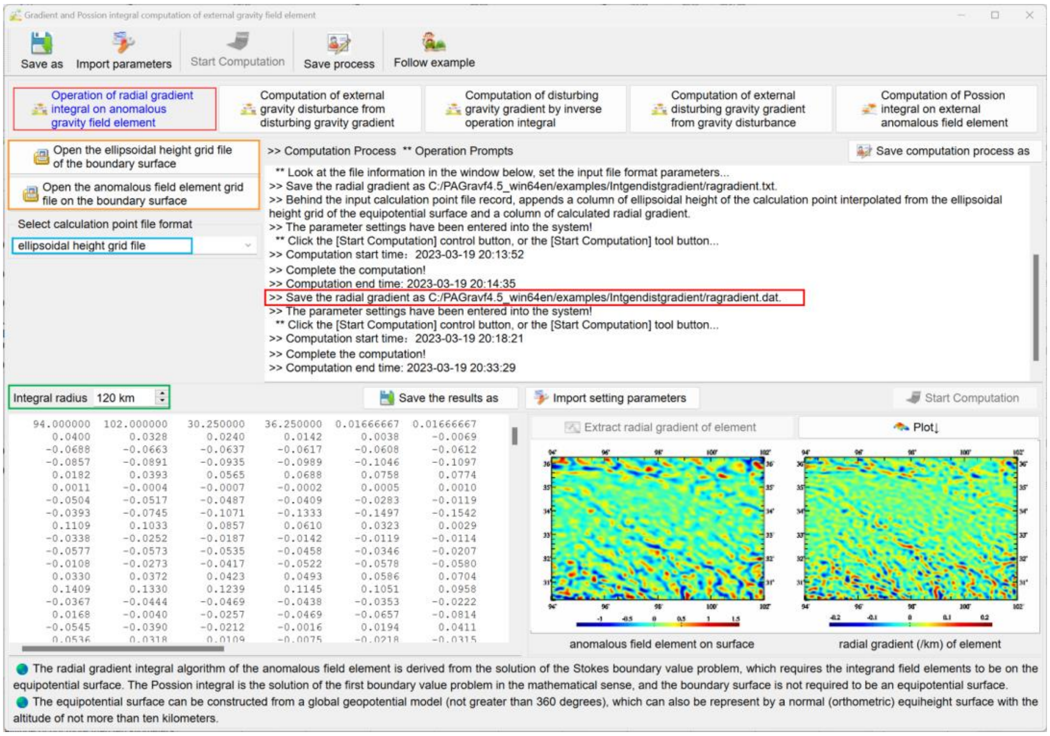
When the ellipsoidal height grid file selected, the program let the ellipsoidal height grid of the equipotential surface as the calculation surface.



[Output file] The radial gradient result file.

When the discrete calculation points file input, the output file record format: Behind the input calculation points file record, appends a column of ellipsoidal height of the calculation point interpolated from the ellipsoidal height grid of the equipotential surface and a column of calculated radial gradient.

When calculated on the calculation surface, the program outputs the radial gradient (/km) grid file with the same grid specifications as the input grid file.



4.4.2 Computation of external gravity disturbance from disturbing gravity gradient

[Function] From the ellipsoidal height grid of the equipotential boundary surface and residual disturbing gravity gradient (E, radial) grid on the surface, compute the residual gravity disturbance (mGal) on the geoid or in its outer space by the numerical integral.

[Input files] The ellipsoidal height grid file of the equipotential surface and residual disturbing gravity gradient grid file on the surface with the same grid specifications, and the calculation point position file or ellipsoidal height grid file of the calculation surface.

The ellipsoidal height grid of the equipotential surface stands for the space position of the equipotential surface, employed to calculate the integral distance.

The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude, ellipsoidal height (m), ...

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

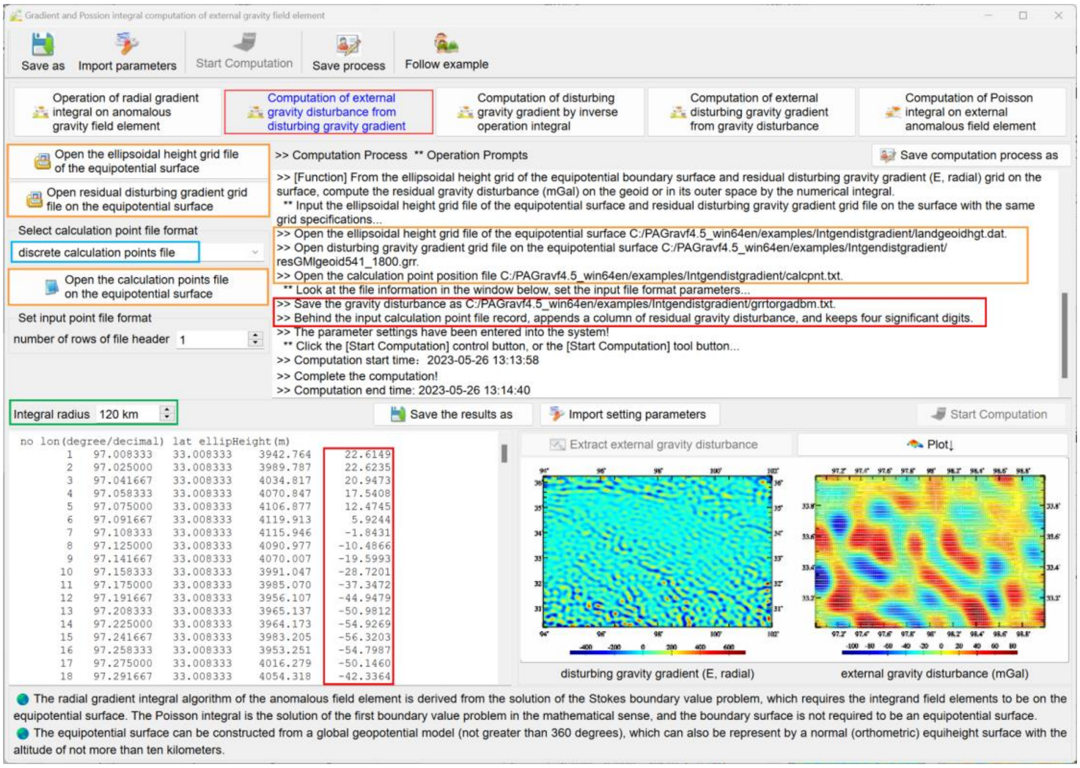
[Output file] The external residual gravity disturbance result file.

When the discrete calculation points file input, the output file record format: Behind the input calculation points file record, appends a column of residual gravity disturbance, and keeps four significant digits.

When the ellipsoidal height grid file input, the program outputs the residual gravity disturbance grid file with the same grid specifications as the input grid file.

In this example, the residual gravity disturbance on geoid is computed from the residual disturbing gravity gradient on geoid, and the integral radius is 120km. After the

2° area of the grid margin with integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual gravity disturbance (regarded as the true reference value) on the ground, and the differences between the results of the integral and model reference values, which can be employed to examine the performance of the algorithm.



The 541 to 1800th degree true reference model value	mean	standard deviation	minimum	maximum
Ground residual gravity disturbances (mGal)	-0.0346	15.7527	-93.9854	66.6638

Differences between Integral and reference value	mean	standard deviation	minimum	maximum
Ground residual gravity disturbances (mGal)	0.0071	4.2456	-18.5325	16.5266

4.4.3 Computation of disturbing gravity gradient by inverse operation integral

[Function] From the ellipsoidal height grid of the equipotential boundary surface and residual gravity disturbance (mGal) grid on the surface, compute the residual disturbing gravity gradient (E, radial) on the surface by the inverse operation integral.

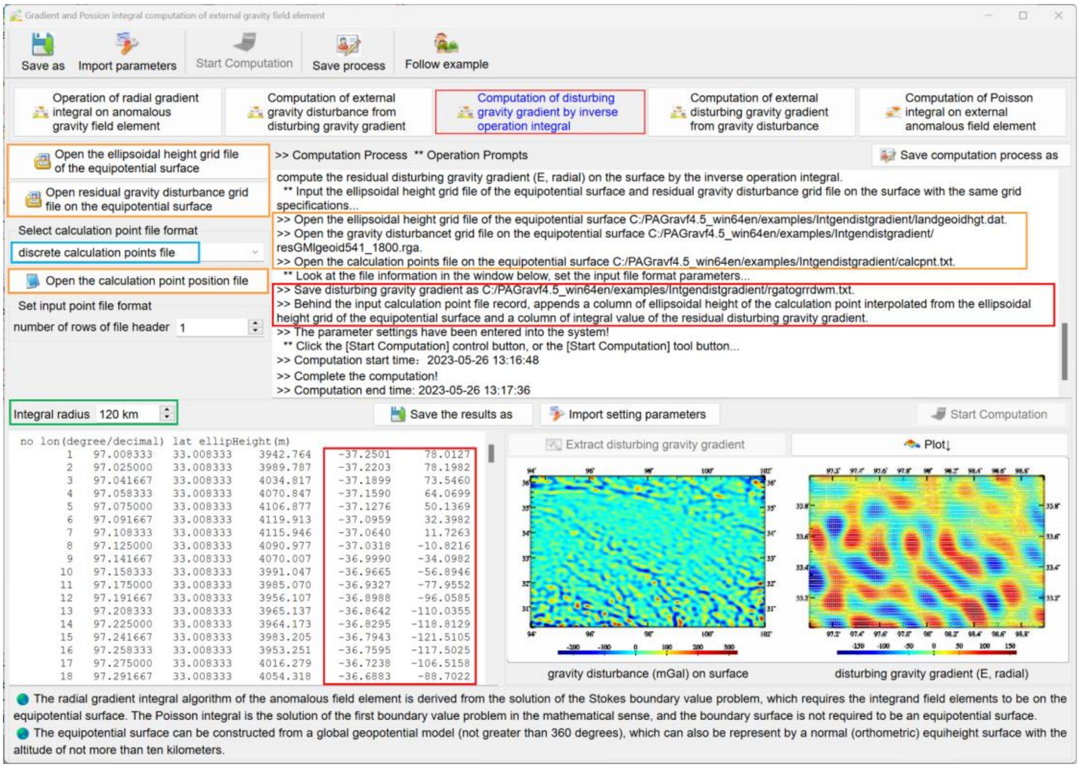
[Input files] The ellipsoidal height grid file of the equipotential surface and residual gravity disturbance grid file on the surface with the same grid specifications, and the calculation point position file on the surface.

The ellipsoidal height grid of the equipotential surface stands for the space position of the equipotential surface, employed to calculate the integral distance.

The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude, ...

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

When the ellipsoidal height grid file selected, the program let the ellipsoidal height grid of the equipotential surface as the calculation surface.

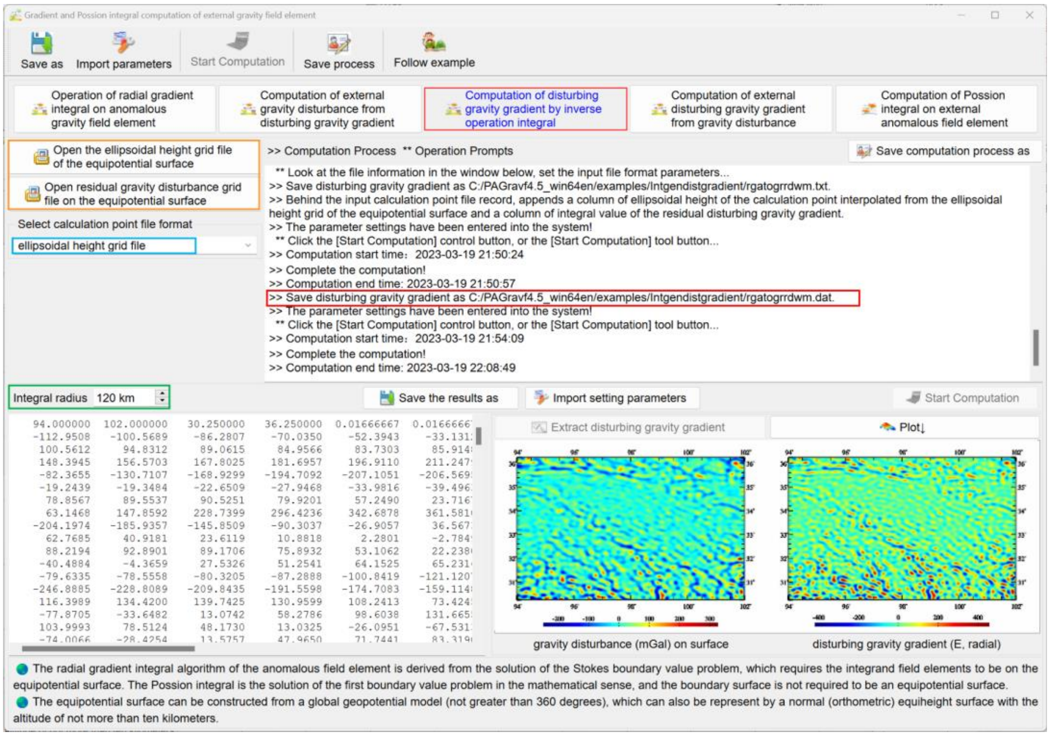


[Output file] The residual disturbing gravity gradient file.

When the discrete calculation points file input, the output file record format: Behind the input calculation points file record, appends a column of ellipsoidal height of the calculation point interpolated from the ellipsoidal height grid of the equipotential surface and a column of integral value of the residual disturbing gravity gradient.

When calculated on the calculation surface, the program outputs the residual disturbing gravity gradient grid file with the same grid specifications as the input grid file.

In this example, the residual disturbing gravity gradient on the geoid is calculated by inverse operation integral from the residual gravity disturbance on the geoid, and the integral radius is 120km. After the 2° area of the grid margin with the integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual disturbing gravity gradient (regarded as the true reference value) on the geoid, and the differences between the results of the integral and model reference values, which can be used to examine the performance of the algorithm.



The 541 to 1800th degree true reference model value	mean	standard deviation	minimum	maximum
Residual disturbing gravity gradients (E, radial) on geoid	0.4763	68.2499	-288.1750	387.7286

Differences between Integral and reference value	mean	standard deviation	minimum	maximum
Residual disturbing gravity gradients (E, radial) on geoid	-0.0562	6.6405	-37.1465	28.3557

4.4.4 Computation of external disturbing gravity gradient from gravity disturbance

[Function] From the ellipsoidal height grid of the boundary surface and residual gravity disturbance grid (mGal) on the surface, compute the residual disturbing gravity gradient (E, radial) on the geoid or in its outer space. The inverse integral of gravity disturbance adopts the combination algorithm with Poisson integral and differentiation, which does not require that the boundary surface should be a gravity equipotential surface.

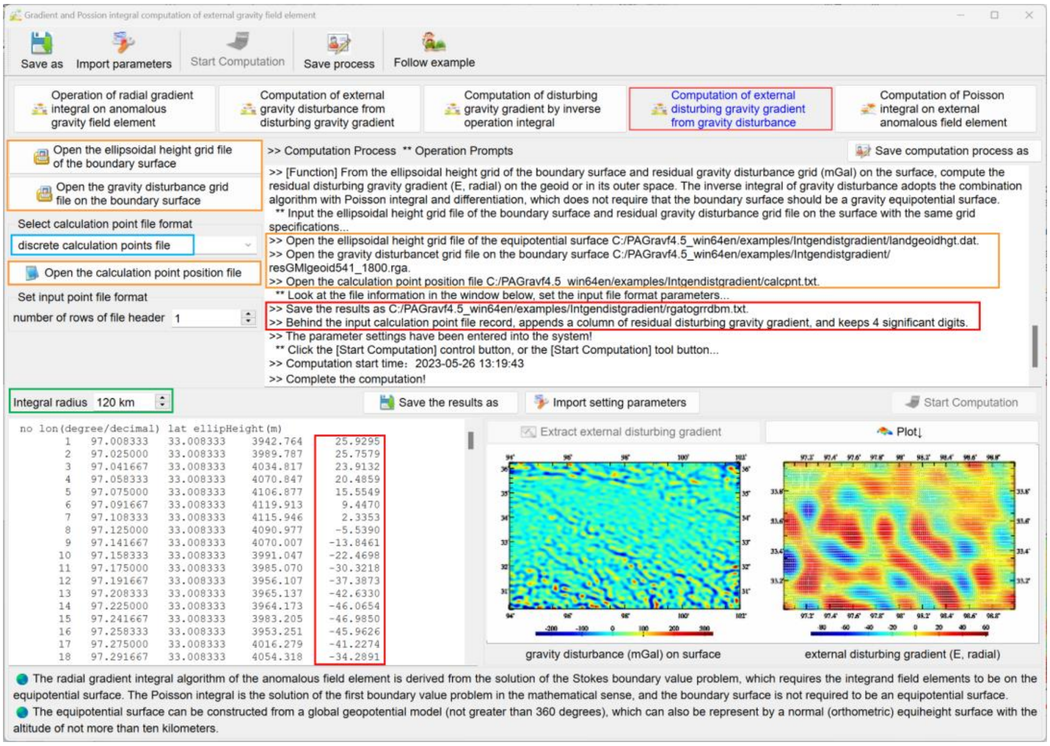
[Input files] The ellipsoidal height grid file of the boundary surface and residual gravity disturbance grid file on the surface with the same grid specifications, and the calculation point position file or the ellipsoidal height grid file of the calculation surface.

The ellipsoidal height grid of the boundary surface stands for the space position of the boundary surface, employed to calculate the integral distance.

The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude, ellipsoidal height (m), ...

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

[Output file] The external residual disturbing gravity gradient result file.



When the discrete calculation points file input, the output file record format: Behind the input calculation points file record, appends a column of residual disturbing gravity gradient, and keeps four significant digits.

When the ellipsoidal height grid file input, the program outputs the residual disturbing gravity gradient grid file with the same grid specifications as the input grid file.

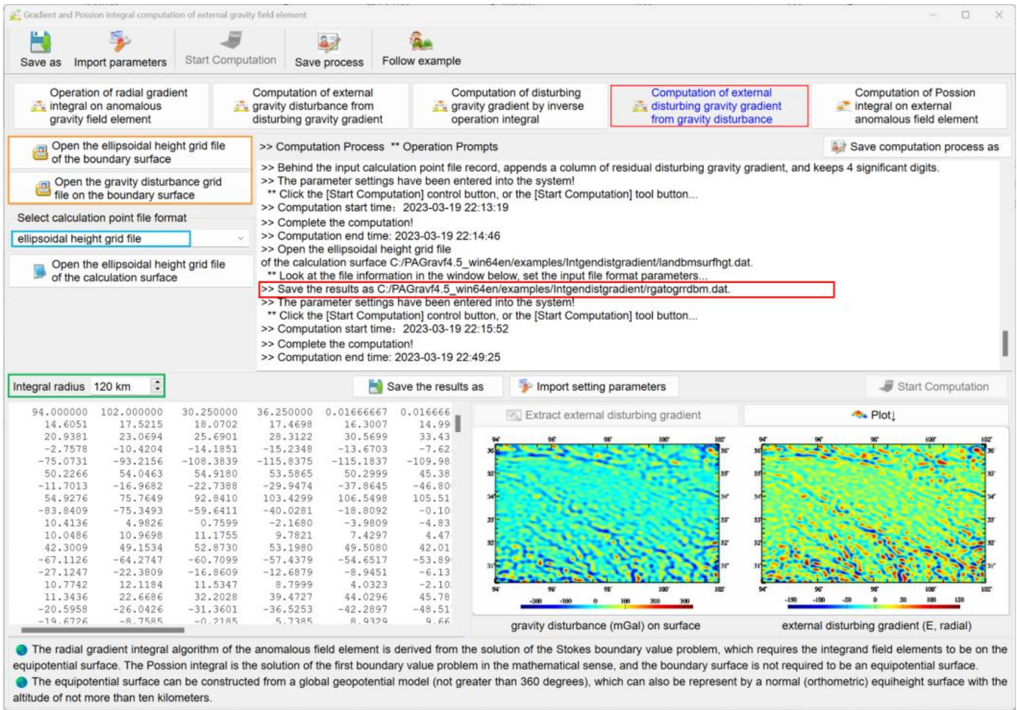
In this example, the ground residual disturbing gravity gradients are computed from the residual gravity disturbance on the geoid, and the integral radius is 120km. After the 2° area of the grid margin with the integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual disturbing gravity gradients (regarded as the true reference value) on the ground, and the differences between the results of the integral and model reference values, which can be used to examine the performance of the algorithm.

The 541 to 1800th degree true reference model value	mean	standard deviation	minimum	maximum
Ground residual disturbing gravity gradients (E, radial)	-0.2872	26.9448	-148.0282	136.7864

Differences between Integral and reference value	mean	standard deviation	minimum	maximum

Ground residual disturbing gravity gradients (E, radial)	-0.0361	2.7361	-14.2089	13.6802

Compared with the 4.4.3 inverse operation integral of gravity disturbance, the performance and accuracy of the inverse integral of gravity disturbance are comparable to those of the inverse operation integral.



4.4.5 Computation of Poisson integral on external anomalous gravity field element

[Function] From the ellipsoidal height grid of the boundary surface and the residual anomalous gravity field element grid on the surface, compute the residual anomalous gravity field element on the geoid or its outer space. The Poisson integral is the solution of the first boundary value problem in the mathematical sense, and the boundary surface need be not an equipotential surface.

[Input files] The ellipsoidal height grid file of the boundary surface and the residual anomalous field element grid file on the surface with the same grid specifications, and the calculation point position file or the ellipsoidal height grid file of the calculation surface.

The ellipsoidal height grid of the boundary surface stands for the space position of the equipotential surface, employed to calculate the integral distance.

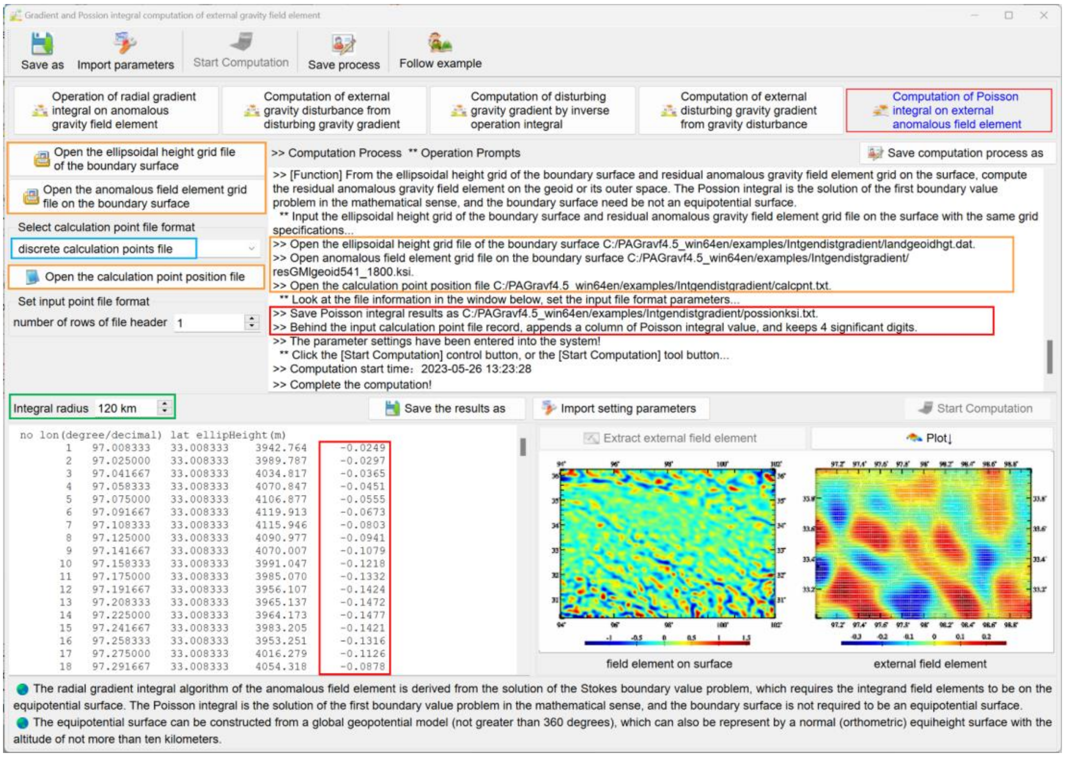
The record format of the discrete calculation point file: point no / point name, longitude (decimal degrees), latitude, ellipsoidal height (m), ...

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

[Output file] The external residual anomalous field element result file.

When the discrete calculation point file input, the output file record format: Behind the input calculation point file record, appends a column of residual anomalous field element, and keeps four significant digits.

When the ellipsoidal height grid file input, the program outputs the residual anomalous field element grid file with the same grid specifications as the input grid file.



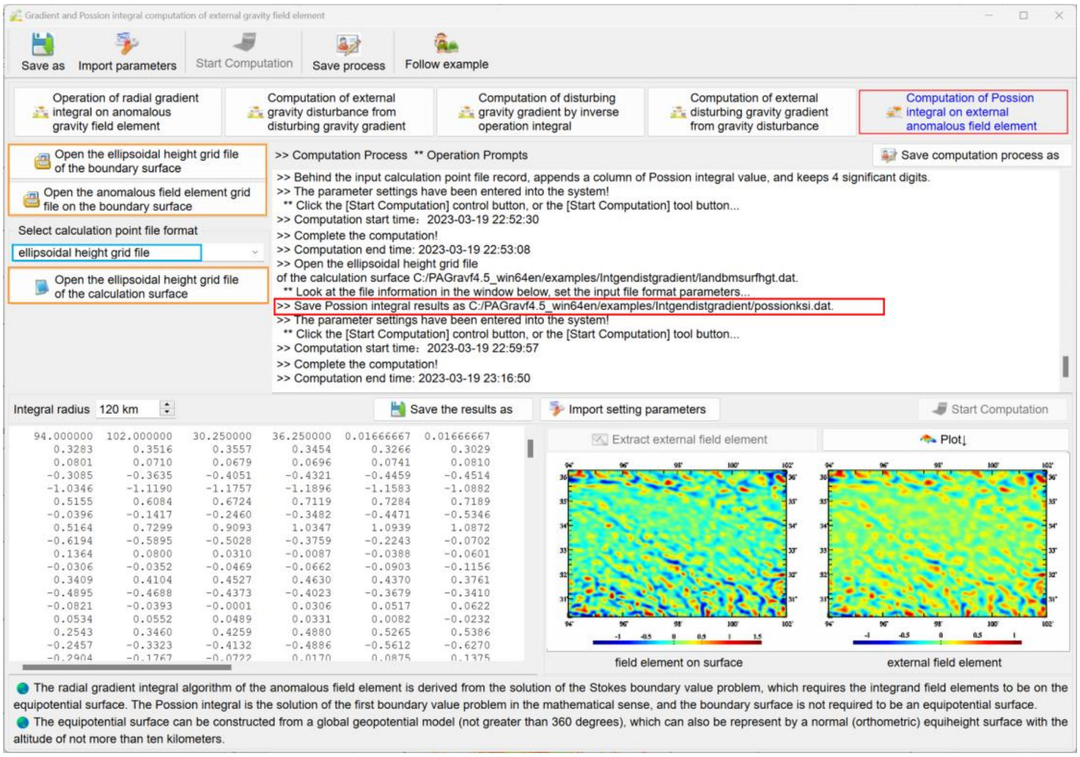
In this example, the ground residual height anomalies and ground gravity disturbances are computed respectively from the residual geoidal height and gravity disturbance on the geoid, and the integral radius is 120km. After the 2° area of the grid margin with the integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual height anomalies and gravity disturbances (regarded as the true reference value) on the ground, and the differences between the results of the integral and model reference values, which can be used to examine the performance of the algorithm.

The 541 to 1800th degree true reference model value	mean	standard deviation	minimum	maximum
Ground residual height anomalies (m)	0.0010	0.1182	-0.6745	0.4760
Ground residual gravity disturbances (mGal)	-0.0346	15.7527	-93.9854	66.6638

Differences between Integral and reference value	mean	standard deviation	minimum	maximum

Ground residual height anomalies (m)	0.0004	0.0074	-0.0326	0.0334
Ground residual gravity disturbances (mGal)	0.0476	1.1919	-5.0173	6.5138

The edge effect of the Poisson integral here is small, which can effectively suppress the attenuation of short-wave signals, and is suitable for upward and downward analytical continuation.



You can call this function repeatedly to iteratively perform the Poisson integral operation. In general, once iteration and at most 3 iterations are sufficient to meet the accuracy requirements in most cases.

4.5 Feature and performance analysis of spherical radial basis functions

[Purpose] Calculate the spatial and spectral curves of four kinds of spherical radial basis functions (SRBF) for various types of anomalous gravity field elements, and then construct an equal area spherical grid according to the specified range, to design the SRBF network and the corresponding approach algorithm of gravity field.

The program does not require an input file.

4.5.1 Spatial and spectral performance analysis of spherical radial basis functions

[Function] Given the action distance (influence radius) and sampling interval of spherical radial basis functions, calculate and display the spatial and spectral curves of spherical radial basis function of the gravity disturbance, height anomaly (disturbing potential), total vertical deflection or disturbing gravity gradient. Continuously adjusting the minimum and maximum degree of SRBF Legendre expansion, buried depth of the reference surface and order number of multipolar SRBF, the spatial and spectral performance and feature for the SRBF of various gravity field elements can be analyzed and revealed.

[Parameter settings] Select the type of anomalous gravity field element, set the spherical radial basis function parameters and plot parameters.

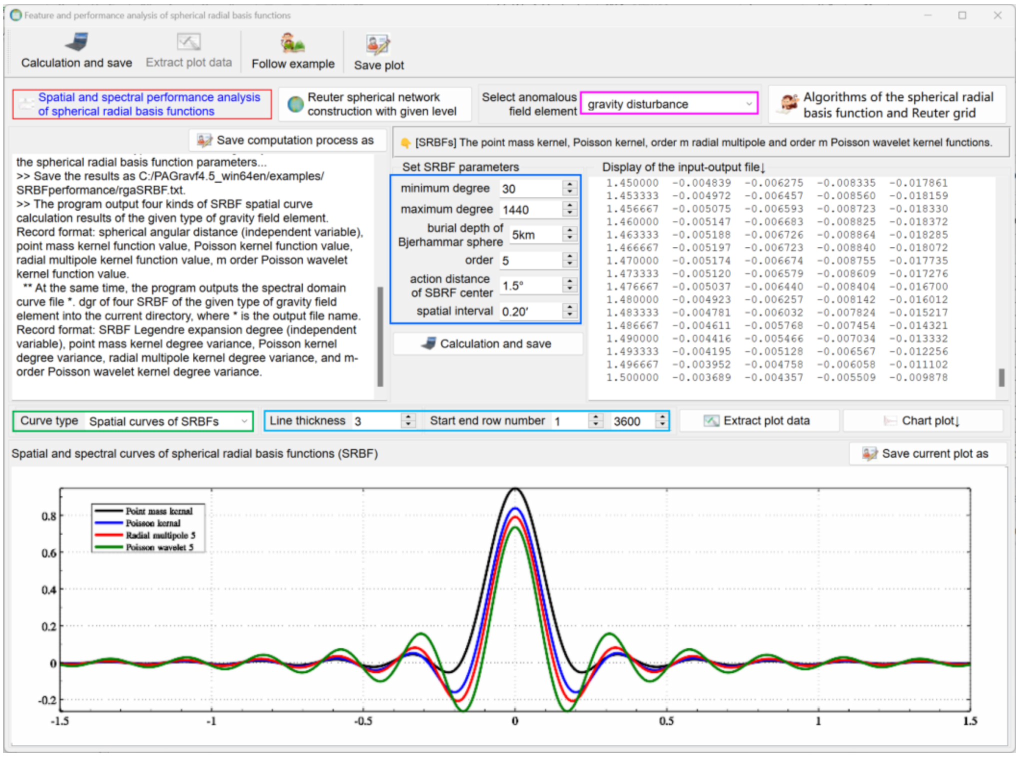
Enter the minimum and maximum degree of SRBF Legendre expansion. Minimum and maximum degree can be employed to adjust SRBF bandwidth.

Enter the burial depth of Bjerhammar sphere: The depth of the Bjerhammar sphere relative to the mean height surface of the observations. Combined with the degree of SRBF Legendre expansion, can be employed to adjust the spectral center and bandwidth of spherical radial basis function. The greater the burial depth, the smoother the SRBF, the smaller the kurtosis namely the wider the spectral bandwidth.

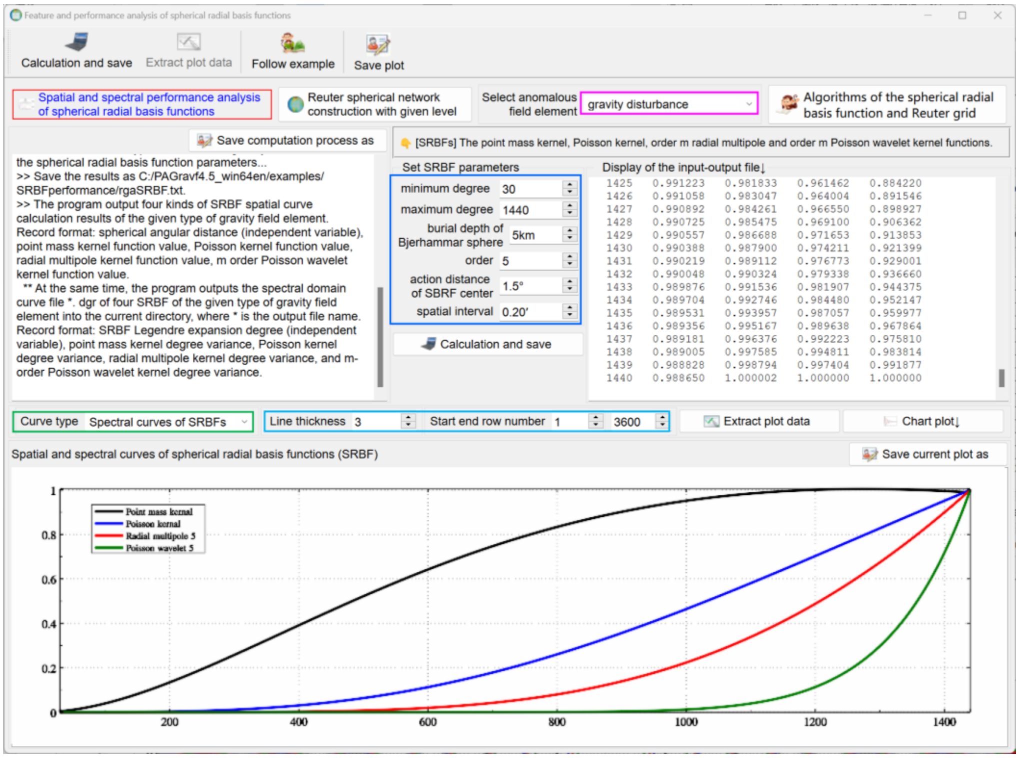
Set the order number m of radial multipole kernel function and Poisson wavelet kernel function. The greater the m, the bigger the kurtosis of SRBF.

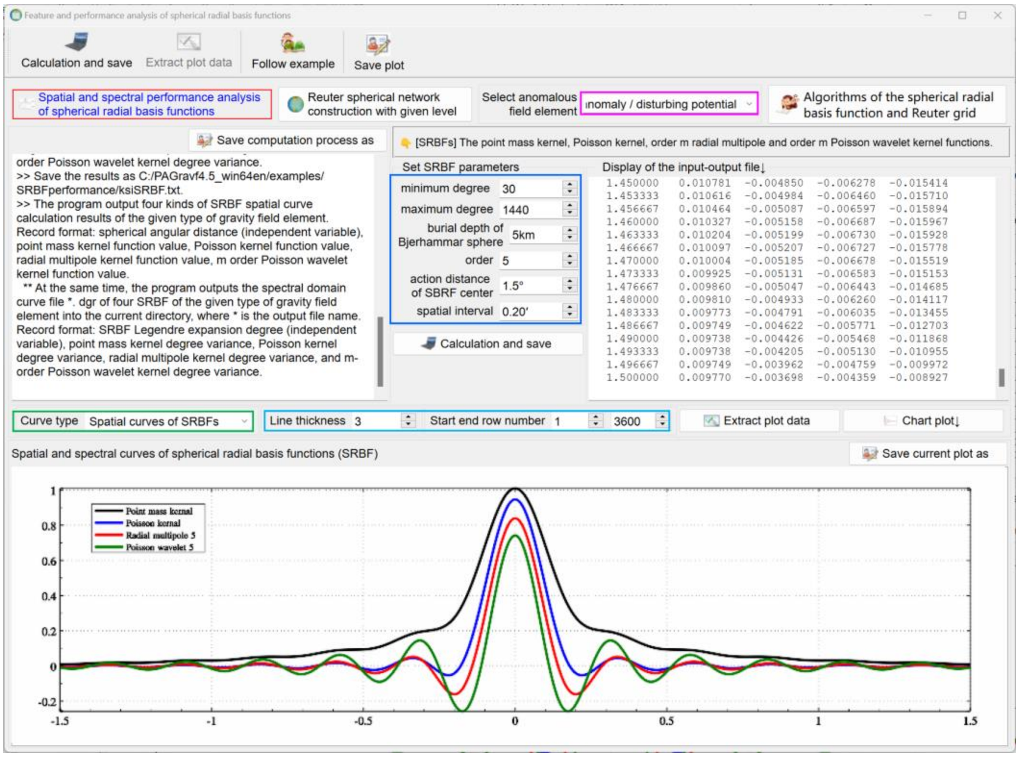
Input the SRBF spatial definition domain represented by spherical angular distance, which is also known as the influence radius, is equivalent to the integral radius of regional gravity field.

Enter Sampling interval of the SRBF spatial curve.

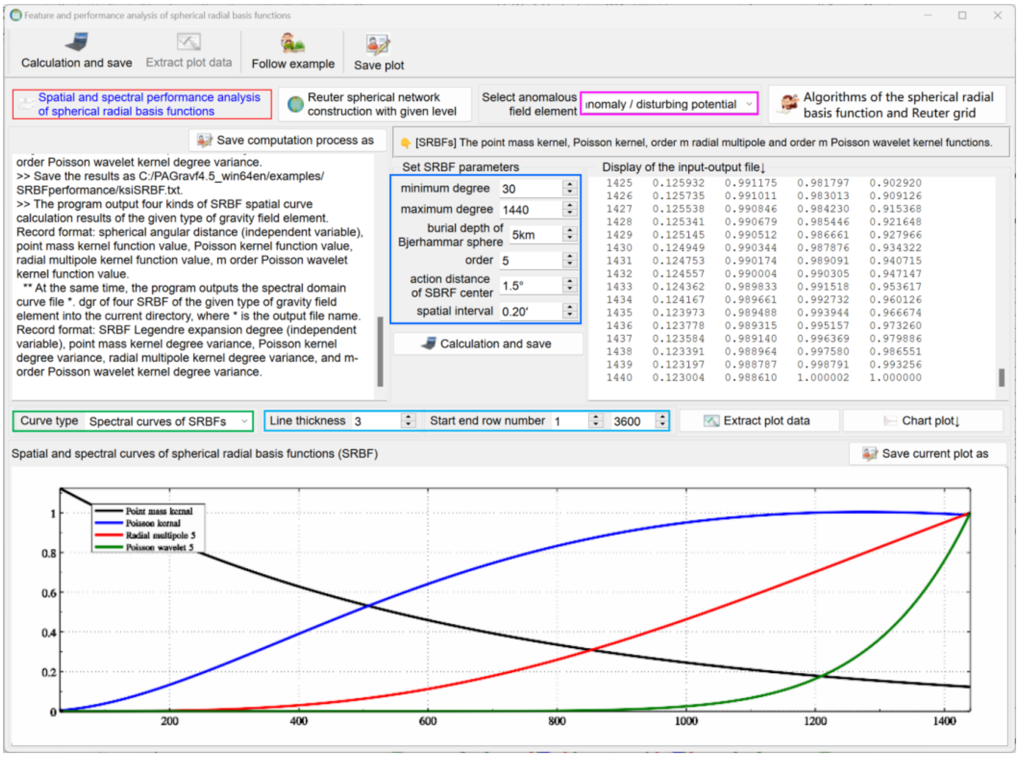


[Output files] The spatial and spectral curves of spherical radial basis functions curves files.





The program output four kinds of SRBF spatial curve calculation results of the given type of gravity field element. Record format: spherical angular distance (independent variable), point mass kernel function value, Poisson kernel function value, radial multipole kernel function value, m order Poisson wavelet kernel function value.



At the same time, the program outputs the spectral domain curve file *. dgr of four SRBF of the given type of gravity field element into the current directory, where * is the output file name. Record format: SRBF Legendre expansion degree (independent variable), point mass kernel degree variance, Poisson kernel degree variance, radial multipole kernel degree variance, and m-order Poisson wavelet kernel degree variance.

4.5.2 Reuter spherical network construction with given level

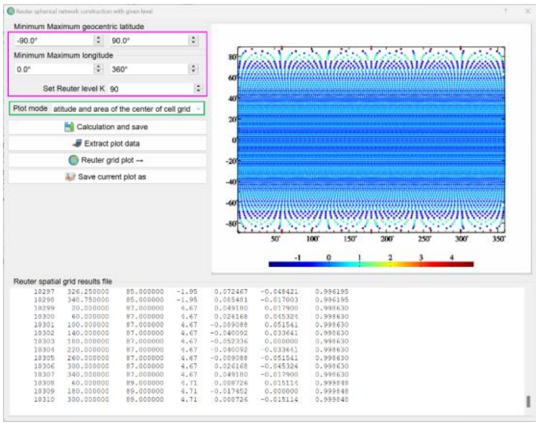
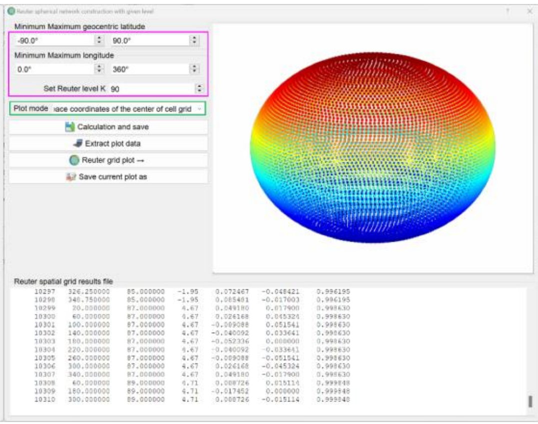
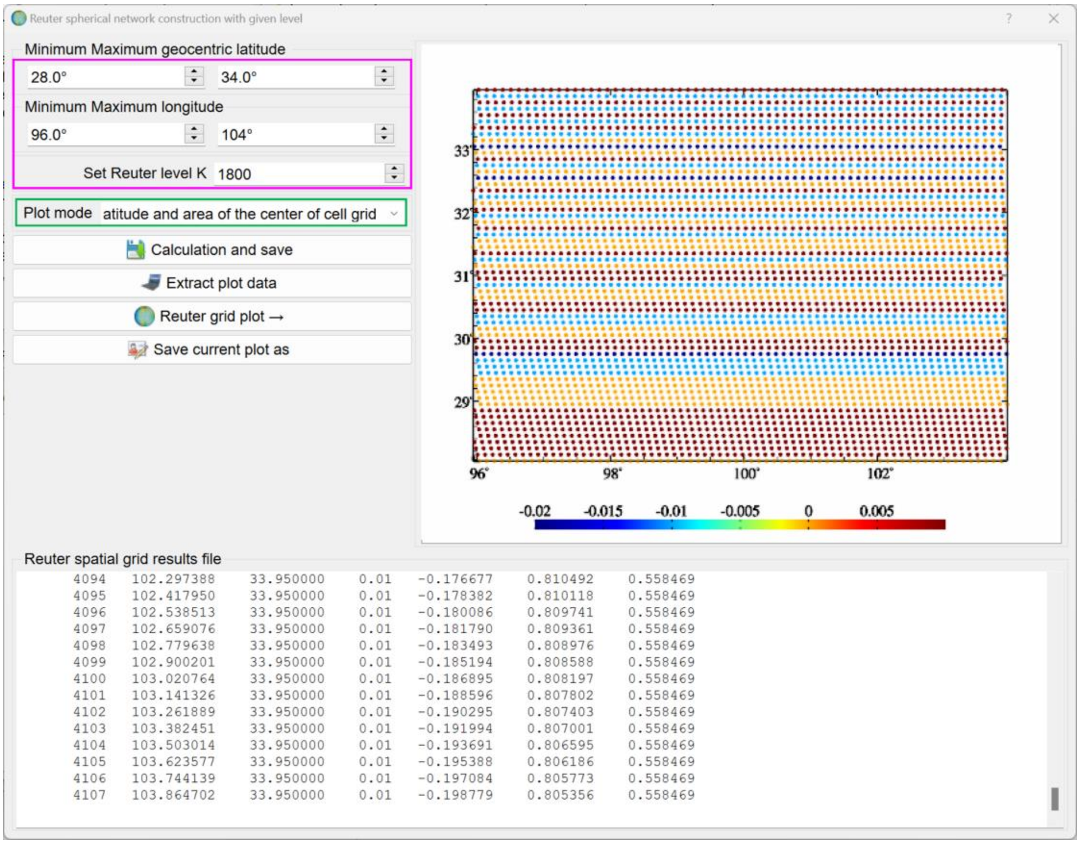
[Function] Given the Reuter network level K and regional latitude and longitude range, construct the global or regional Reuter spherical coordinate grid. The spatial resolution of the grid on the unit sphere is about π/K, the cell grid area at the equator is $A = \pi^2/K^2$, and the cell grid area at the poles is equal to πA/4.

Reuter network level K: the spherical surface is divided into K prime vertical circles, and the latitude interval is π/K. The larger the K value, the greater the spatial resolution of the spherical Reuter network. When K = 360, the spatial resolution is 30 ′, and when K = 1800, the spatial resolution is 6′.

[Parameter settings] Enter the latitude and longitude range and Reuter grid level K and set plot parameters.

[Output file] The spherical coordinate grid file of the Reuter unit spherical.

Record format of the result file: cell grid no, central longitude, geocentric latitude, percentage of area deviation, rectangular coordinates X, Y, Z. Global Reuter network does not contain bipolar cell grid. The percentage of cell grid area deviation is equal to the difference between the cell grid area and the cell grid area A at the equator, divided by the cell grid area A at the equator, multiplied by 100.

The program output Reuter network parameter file *.par into the current directory, where * is the output file name. The file header format: the latitude and longitude range, Reuter grid level, and total number of points. The record format: point no, prime vertical circle direction grid center latitude (°), prime vertical circle direction grid number, longitude interval ('), cell grid area deviation percentage.

4.6 Gravity field approach using SRBFs in spectral domain and performance test

[Function] From a single type of observations selected from the residual gravity disturbance (mGal), height anomaly (m), gravity anomaly (mGal), disturbing gravity gradient (E, radial) or vertical deflection (″), and a kind of spherical radial basis function (SRBF) selected from the point mass Kernel function, Poisson kernel function, m-order radial multipole kernel or m-order wavelet kernel function, estimate the residual gravity disturbance, height anomaly, gravity anomaly, disturbing gravity gradient or vertical deflection on the geoid or in its outer space.

Selecting different type of observations and constructing different figure of SRBF, we can calculate different type of target field elements, and then fully verify and analyze the spatial and spectral properties of gravity field approach algorithms using SRBFs.

Setting the observation weight of the target field element to be evaluated to zero, or directly taking the measuring point of the target field element to be evaluated as the calculation point, we can effectively detect the gross error of the target observations and measure their external accuracy indexes.

The program itself can be employed for analytical continuation, griding, type conversion and full element modelling on gravity field from various single type of observed field elements.

[Input files] The discrete residual anomalous field element observation file and ellipsoidal height grid file of the calculation surface.

The record agreed format of the observation file: ID, longitude (decimal degrees), latitude, …, ellipsoidal height (m), ..., observation, ..., weight, ...

There is no limit to the grid resolution of the calculation surface.

When selecting 'Synchronous calculation of elements at discrete points', the program requires input the calculation point space location file. The header file occupies 1 row, and the agreed format of the file record: ID, longitude (degree decimal), latitude, ellipsoidal height (m), ...

If the measuring points of the target observations to be evaluated are taken as the calculation points, the field elements at the measuring points can be estimated from the observations input by the program, and then we can effectively detect the gross error of the target observations to be evaluated and measure their external accuracy indexes.

[Parameter settings] Select the type of the observations and type of unknown target field element, set the input observation file format parameters, SRBF parameters and algorithm parameters.

When the column ordinal number of the weight attribute is less than 1, exceeds the column number of the record, or the weight is less than zero, the program makes the weight equal to 1.

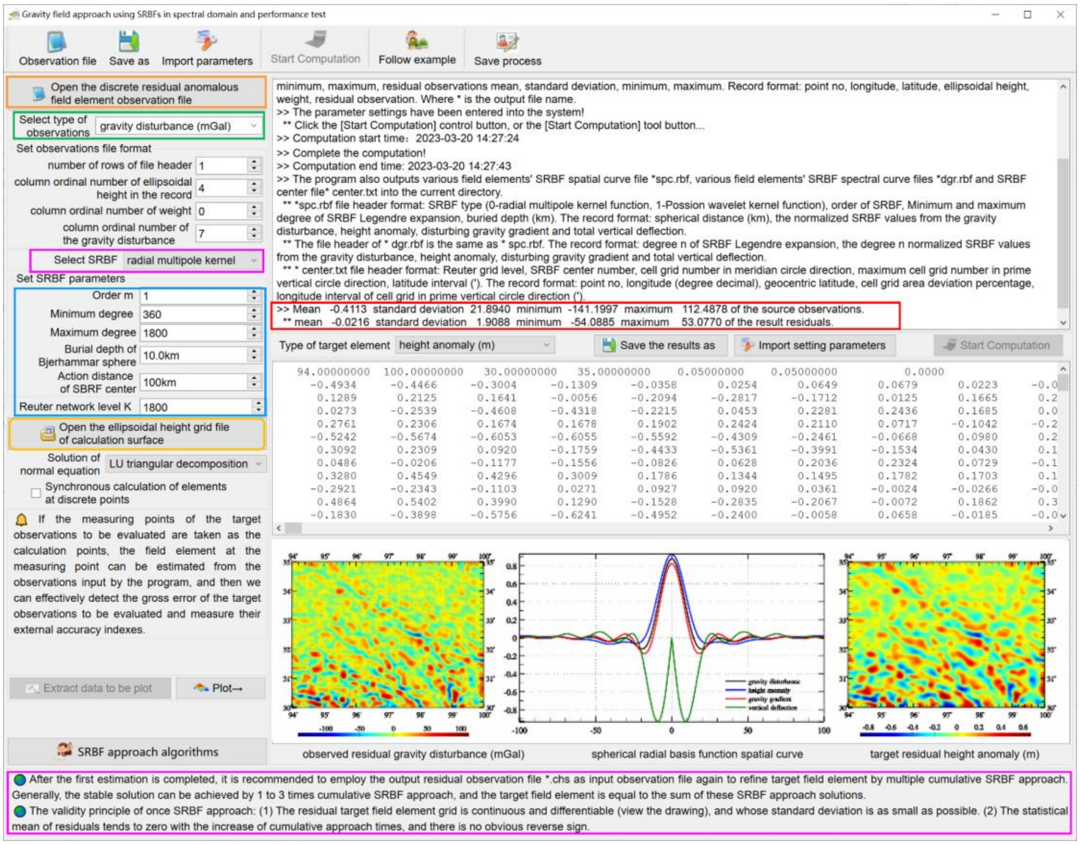
When the weight in the file record is equal to zero, the observation will not participate in the estimation of the SRBF coefficient, and the program can be employed to measure the external accuracy index of the observation.

Select spherical radial basis function: radial multipole kernel function, Poisson wavelet kernel function. The zero-order radial multipole kernel function is the point mass kernel function, and the zero-order Poisson wavelet kernel function is the Poisson kernel function.

Enter the order m. The order number m of radial multipole kernel function and Poisson wavelet kernel function. The greater the m, the bigger the kurtosis of SRBF.

Input the Bjerhammar sphere burial depth: The depth of the Bjerhammar sphere relative to the mean height surface of the observations. which can be employed to adjust the spectral center and bandwidth of SRBF when combined with the degree of SRBF Legendre expansion.

The greater the burial depth, the smoother the SRBF, the smaller the kurtosis namely the wider the spectral bandwidth.

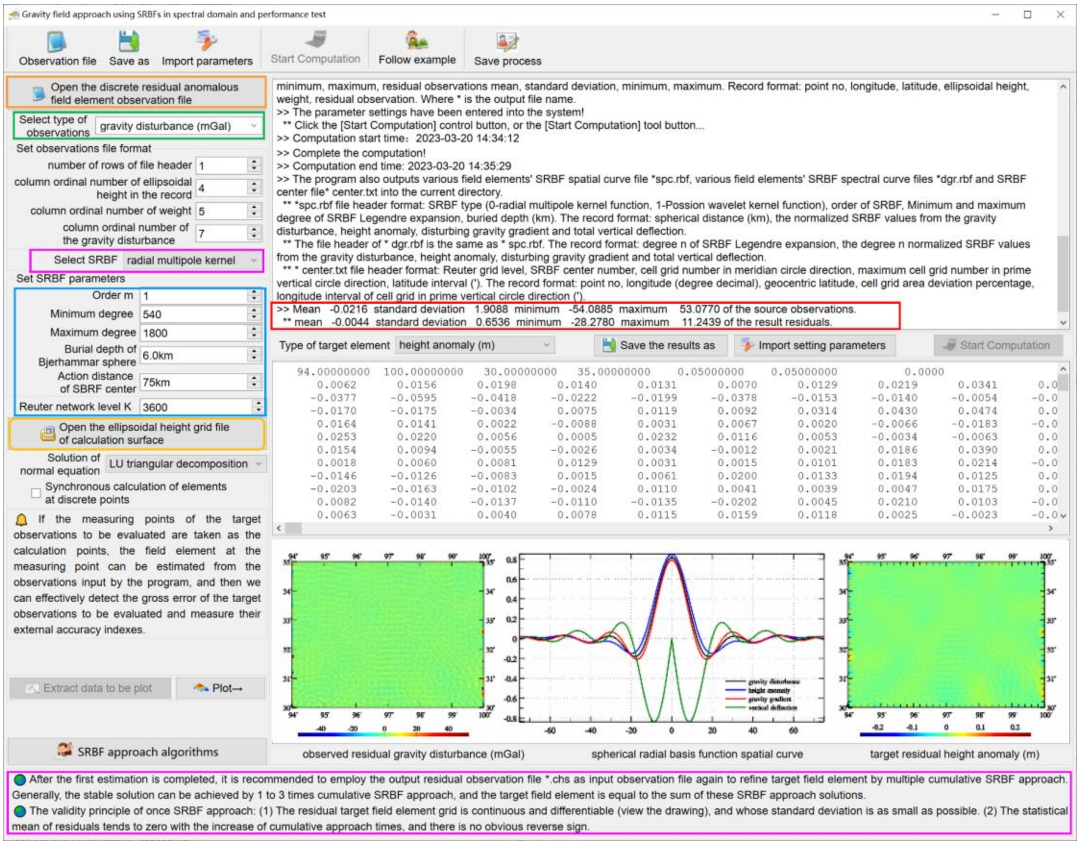


Enter the action distance of SRBF center. The action distance is also called as the influence radius = spherical angular distance × the Bjerhammar spherical radius, which is equivalent to the integral radius of regional gravity field.

A fixed action distance is adopted to ensure the coordination and consistency of the spatial and spectral figure of regional gravity field.

The suitable burial depth is about 1/20~1/5 of the action distance of SRBF center.

Set the Reuter network level K: The spherical surface is divided into K prime vertical circles, and the latitude interval is 180°/K. The larger the K value, the greater the spatial resolution of the spherical Reuter network. The suitable 180°/K is approximately equal to the average distance between observation points.



PAGravf4.5 adopts the Reuter grid fitting algorithm to quickly determine the effective number J of observations in the cell grid where each SRBF center (node) is located. When J is less than 1, the SRBF center is eliminated to ensure that the space distribution of observations is consistent with the space distribution of SRBF centers everywhere.

If the distribution of observations is uniform, the SRBF centers will also be uniformly distributed, and if the distribution of observations is irregular, the distribution of SRBF centers will also be irregular.

Enter minimum and maximum degree of SRBF Legendre expansion. Minimum and maximum degree can be employed to adjust the SRBF bandwidth.

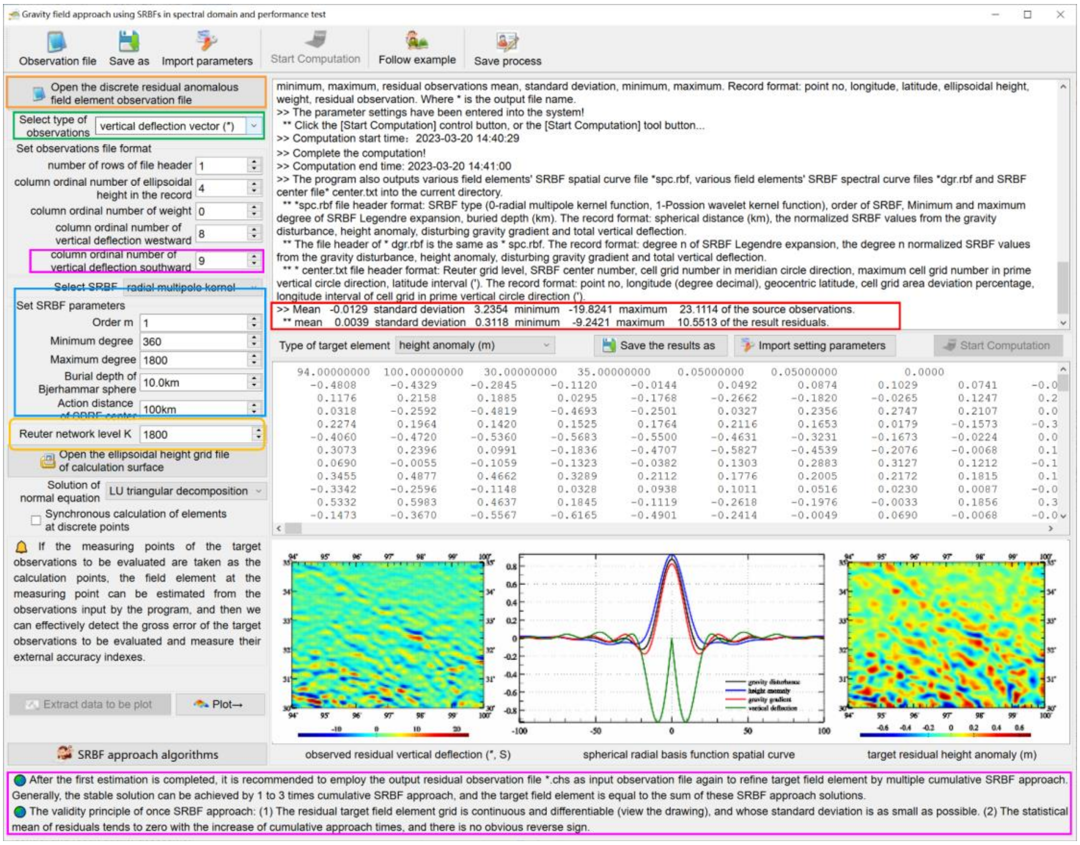
The minimum degree has a great influence on the statistical mean of the residual observations from the calculation results. Maximum degree, Reuter grid level K have some influence on the statistical standard deviation of the residuals. The influence of burial depth D has no obvious regularity, and the sensitivity is not large.

Select the solution of normal equation. LU triangular decomposition method, Cholesky decomposition, smallest norm solution.

The normal equation no longer needs regularization and iterative computation.

[Output file] The approached target field element grid file.

'When 'synchronous calculation of the field elements at discrete points' selected, the program outputs the target type of field element file &.tgt of the calculation points into the current directory, where & is the output file name.



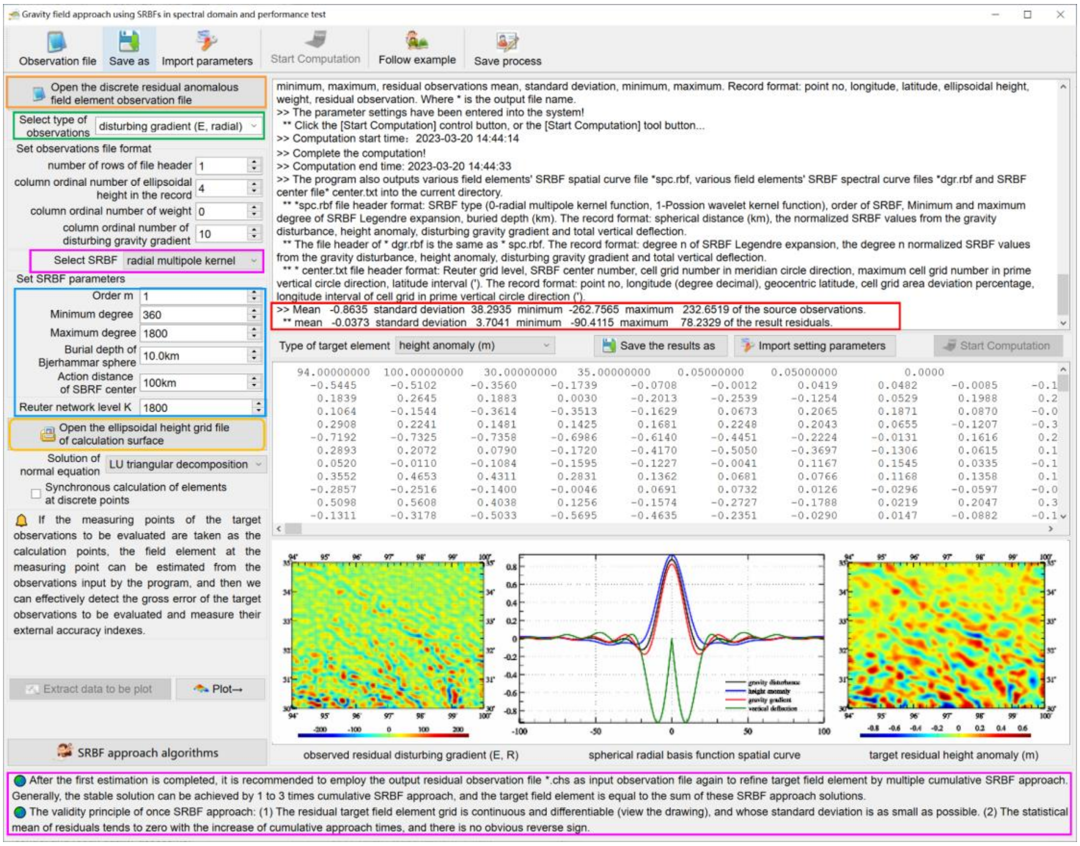
The program outputs the residual observation file *. chs into the current directory. The file header format: source observations mean, standard deviation, minimum, maximum, residual observations mean, standard deviation, minimum, maximum. The record format: ID, longitude, latitude, ellipsoidal height, weight, residual observation, where * is the input calculation point space location file name.

The program also outputs the various field elements' SRBF spatial curve file *spc.rbf, various field elements' SRBF spectral curve files *dgr.rbf and SRBF center file* center.txt into the current directory.

*spc.rbf file header format: SRBF type (0-radial multipole kernel function, 1-Poisson wavelet kernel function), order of SRBF, Minimum and maximum degree of SRBF Legendre expansion, buried depth (km). The record format: spherical distance (km), the normalized SRBF values from the gravity disturbance, height anomaly, disturbing gravity gradient and total vertical deflection.

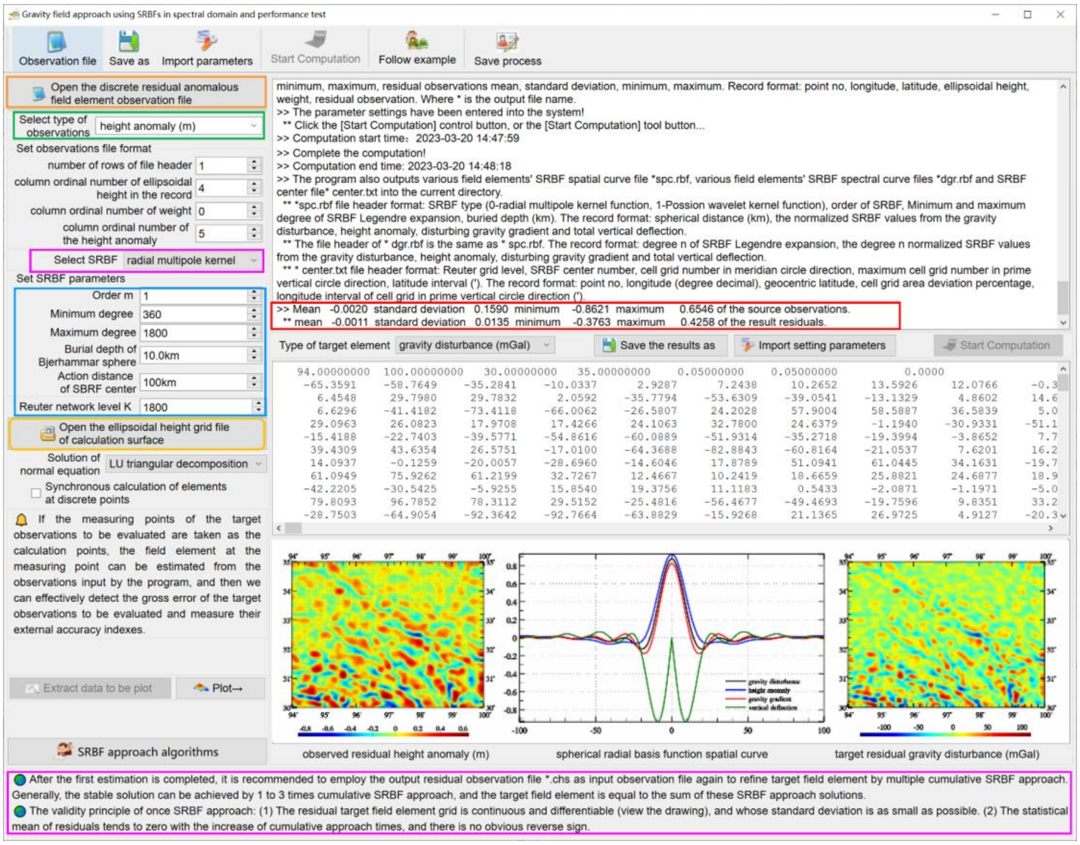
The file header of * dgr.rbf is the same as * spc.rbf. The record format: degree n of SRBF Legendre expansion, the degree n normalized SRBF values from the gravity disturbance, height anomaly, disturbing gravity gradient and total vertical deflection.

* center.txt file header format: Reuter grid level, SRBF center number, cell grid number in meridian circle direction, maximum cell grid number in prime vertical circle direction, latitude interval ('). The record format: point no, longitude (degree decimal), geocentric latitude, cell grid area deviation percentage, longitude interval of cell grid in prime vertical circle direction (').



PAGravf4.5 proposes an algorithm to improve the performance of parameter estimation by suppressing edge effects. When the SRBF center is located at the margin of the calculation area, let that the SRBF coefficient is equal to zero as the observation equation to improve the stability and reliability of unknown SRBF coefficient estimation. After the edge effect suppressed, the normal equation no longer need regularization to effectively avoid the influence of the observation error on the gravity field approach algorithm.

The target field elements are equal to the convolution of the observations and the filter SRBF. When the target field elements and the observations are of different types, it is difficult for one SRBF to effectively match the spectral center and bandwidth of the observations and the target field element at the same time, which would make the spectral leakage of the target field element. In addition, the SRBF type, minimum and maximum degree of Legendre expansion and SRBF center distribution also all affect the approach performance of gravity field. Therefore, only the optimal estimation of the SRBF coefficients with the burial depth as the parameter is not enough to ensure the best approach of the gravity field.

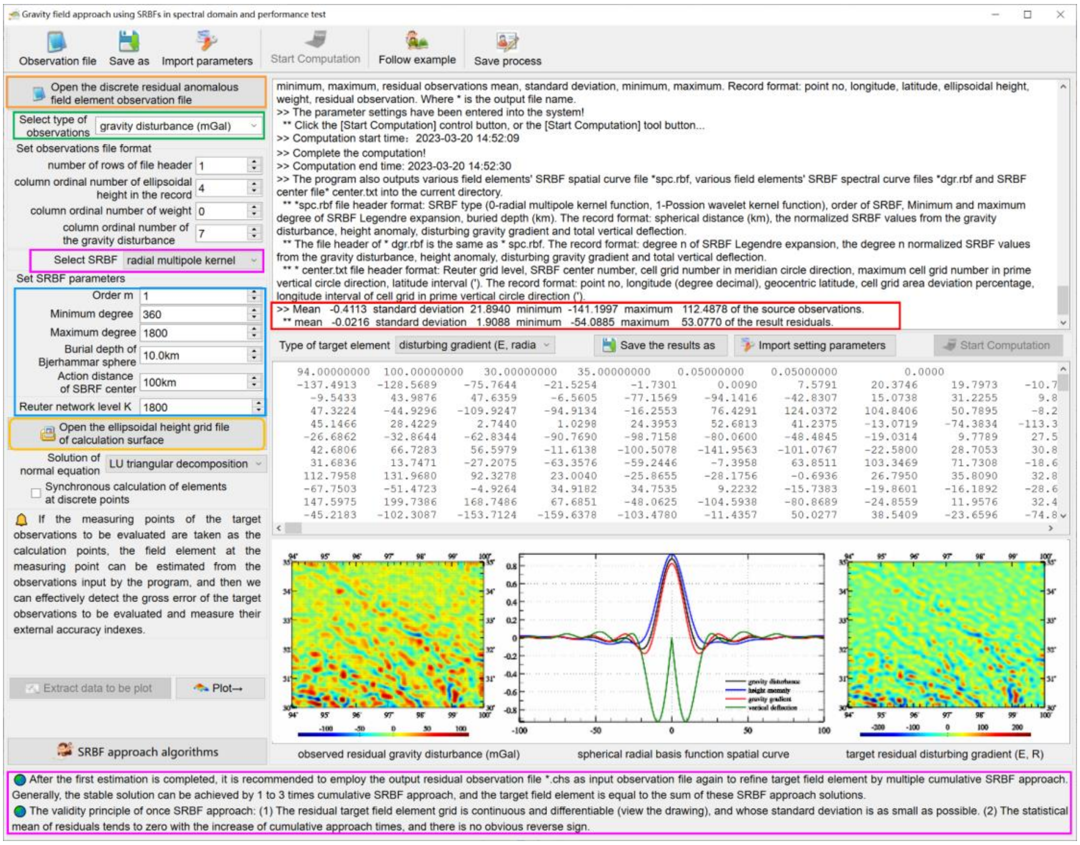


PAGravf4.5 proposes a multiple cumulative SRBF approach scheme according to the linear additivity of the gravity to solve this key problem. When each SRBF approach of gravity field adopts SRBF with different spectral figure, the cumulative SRBF approach can fully resolve the spectral domain signal of the target field element by combining multiple SRBF spectral centers and bandwidths, and then optimally restore the target field element in space domain.

In the cumulative SRBF approach scheme, it is not necessary to determine the optimal burial depth. Generally, the 1 to 3 cumulative SRBF approach of gravity field can obtain the stable results.

PAGravf4.5 gives the validity principle of once SRBF approach of gravity field: (1) the target field element grid is continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input observations file again to refine target field element by the multiple cumulative SRBF approach. Generally, the stable solution can be achieved by accumulating 1-3 times SRBF approaches, and the target field element is equal to the sum of these SRBF approach solutions.



In this example, from one of the residual ground height anomaly, gravity disturbance, vertical deflection and disturbing gravity gradient, one type residual element in ground gravity field is estimated using SRBFs with the action distance of SRBF center is 120~150km. After the 1° area of the grid margin with edge effect deducted, statistically analyze the 541 to 1800th degree model residuals (regarded as the true reference value), and the difference between the results of the SRBF approach and model reference value, which can be employed to examine the performance of the algorithm.

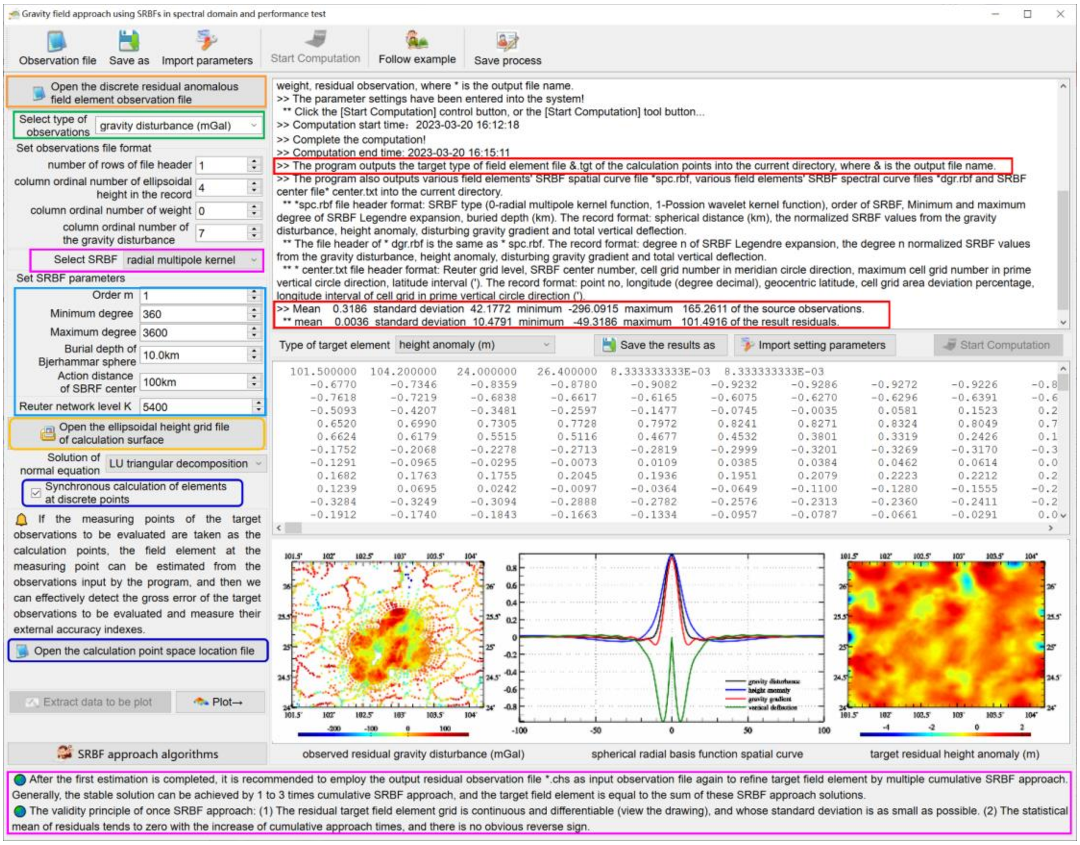
The 541~1800th degree true reference model value	mean	standard deviation	minimum	maximum
residual geoidal heights (m)	-0.0020	0.1590	-0.8621	0.6546
residual gravity disturbances (mGal)	-0.4113	21.8940	-141.1997	112.4878

Difference between SRBF approach and reference model value	First SRBF approach	Second SRBF approach

residual observation	target residual	mean	standard deviation	mean	standard deviation
gravity disturbances	ground height anomaly (m)	0.0201	0.0185	0.0189	0.0162
vertical deflection		0.0058	0.0147	0.0058	0.0072
disturbing gravity gradient		-0.0055	0.0202	-0.0056	0.0185
height anomaly	ground gravity disturbance (mGal)	0.1731	1.4421	0.2075	1.4382
vertical deflection		0.3550	1.6905	0.3566	1.6845
disturbing gravity gradient		0.0840	1.5780	0.0906	1.5298

It can be found that the performance of the primary SRBF approach is slightly better than that of gravity field integral, and the secondary cumulative SRBF approach is significantly better than gravity field integral.

The program itself has a strong capacity to detect the gross errors of discrete observations and directly measure the external accuracy indexes. In this example, only from discrete disturbance gravity observations, the gross errors of observed GNSS-levelling geoidal heights are detected, and the external accuracy index can be measured directly using SRBFs.

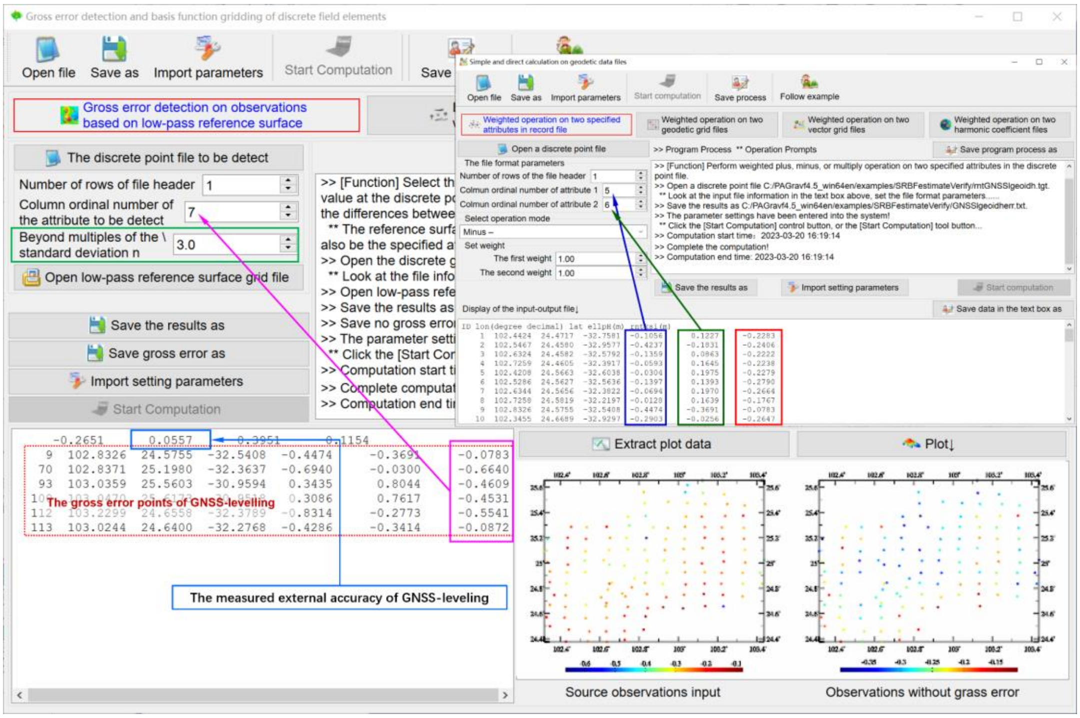


Firstly, calculate and remove the 2~540th degree EGM2008 model gravity field respectively from the observed gravity disturbances and observed GNSS-levelling geoidal heights to get the observed residual gravity disturbance file rntobsdistgrav.txt as the input observation file and get the observed residual GNSS-levelling geoidal height file rntGNSSlgeoidh.txt as the input calculation point space location file. Calculate the 2~180th degree EGM2008 model geoidal height grid in target area as the input ellipsoidal height grid file mdlgeoidh30s.dat of the calculation surface. Then call the program to output the residual GNSS-levelling geoidal height file rntGNSSlgeoidh.tgt into the current directory.

Finally, subtract the observed geoidal height from the estimated geoidal height in record of the file rntGNSSlgeoidh.tgt to obtain the GNSS-levelling remaining residuals, which is the statistical attribute, and then detect and remove the gross error points beyond 3 times of standard deviation range of the GNSS-levelling remaining residuals. Before and after gross error removed, the statistical results are as follows.

Observed GNSS-levelling residuals (m)	number of points	mean	standard deviation	minimum	maximum
Original residuals	125	-0.3510	0.2774	-0.9982	0.3435
Residuals without error	119	-0.3540	0.2647	-0.9982	0.3136
Remaining residuals	119	-0.2651	0.0557①	-0.3951	-0.1154

In the table, 0.0557①m is the external accuracy index of GNSS-levelling expressed as standard deviation, that is, 5.57 cm, which indicates that the external accuracy of GNSS-leveling is not bad than 5.57 cm (standard deviation).



In general, it is necessary for 1 to 2 times cumulative SRBF approach to obtain the minimum of the standard deviation of the GNSS-levelling remaining residuals as the external accuracy index, and this process is omitted in this example.

4.7 Full element modelling on gravity field using SRBFs from heterogeneous observations

[Function] From various heterogeneous observations which can be the residual gravity disturbance, height anomaly, gravity anomaly, disturbing gravity gradient, or vertical deflection, determinate the residual gravity disturbance, height anomaly, gravity anomaly, disturbing gravity gradient, and vertical deflection outside geoid using spherical radial basis functions (SRBFs), to realize the unified modelling on regional gravity field and geoid.

The program is a high performance and adaptable modelling tool on gravity field. Various observations with heterogeneity, different altitudes, cross-distribution, and land-sea coexisting can be directly employed to estimate the full element models of gravity field without reduction, continuation, and gridding.

The program has strong capacity on the detection of observation gross errors, measurement of external accuracy indexes and control of computation performance.

[Input files] The heterogeneous observations file and ellipsoidal height grid file of the calculation surface.

The agreed format of the observation file record: ID (point no / station name), longitude (degree decimal), latitude, ellipsoidal height (m), observation, ..., observation type (0 ~ 5), weight, ... The order of the first five attributes is fixed by convention.

The observation types and units: 0 - residual gravity disturbance (mGal), 1 - residual height anomaly (residual geoidal height, m), 2 - residual gravity anomaly (mGal), 3 - residual disturbing gravity gradient (E, radial), 4 - residual vertical deflection southward component (″), 5 - residual vertical deflection westward component (″).

The weights are only used to distinguish the observation errors at different measuring points of the same type, while it is regardless of the differences between observation errors of different types.

There is no limit to the grid resolution of the calculation surface.

[Parameter settings] Select the type of the observations and type of unknown target field element, set the input observation file format parameters, SRBF parameters and algorithm parameters.

When the column ordinal number of the weight attribute is less than 1, exceeds the column number of the record, or the weight is less than zero, the program makes the weight equal to 1.

When the weight in the file record is equal to zero, the observation will not participate in the estimation of the SRBF coefficient, and the program can be employed to measure the external accuracy index of the observation.

Select the spherical radial basis function: the radial multipole kernel function or

Poisson wavelet kernel function. The zero-order radial multipole kernel function is the point mass kernel function, and the zero-order Poisson wavelet kernel function is the Poisson kernel function.

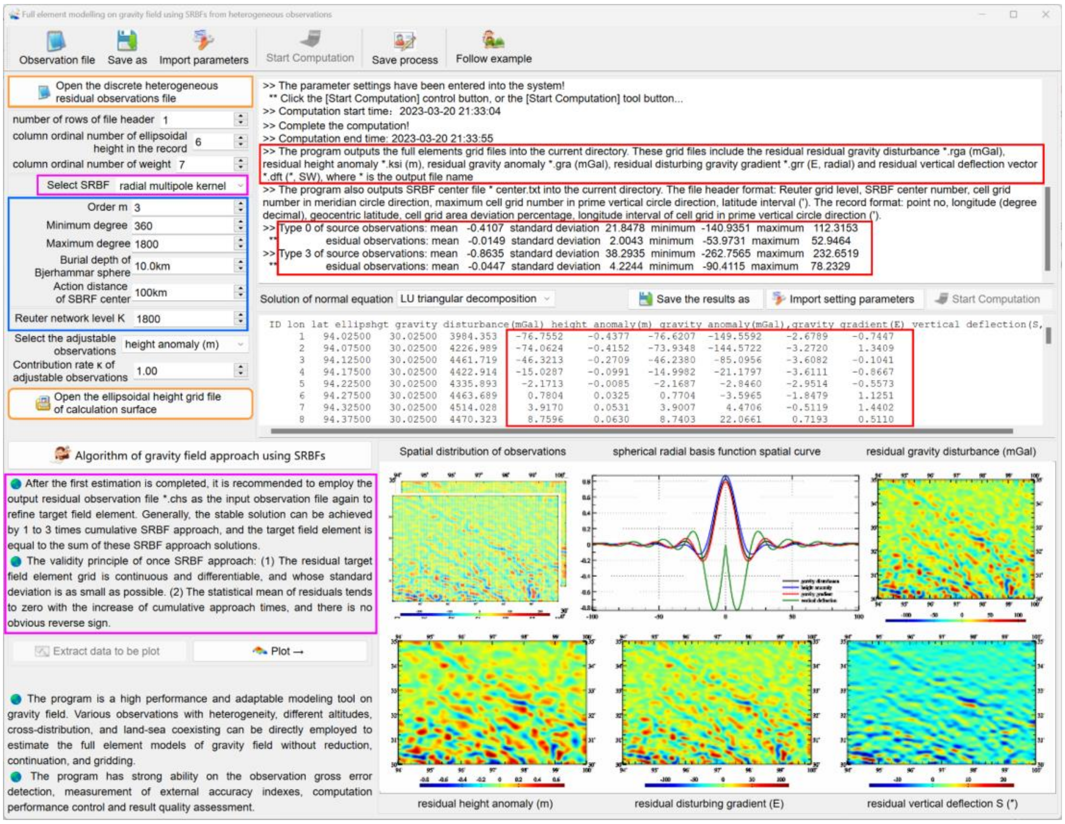
Enter the order m. The order number m of radial multipole kernel function and Poisson wavelet kernel function. The greater the m, the bigger the kurtosis of SRBF.

Input the Bjerhammar sphere burial depth: The depth of the Bjerhammar sphere relative to the mean height surface of the observations. which can be employed to adjust the spectral center and bandwidth of SRBF when combined with the degree of SRBF Legendre expansion.

The greater the burial depth, the smoother the SRBF, the smaller the kurtosis namely the wider the spectral bandwidth.

Enter the action distance of SRBF center. The action distance is also called as the influence radius = spherical angular distance × the Bjerhammar spherical radius, which is equivalent to the integral radius of regional gravity field.

A fixed action distance is adopted to ensure the coordination and consistency of the spatial and spectral figure of regional gravity field.



Set the Reuter network level K: The spherical surface is divided into K prime vertical circles, and the latitude interval is 180°/K. The larger the K, the greater the spatial resolution of the spherical Reuter network. The suitable 180°/K is approximately equal to the average distance between observation points.

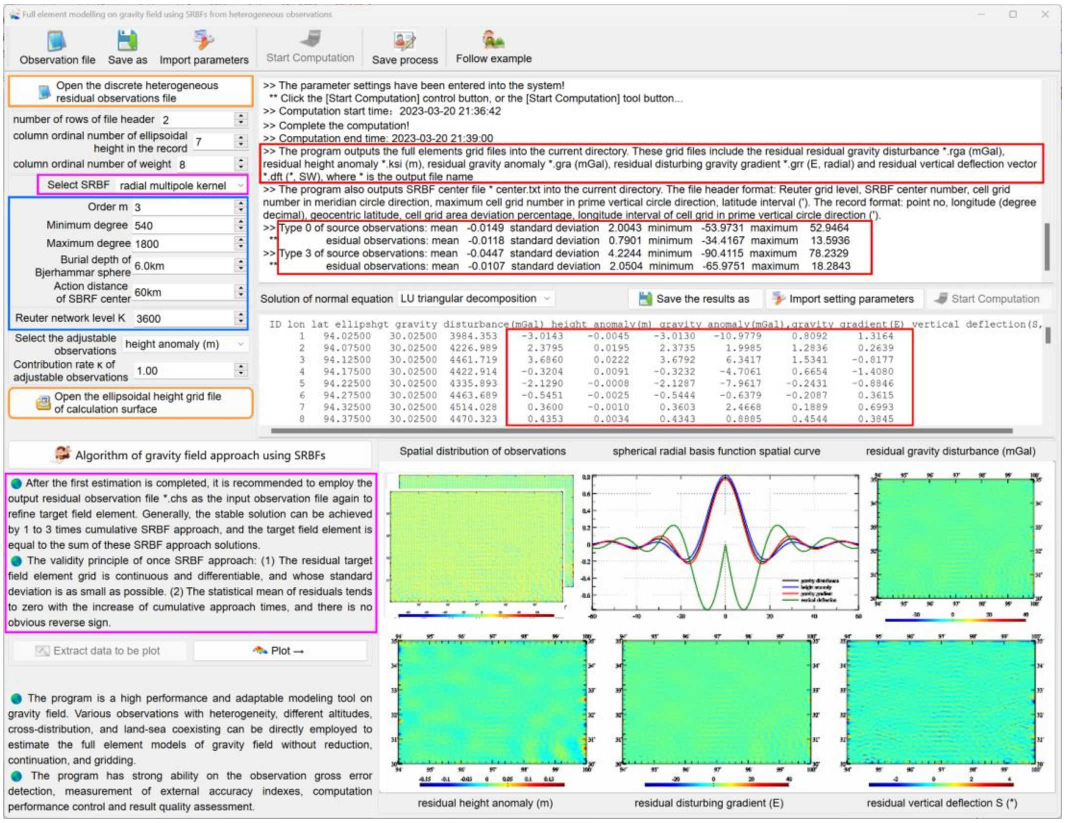
Enter minimum and maximum degree of SRBF Legendre expansion. Minimum and maximum degree can be employed to adjust SRBF bandwidth.

Select the type of the adjustable observations and set the contribution rate κ of the adjustable observations.

The program multiplies the normal equation coefficient matrix and constant matrix of the adjustable observations by κ, respectively, to increase ($\kappa > 1$) or decrease ($\kappa < 1$) the contribution of the adjustable observations. When $\kappa = 1$, it means that there are not any adjustable observations selected. When $\kappa = 0$, the adjustable observations do not participate in the estimation of SBRF coefficients.

Select the solution of normal equation. LU triangular decomposition method, Cholesky decomposition, smallest norm solution.

The normal equation no longer needs regularization and iterative computation.



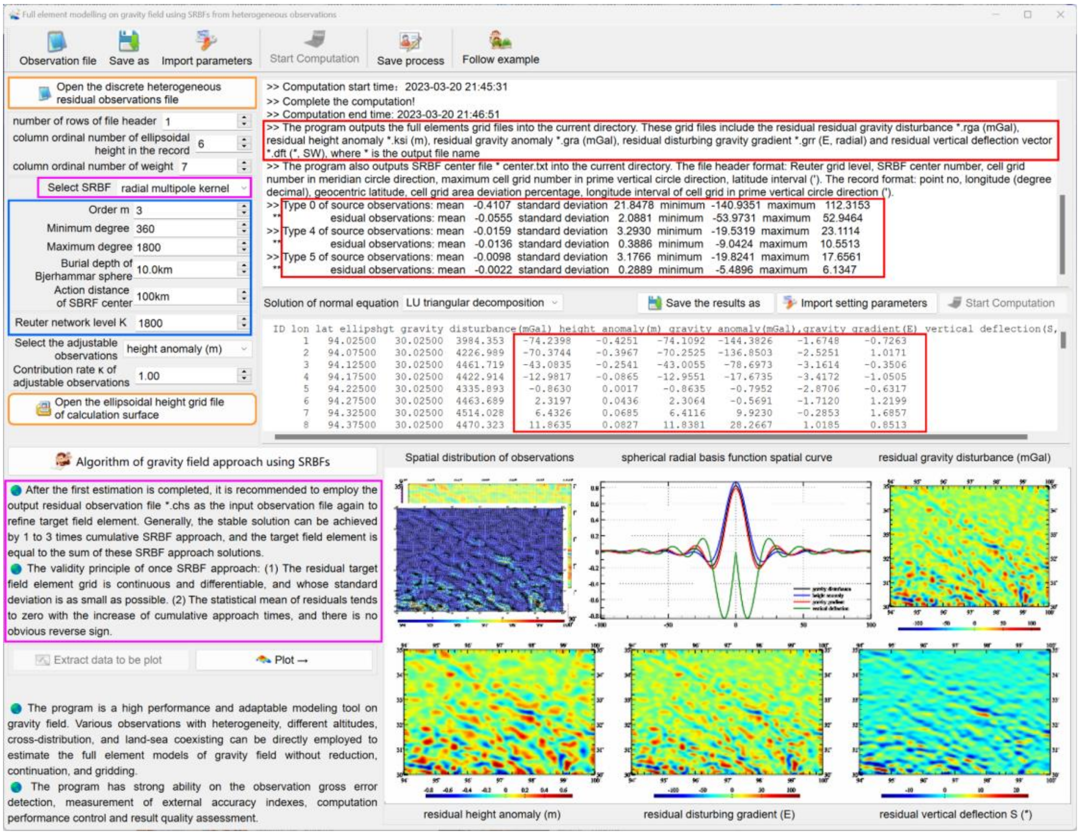
[Output file] The full elements of gravity field file.

The file record: ID, longitude (degree decimal), latitude, ellipsoidal height (m) of calculated point, residual gravity disturbance (mGal), residual height anomaly (m), residual gravity anomaly (mGal), residual disturbing gravity gradient (E, radial) and residual vertical deflection (″, SW).

The program outputs the full element grid files into the current directory. These grid files include the residual gravity disturbance *.rga (mGal), residual height anomaly *.ksi (m), residual gravity anomaly *.gra (mGal), residual disturbing gravity gradient *.grr (E, radial) and residual vertical deflection vector *.dft (″, SW), where * is the output file name, and whose grid specifications are the same as the input ellipsoidal height grid of calculation surface.

The program also outputs the residual observation file *.chs into the current directory. The statistical results of each type of observations occupies a row of file header, whose format: observation type (0~5), source observation mean, standard deviation, minimum, maximum, residual observation mean, standard deviation, minimum, maximum. The record format: ID, longitude, latitude, ellipsoidal height, residual observation, source observation, observation type, weight.

The program also outputs SRBF center file * center.txt into the current directory. The file header format: Reuter grid level, SRBF center number, cell grid number in meridian circle direction, maximum cell grid number in prime vertical circle direction, latitude interval ('). The record format: ID, longitude (degree decimal), geocentric latitude, cell grid area deviation percentage, longitude interval of cell grid in prime vertical circle direction (').



PAGravf4.5 proposes a cofactor matrix diagonal standard deviation method to combinate different types of heterogeneous observations for estimation of the SRBF coefficients. This method replaces the residual observations variance in the iterative process of the variance component estimation with the diagonal standard deviation of the cofactor matrix, so that the properties of the parameter estimation solution are only

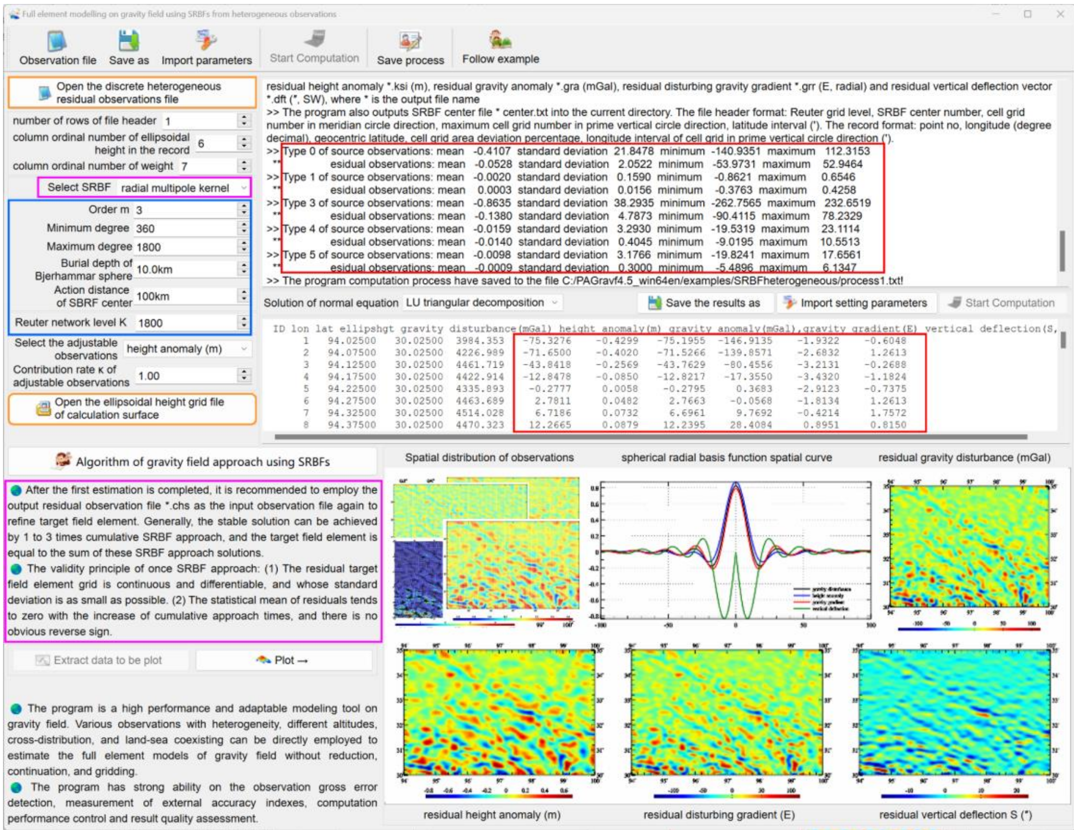
related to the space distribution of the observations without influence of observation error. Which is conducive to combination of various types of observations with extreme differences in space distribution, such as a very small number of astronomical vertical deflections or GNSS-levelling data.

In this case, the normal equation does not also need to be iteratively resolved, which conducive to improve the analytical nature of SRBF approach algorithm.

PAGravf4.5 gives the validity principle of once SRBF approach: (1) The residual target field element grid is continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

Generally, 1 to 3 cumulative SRBF approaches on gravity field can obtain the stable approach results.

Selecting the adjustable observation and its contribution rate κ, we can effectively deal with the problem of high-precision gravity field approach from heterogeneous observations with extreme differences in space distribution, quality, and accuracy. With $\kappa = 0$ for some a type of observations, we can effectively detect the gross error and evaluate the quality and external accuracy. With $\kappa > 1$ for several high-precision observations, we can effectively improve the contribution of the several observations such as the astronomical vertical deflection or GNSS-levelling observations.



In this example, from the residual ground height anomaly, gravity disturbance, vertical deflection or disturbing gravity gradient, the residual ground height anomaly is calculated using SRBFs with the action distance of SRBF center 150 km. After the 1° area of the grid margin with edge effect deducted, statistically analyze the 541~1800th degree model residuals (regarded as the true reference value), and the difference between the results of the SRBF approach and the model reference value, which can be employed to examine the performance of the algorithm.

The 541~1800th degree true reference model value	mean	standard deviation	minimum	maximum
residual geoidal heights (m)	-0.0020	0.1590	-0.8621	0.6546
residual gravity disturbances (mGal)	-0.4113	21.8940	-141.1997	112.4878
residual disturbing gravity gradients (E)	-0.8635	38.2935	-262.7565	232.6519

<table>
<tr><th colspan="2">Difference between SRBF approach and reference model value</th><th colspan="2">First SRBF approach</th><th colspan="2">Second SRBF approach</th></tr>
<tr><td>residual observation</td><td>target residual</td><td>mean</td><td>standard deviation</td><td>mean</td><td>standard deviation</td></tr>
<tr><td>gravity disturbance</td><td rowspan="2">ground height anomaly (m)</td><td rowspan="2">0.0023</td><td rowspan="2">0.0095</td><td rowspan="2">0.0005</td><td rowspan="2">0.0093</td></tr>
<tr><td>height anomaly</td></tr>
<tr><td>gravity disturbance</td><td rowspan="2">ground height anomaly (m)</td><td rowspan="2">0.0036</td><td rowspan="2">0.0190</td><td rowspan="2">0.0031</td><td rowspan="2">0.0158</td></tr>
<tr><td>disturbing gravity gradient</td></tr>
</table>

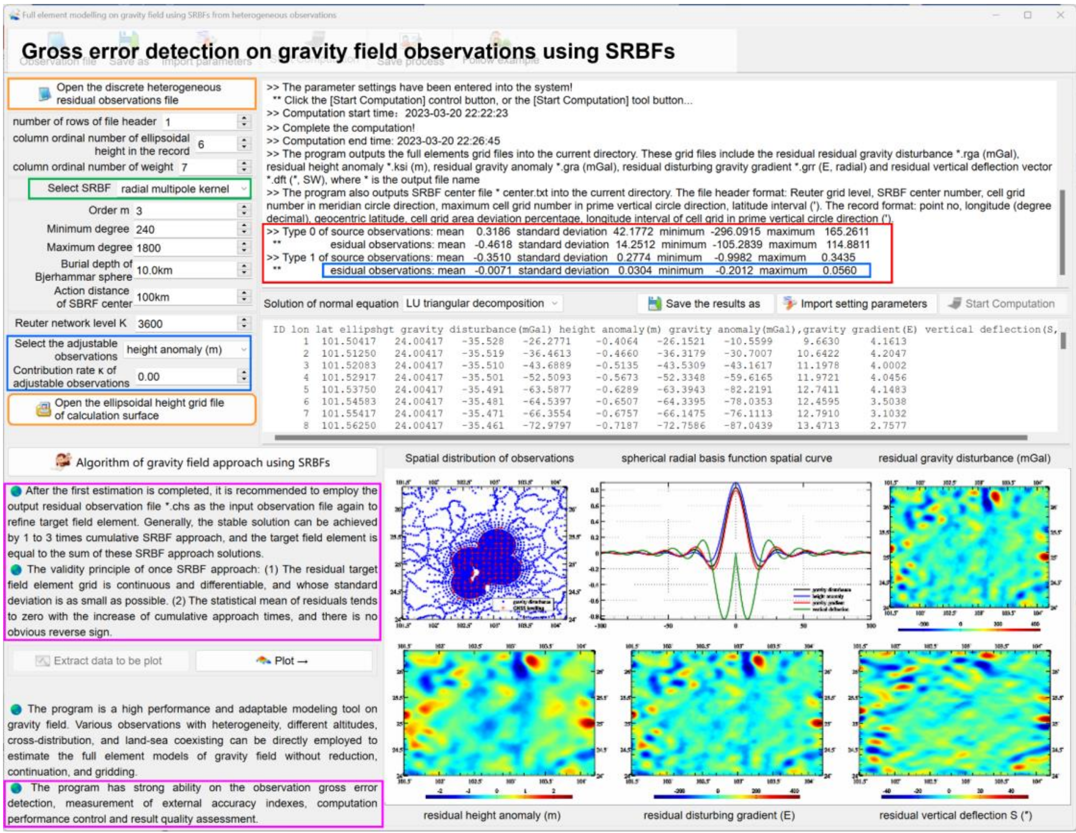
In addition, by setting the zero contribution rate ($\kappa = 0$) or zero weight ($p = 0$) of the observations to be investigated, and making full use of the residual observation files *.chs output during the operation of the program, we can effectively deal with the typical problems such as the observation gross error detection, measurement of external accuracy indexes, computation performance control and result quality assessment in various complex cases.

In this example, only from discrete disturbance gravity observations, the gross error of observed GNSS-levelling geoidal heights is detected, and its external accuracy index can be measured directly using SRBFs. The process is shown in process2.txt.

Firstly, calculate and remove the 2~540th degree EGM2008 model gravity field respectively from the observed gravity disturbances and observed GNSS-levelling geoidal heights, and generate the heterogeneous residual observation file obsresiduals0.txt. Calculate the 2~180th degree EGM2008 model geoidal height grid as the ellipsoidal height grid file mdlgeoidh30s.dat of calculation surface. Then call the program, select the height anomaly as the adjustable observation, let the contribution rate $\kappa = 0$ to get the remaining residual file GNSSlerrpk0.chs.

Separate the remaining residual records of the observed GNSS-leveling and observed gravity disturbance from the remaining residual file GNSSlerrpk0.chs, detect and remove the observation gross error points beyond 3 times of standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 of times standard deviation range for the disturbance gravity points, and then reconstruct the new heterogeneous observation residual file obsresidnoerr.txt.

Replace the input file obsresiduals0.txt with the new heterogeneous observation residual file obsresidnoerr.txt and call the program again to get the new remaining residual file GNSSlerrpk02.chs.

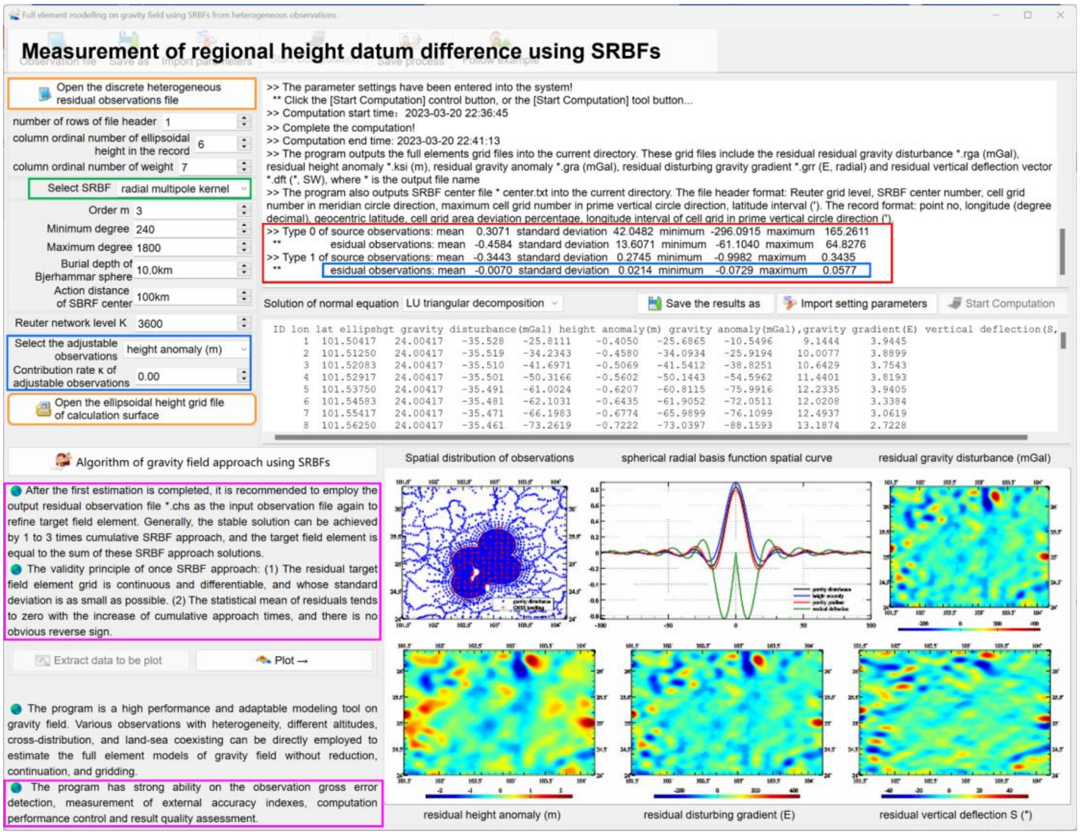


Since the contribution rate of GNSS-levelling $\kappa = 0$ is set in advance, it is essentially here to directly determine the external accuracy index of the observed GNSS levelling using only discrete gravity disturbance observations. Before and after gross error removed, the statistical results on the observation residuals are as follows.

		number of points	mean	standard deviation	minimum	maximum
Gravity disturbance (mGal)	Original residuals	4219	0.3186	42.1772	-296.0915	165.2611
	Residuals without error	4213	0.3071	42.0482	-296.0915	165.2611
	Remaining residuals	4213	-0.4584	13.6071	-61.1040	64.8276
GNSS levelling	Original residuals	125	-0.3510	0.2774	-0.9982	0.3435

geoidal height (m)	Residuals without error	123	-0.3443①	0.2745③	-0.9982	0.3435
	Remaining residuals	123	-0.0070②	0.0214④	-0.0729	0.0577

The statistical mean ① minus ② of the GNSS-levelling remaining residuals in the table, that is, -0.3443①- (-0.0070②) = -0.3373m, is the difference between the regional height datum and the global height datum (gravimetric geoid). Here provides the SRBF measurement method for regional height datum difference.



In the table, 0.2745③m = 27.45cm can represent the accuracy index of the model geoidal height from the 2~540th degrees EGM2008 model.

0.0214④m is the external accuracy index of GNSS-levelling expressed as standard deviation, that is, 2.14 cm. Here provides the SRBF measurement method for the external accuracy index of GNSS-leveling. The result indicates that the external accuracy of GNSS-leveling is not bad than 2.14 cm (standard deviation).

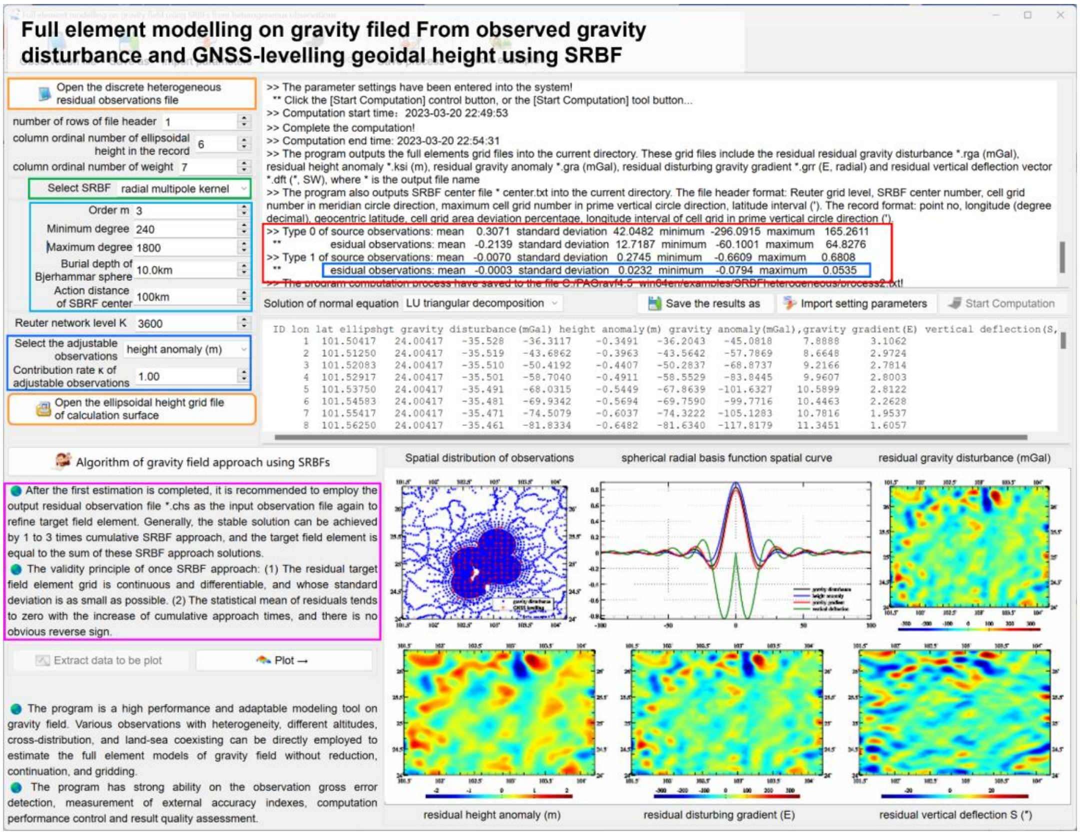
● The typical technical features of the program

① The analytical function relationships between gravity field elements are strict, and the SRBF approach performance has nothing to do with the observation errors.

② Various heterogeneous observations in the different altitudes, cross-distribution, and land-sea coexisting cases can be directly employed to estimate the full element models of gravity field without reduction, continuation, and griding.

③ Can integrate very little astronomical vertical deflection or GNSS-levelling data, and effectively absorb the edge effect.

④ Has the strong capacity in detection of observation gross errors, measurement of external accuracy indexes, and control of computational performance.



4.8. Modelling process exercise of regional gravity field and geoid

4.8.1 Computation process demo of full element modelling on gravity field by integral method

From the ground digital elevation model and observed gravity disturbances from EGM2008 geopotential model, using a remove-integral-restore scheme combination with residual terrain effects and EGM2008 model, compute the ground height anomaly, ground gravity disturbance and ground disturbing gravity gradient as well as the geoidal height, gravity disturbance and disturbing gravity gradient on the geoid, to show the key problem and computation process of full element modelling on regional gravity field by integral method in space domain.

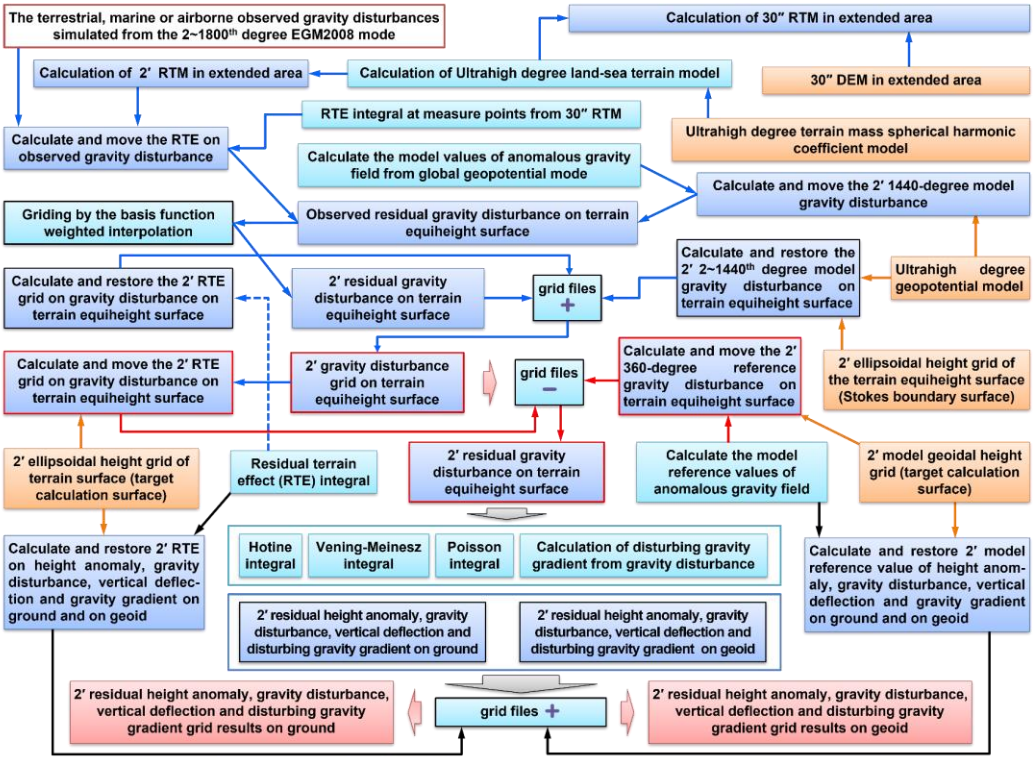
● **Input and output data and related terrain models**

Let terrain data range (extended area, E104.0~111.0°, N24.0~29.0°) ⊃ Calculation area (observed gravity points distribution / the boundary surface range, E104.5~110.5°, N24.5~28.5°) ⊃ Results area (regional gravity field model results range, E105.0~110.0°, N25.0~28.0°) to absorb the edge effect of integral.

(1) The observed gravity disturbance file Obsgrav.txt.

The gravity disturbances at the measurement points are simulated from the 2~1800th degree EGM2008 model. PAGravf4.5 employs the exact same algorithms to process all kinds of terrestrial, marine, and airborne gravity data in an unified way, and there is no need to distinguish whether the measurement point is on the ground, at the air altitude or in the sea area.

The format of the file record: ID (point no/name), longitude (°), latitude (°), ellipsoidal height (m), gravity disturbance (mGal). The distribution of observations is shown in Fig.



Computation process demo of full element modeling on gravity field by integral method

(2) The 3600-degree terrain mass spherical harmonic coefficient model file ETOPOcs3600.dat and 2190-degree geopotential coefficient model file EGM2008.gfc.

The two model files are stored in the directory C:\PAGravf4.5_win64en\data. The 3600-degree global land-sea terrain mass spherical harmonic coefficient model ETOPOcs3600.dat is generated by the PAGravf4.5 function [Ultrahigh degree spherical harmonic analysis of global land-sea terrain model] from the global 2′×2′ land-sea terrain model ETOPO2v2g.

(3) The ground digital elevation model (DEM)

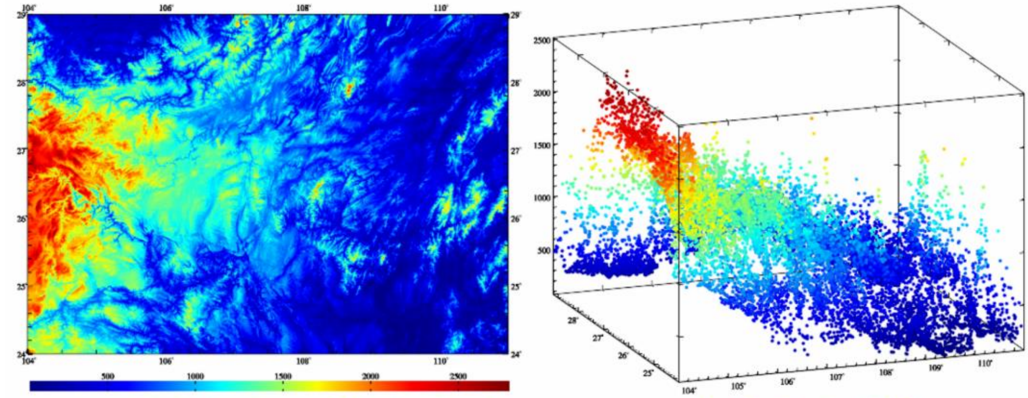
Two resolutions of DEM are required. The high-resolution is employed for observation reduction, that is, to calculate and remove the residual terrain effects on the observations. The other resolution is consistent with the target result resolution and is employed to restore the residual terrain effects on the target field elements. In this example, they are 30" and 2′ respectively, and the corresponding files are extdtm30s.dat and extdtm2m.dat.

(4) The 30″/2′ ellipsoidal height grid files equihgt30s.dat/equihgt2m.dat of terrain equiheight surface.

The ellipsoidal height of the terrain equiheight surface is equal to the sum of the 2~360th degree EGM2008 model height anomaly and mean of the ground normal height. In this example, the terrain equiheight surface is the reduction surface of the ground observed gravity disturbances, which is also the equipotential boundary surface for solving the Stokes boundary value problem.

When the normal (orthometric) heights of the surface are zero, namely whose ellipsoidal height are geoidal height, the reduction surface and the boundary surface are the geoid in the traditional sense.

One of the main purposes of using the terrain equiheight surface as the boundary surface of the Stokes boundary value problem is to make the ground measurement points as close as possible to the boundary surface to suppress the attenuation of the ultrashort-wave signal of gravity field.



digital elevation model (m) and gravity measurement point distribution (BLH)

(5) The 2′ geoidal height grid file geoidhgt2m.dat and 2′ ground ellipsoidal height grid file surfhgt2m.dat.

The geoidal height grid is employed to stand for the space position of the gravity field element grid on the geoid, calculated from the 2~360th degree EGM2008 model.

The ground ellipsoidal height grid is the sum of the 2~360th degree model ground height anomaly grid and ground digital elevation model grid, which is employed to stand for the space position of the ground gravity field element grids.

(6) Output a series of result grid models of the regional gravity field.

Various anomalous gravity field element grid models on the geoid. The 2′×2′ geoidal height, gravity disturbance, vertical deflection vector and disturbing gravity gradient grid models. The geoidal height grid model here also stands for the space position of the anomalous gravity field element grid.

Various anomalous gravity field element grid models on the ground. The 2′×2′ ground height anomaly, gravity disturbance, vertical deflection vector and disturbing gravity gradient gid models, as well as DEM or ground ellipsoidal height grid model which is employed to stand for the space position of the ground anomalous gravity field element grid.

● **Programs call and input-output data flow**

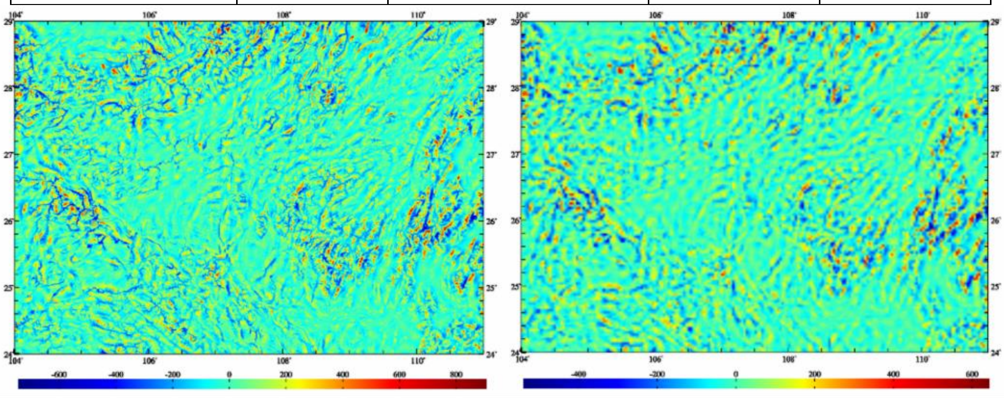
(1) Calculate and remove the model terrain height value, and then construct the 30″ and 2′ residual terrain model (RTM) grids.

Call the function [Calculation of model value for complete Bouguer or residual terrain effects] with the minimum degree 1 and maximum degree 1800, select the calculation type ‘terrain height/sea depth (m)’, and input the land-sea terrain mass spherical harmonic coefficient model file ETOPOcs3600.dat, respectively generate 30″ and 2′ model terrain height grid files mdldtm30s.dat and mdldtm2m.dat from extdtm30s.dat and extdtm2m.dat.

After extdtm30s.dat minus mdldtm30s.dat and extdtm2m.dat minus mdldtm2m.dat, the residual terrain models (RTM) resdtm30s.dat and resdtm2m.dat in the extended area are obtained, as shown in the figure.

When the program output file name is inconsistent with the file name given here, the program output file name need be renamed, and the subsequent ones are the same.

	mean	standard deviation	minimum	maximum
30″ RTM (m)	-0.4626	137.2485	-746.0400	908.8900
2′ RTM (m)	-0.8250	97.5569	-541.2900	645.0400



30″ and 2′ residual terrain model (RTM, m)

(2) Calculate and remove the ultrahigh-degree model gravity disturbances at the measurement points.

Call the function [Calculation of gravity field elements from global geopotential model] with the minimum degree 2 and maximum degree 1440, input the file EGM2008.gfc and observation file Obsgrav.txt, select the type ‘gravity disturbance’, and generate the model gravity disturbance file Obsgravmdl.txt (columns 6) at the measurement points.

Subtract the observed gravity disturbance (column 5) and model gravity disturbance (column 6) in Obsgravmdl.txt, to generate the model residual gravity disturbances (column 7) file Obsgravmdlresd.txt at the measurement points.

Table 2 shows the statistical results of the gravity disturbances at the measurement points before and after the 1440-degree model values removed.

Measurement points (mGal)	mean	standard deviation	minimum	maximum
Observed gravity disturbance	-27.7853	28.7143	-147.4878	74.7074
Model residual of gravity disturbance	-0.3743	6.4755	-35.8263	27.4789

(3) Calculate and remove the residual terrain effects on the gravity disturbances at the measurement points.

Call the function [Numerical integral of land-sea residual terrain effects on various gravity field elements], input the observation file (for the convenience of calculation, here adopts the geodetic coordinates of the measurement points in Obsgravmdlresd.txt), high-resolution DEM extdtm30s.dat, low-pass DEM mdldtm30s.dat and ground ellipsoidal height grid file surfhgt30s.dat, set the integral radius 60km, and generate the residual terrain effect (RTE) on the gravity disturbance, Obsgravresdtm.txt (column 8).

Subtract the model residual gravity disturbance (column 7) and its residual terrain effect (column 8) in Obsgravresdtm.txt, to generate the residual gravity disturbances (column 9) file Obsgravresidual.txt at the measurement points.

After the residual terrain effects removed, the residual gravity disturbances statistical results at the measurement points are shown in Table 3.

Measurement points (mGal)	mean	standard deviation	minimum	maximum
Residual gravity disturbance	6.4474	9.7051	-28.6215	79.6853

In this example, the analytical continuation step using the residual radial gradient (within a height difference of 1000m, the value is small) is omitted. In this case, the residual gravity disturbance at the measurement point is equal to that on the equipotential surface.

So far, the reduction processing of the gravity disturbance from the measurement points to the terrain equiheight surface has been complete.

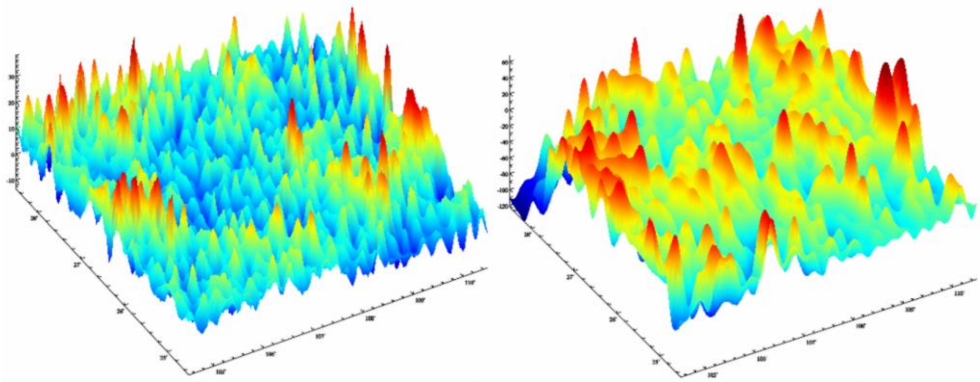
The basic purpose of the statistics in Tables 1 to 3 is to improve the residual terrain effect algorithm and related parameters according to the griding optimization criteria. Since the simulated data lacks sufficient ultrashort wave information of the real gravity field, the optimization criterion analysis process is omitted in this example.

(4) Grid the residual gravity disturbance into 2′×2′ grid on terrain equiheight surface.

Call the function [Gridding of heterogeneous data by basis function weighted interpolation], select ‘equal weights of observations’ (the weights can be estimated with the residual terrain effect as the reference attribute in advance), and grid the 13th and 10th column of attributes (from the file Obsgravresidual.txt), to generate 2′ residual gravity disturbance grid file distgravresidual.dat on the terrain equiheight surface.

(5) Calculate the 2′ EGM2008 1440-degree model value grid of the gravity disturbance on the terrain equiheight surface.

Call the function [Calculation of gravity field elements from global geopotential model] with the minimum degree 2 and maximum degree 1440, input the file EGM2008.gfc and ellipsoidal height grid file (equihgt2m01.dat) of the terrain equiheight surface, select the calculation type ‘gravity disturbance’, to generate 2′ model gravity disturbance grid file distgravmdl.dat on the terrain equiheight surface.



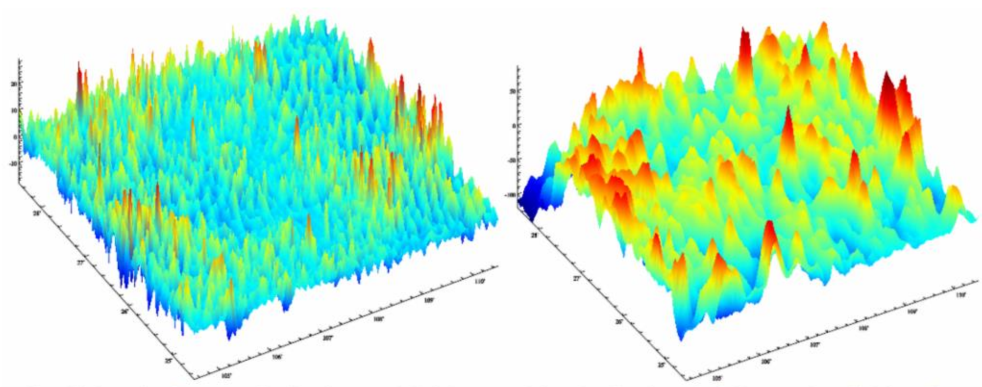
2′ residual gravity disturbance (mGal) and 1440-degree model gravity disturbance (mGal) on terrain equiheight surface

Here, the geopotential model and the minimum and maximum degree are required to be the same as in step (2).

Step (2) removes the model gravity disturbance at the ground measurement point, and step (5) restores the model gravity disturbance grid on the reduction surface. The purpose of this remove-store process is to carry out analytical continuation with the help of the ultrahigh-degree geopotential model.

(6) Calculate the 2′ residual terrain effect grid on the gravity disturbance on the terrain equiheight surface.

Call the function [Numerical integral of land-sea residual terrain effects on various gravity field elements] with extdtm2m.dat as the high-resolution DTM and the 2′ terrain model grid mdldtm2m.dat as low-pass DTM, input the 2′ ground ellipsoidal height grid file surfhgt2m.dat (representing the 2′ residual terrain mass position) and the ellipsoidal height grid file equihgt2m01.dat of the terrain equiheight surface, select the type ‘gravity disturbance’, set the integral radius 60km, and generate the 2′ residual terrain effect grid file distgravresidtm.dat of the gravity disturbance on the terrain equiheight surface.



2′ residual terrain effect on gravity disturbance and 1440-degree model gravity disturbance (mGal) on terrain equiheight surface

Here, the integral radius is required to be the same as in step (3).

(7) Generate the 2′ gravity disturbance grid model on the terrain equiheight surface.

Sum up the residual grid distgravresidual.dat, the ultrahigh degree model value grid distgravmdl.dat and the residual terrain effect grid distgravresidtm.dat three grid files of the gravity disturbance with the same grid specifications, to generate the 2′ gravity disturbance grid model on the terrain equiheight surface.

Steps (1) to (7) are the process of regional gravity field data reduction and processing, and the subsequent steps are the gravity field approach and modelling process according to the geodetic boundary value theory.

(8) According to the requirements of the gravity field approach, select the residual terrain model bandwidth (in this example, the maximum degree of the model terrain is 1440), recalculate the 2' residual terrain model resdtm2m1.dat, and then call the [Numerical integral of land-sea residual terrain effects on various gravity field elements] program, set the integral radius 60km, and calculate the 2′ residual terrain effects on gravity disturbance on the terrain equiheight surface equalsgravrtm.dat.

The terrain effect optimization criteria for gravity field approach are different from that for gridding of anomalous gravity field elements, so those residual terrain models are generally constructed differently. However, it is difficult to process and analyze these problems from the simulated data, and the maximum degree of the model terrain here is directly taken as 1440.

(9) Calculate the 2~720th degree model reference value grid of the gravity disturbance on the terrain equiheight surface.

Call the function [Calculation of gravity field elements from global geopotential model] with the minimum degree 2 and maximum degree 720, input the file EGM2008.gfc and the ellipsoidal height grid file (equihgt2m01.dat) of the terrain equiheight surface, select the calculation type ‘gravity disturbance’, and generate 2′ model gravity disturbance grid file equdisgravmdl.dat on the terrain equiheight surface.

The ultrahigh-degree geopotential model (1440 degree) in steps (2) and (5) is employed for analytical continuation, and the 2~720th degree geopotential model here is employed as the reference gravity field for regional gravity field integral.

(10) Calculate the 2′ residual gravity disturbance grid on the terrain equiheight surface (remove the residual terrain effect and model reference value form gravity disturbance grid).

From the 2′ gravity disturbance grid equdistgrav.dat on the terrain equiheight surface, subtract the 2′ residual terrain effects equdisgravrtm.dat, and then subtract the 2~720th degree model reference value grid equdisgravmdl.dat, to generate the 2' residual gravity disturbance grid equgravresidual.dat on the terrain equiheight surface.

Terrain equiheight surface (mGal)	mean	standard deviation	minimum	maximum
2′ gravity disturbance grid	-17.7675	23.3340	-119.4421	86.6984
Residual terrain effects	-1.6478	4.5234	-31.5185	22.3032
720-degree model values	-23.5523	19.5560	-112.3400	31.4088
Residual gravity disturbances	7.4326	13.1966	-56.9378	71.9857

The basic purpose of statistics here is to improve the residual terrain effect algorithm and parameters according to the gravity field approach optimization criterion (the optimization goal: the standard deviation of residual gravity disturbances in Table 4 is the smallest, and the statistical mean is close to zero). The simulated data lacks sufficient ultrashort wave information of the real gravity field, so the optimization criterion analysis process is omitted in this example.

(11) Call the relevant gravity field integral function to calculate various types of residual anomalous gravity field elements on the ground and on the geoid, respectively.

When calculating the ground residual field elements, input the ground ellipsoidal height grid surfhgt2m2.dat, and when calculating the residual field elements on the geoid, input the model geoidal height grid geoidhgt2m2.dat calculated from the 360-degree EGM2008 model. The integral radius is 90km.

· Call the function [External height anomaly computation using generalized Hotine integral] to calculate the ground residual height anomaly and residual geoidal height grid surfksiresidual.dat and geoidhgtresidual.dat, respectively.

· Call the function [Computation of external vertical deflection from gravity disturbance] to calculate the ground residual vertical deflection grid surfdftresidual.dat and geoidal residual vertical deflection grid geoiddftresidual.dat, respectively.

· Call the function [Computation of Poisson integral on external anomalous gravity field element] to calculate the ground residual gravity disturbance grid surfrgaresidual.dat and geoidal residual gravity disturbance grid geoidrgaresidual.dat, respectively.

· Call the function [Computation of external disturbing gravity gradient from gravity disturbance] to calculate the ground residual disturbing gravity gradient grid surfgrresidual.dat and geoidal residual disturbing gravity gradient grid geoidgrresidual.dat, respectively.

(12) Calculate the residual terrain effects of various anomalous field elements on the ground and on the geoid, respectively (to restore the residual terrain effects of various field elements).

Call the function [Numerical integral of land-sea residual terrain effects on various gravity field elements] with simultaneously selecting the height anomaly, gravity disturbance, vertical deflection and disturbing gravity gradient as the field element types. When calculating the residual terrain effect on the ground field elements, input the ground ellipsoidal height grid surfhgt2m.dat, and when calculating the residual terrain effect on the geoidal field elements, input the model geoidal height grid geoidhgt2m.dat.

Here, the integral radius and RTM resdtm2m.dat are required to be the same as in step (8).

The output grid files of the residual terrain effect (RTE) on the ground field elements include the RTE on height anomaly surfhgt2mrtm.ksi, RTE on gravity disturbance surfhgt2mrtm.rga, RTE on vertical deflection vector surfhgt2mrtm.dft, and RTE on disturbing gravity gradient surfhgt2mrtm.grr.

The output grid files of the residual terrain effect (RTE) on the geoidal field elements include the RTE on geoidal height geoidhgtrtm.ksi, RTE on gravity disturbance geoidhgtrtm.rga, RTE on vertical deflection vector geoidhgtrtm.dft, and RTE on disturbing gravity gradient geoidhgtrtm.grr.

(13) Calculate the 2~720th degree model reference value of various anomalous field elements on the ground and on the geoid, respectively (to restore the model reference value of various field elements).

Call the function [Calculation of gravity field elements from global geopotential model] with simultaneously selecting the height anomaly, gravity disturbance, vertical deflection and disturbing gravity gradient as the field element types. When calculating the model reference value of the ground field elements, input the ground ellipsoidal height grid surfhgt2m.dat, and when calculating the model reference value of the geoidal field elements, input the model geoidal height grid geoidhgt2m.dat.

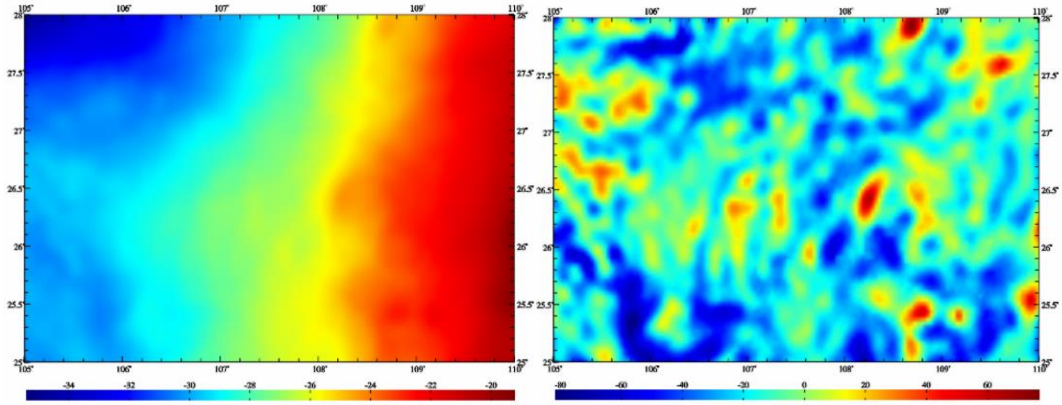
Here, the minimum degree is 2 and the maximum degree is 720, which to be the same as in step (9).

The output grid files of the model reference value (MRV) of the ground field elements include the MRV of height anomaly surfhgt2mgm720.ksi, MRV of gravity disturbance surfhgt2mgm720.rga, MRV of vertical deflection vector surfhgt2mgm720.dft, and MRV of disturbing gravity gradient surfhgt2mgm720.grr.

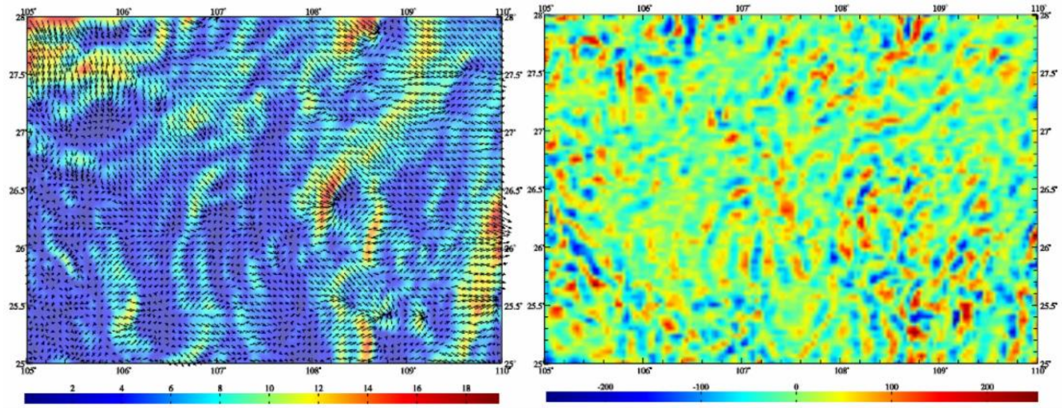
The output grid files of the model reference value (MRV) of the geoidal field elements include the MRV of geoidal height geoidh2mgm720.ksi, MRV of gravity disturbance geoidh2mgm720.rga, MRV of vertical deflection vector geoidh2mgm720.dft, and MRV of disturbing gravity gradient geoidh2mgm720.grr.

(14) Generate the target result grid of various anomalous gravity field elements on the ground.

The residual field element grid, residual terrain effect grid and model reference value grid of various gravity field elements on the ground are respectively summed together to generate the 2′ ground height anomaly grid surfhgtksi2m.dat, ground gravity disturbance grid surfhgtrga2m.dat, ground vertical deflection vector grid surfhgtdft2m.dat and ground disturbing gravity gradient grid surfhgtgrr2m.dat grid, as well as the ground ellipsoidal height grid (indispensable) which is employed to specify the space position of the ground anomalous gravity field element grid.



2′ ground height anomaly (m) and ground gravity disturbance (mGal)



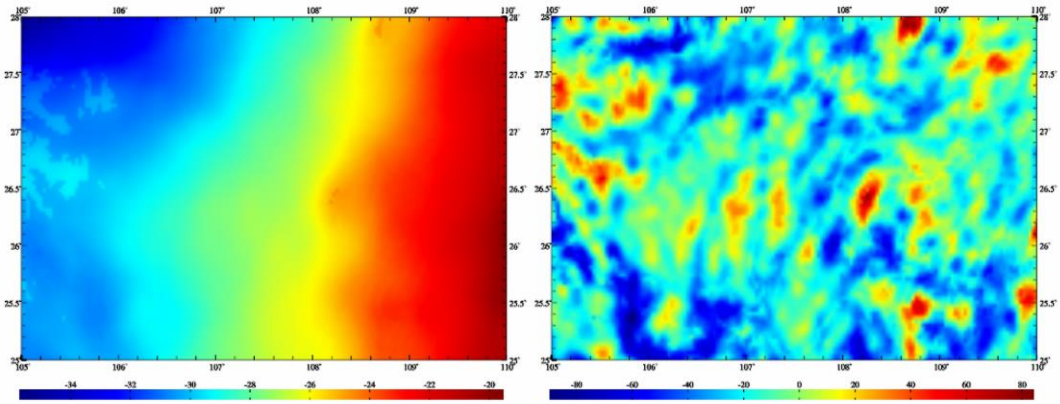
2′ ground vertical deflection (″) and ground disturbing gravity gradient (E)

Element type	Component	mean	standard deviation	minimum	maximum
Ground height anomaly (m)	MRV	-27.4261	3.2927	-36.0901	-20.1879
	RTE	-0.0008	0.0314	-0.1219	0.1363
	Residual	0.6136	0.1608	0.2218	1.2427
	Total	-26.8133	3.3149	-35.4331	-19.1927

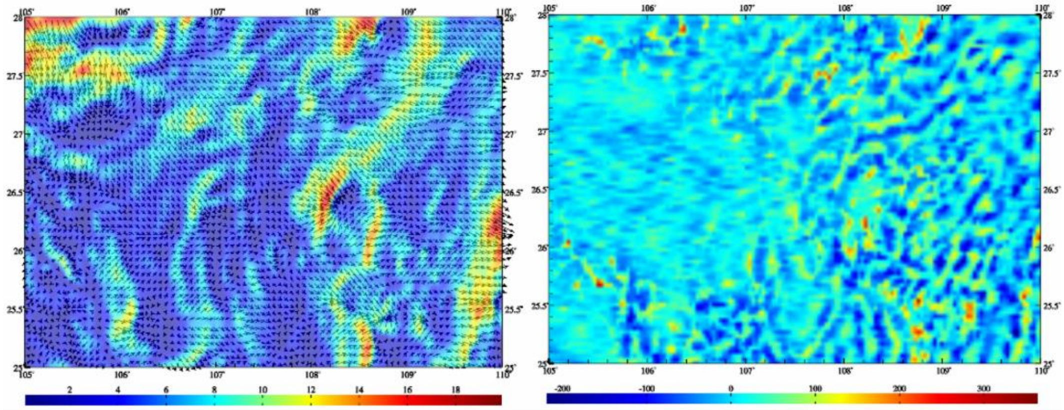
Ground gravity disturbance (mGal)	MRV	-22.4136	15.7881	-76.2475	18.5218
	RTE	-0.7256	0.6856	-7.8832	1.8187
	Residual	6.8912	11.4588	-35.1321	65.7856
	Total	-16.2480	18.7702	-81.8505	77.2212
Ground vertical deflection (″, S)	MRV	1.5543	3.0688	-5.7396	14.0662
	RTE	0.1340	0.7770	-3.5615	3.6938
	Residual	-0.0232	1.4285	-7.3171	6.1334
	Total	1.6649	3.2346	-9.5143	16.4299
Ground vertical deflection (″, W)	MRV	-4.3753	2.7632	-12.4916	5.1844
	RTE	-0.0525	0.9824	-4.3215	5.2168
	Residual	0.0222	1.6118	-7.6123	6.2307
	Total	-4.4050	3.2895	-18.4035	6.0986
Ground disturbing gravity gradient (E)	MRV	-0.2116	9.6634	-34.2497	32.3422
	RTE	-0.0986	49.3177	-269.7075	232.3091
	Residual	-0.0622	15.1553	-62.7129	112.4539
	Total	-0.3736	52.1555	-262.5373	253.7105

(15) Generate the target result grid of various anomalous field elements on geoid.

The residual field element grid, residual terrain effect grid and model reference value grid of various gravity field elements on the geoid are respectively summed together to generate the 2′ geoidal height grid geoidhksi2m.dat, geoidal gravity disturbance grid geoidhrga2m.dat, geoidal vertical deflection vector grid geoidhdft2m.dat and geoidal disturbing gravity gradient grid geoidhgrr2m.dat.



2′ geoidal height (m) and gravity disturbance (mGal)

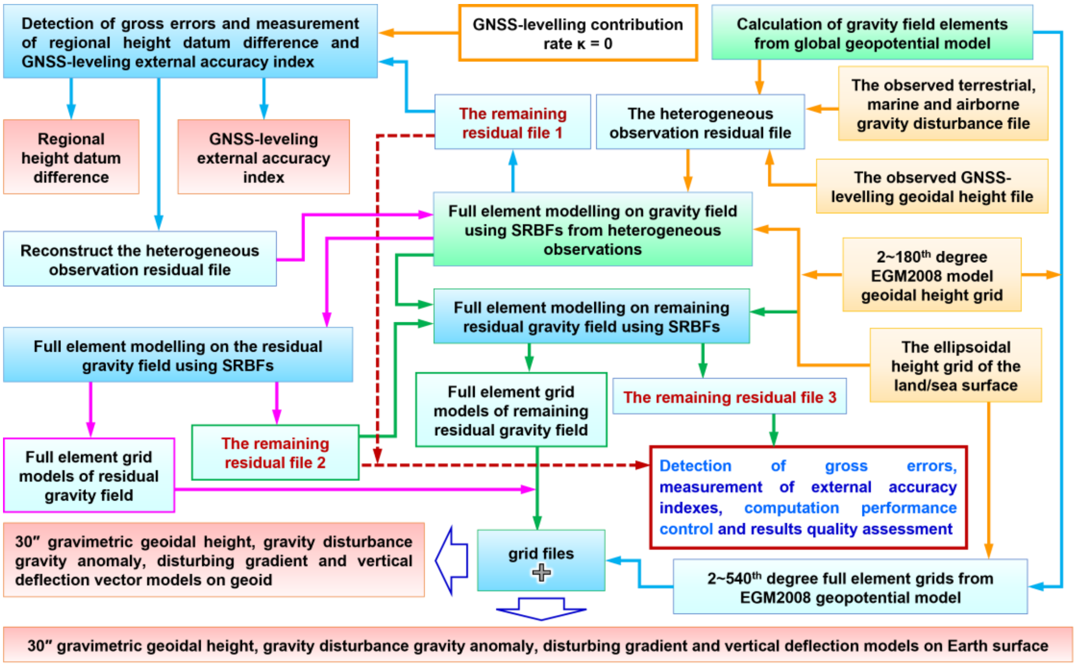


2′ vertical deflection (″) and disturbing gravity gradient (E) on the geoid

4.8.2 Simple process demo of full element modelling on gravity field using SRBFs in orthometric height system

Exercise purpose: From the observed terrestrial, marine and airborne gravity disturbances and GNSS-leveling geoidal heights in orthometric height system, make the full element models on gravity field using spherical radial basis functions (SRBFs) in six steps, in which all the terrain effects are not processed, to quickly master the essentials in observation analysis, computation quality control and full element modelling on regional gravity field.

After the terrain effect processing omitted, SRBF approach process of gravity field is very simple because there is no need for additional continuation reduction, gridding and GNSS leveling fusion process.



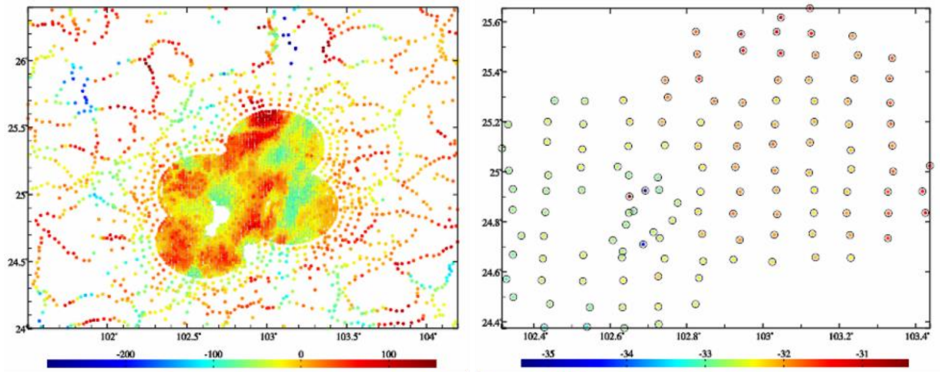
Simple process demo of full element modelling on gravity filed using SRBFs in orthometric height system

● **The observed gravity disturbance and GNSS-levelling data**

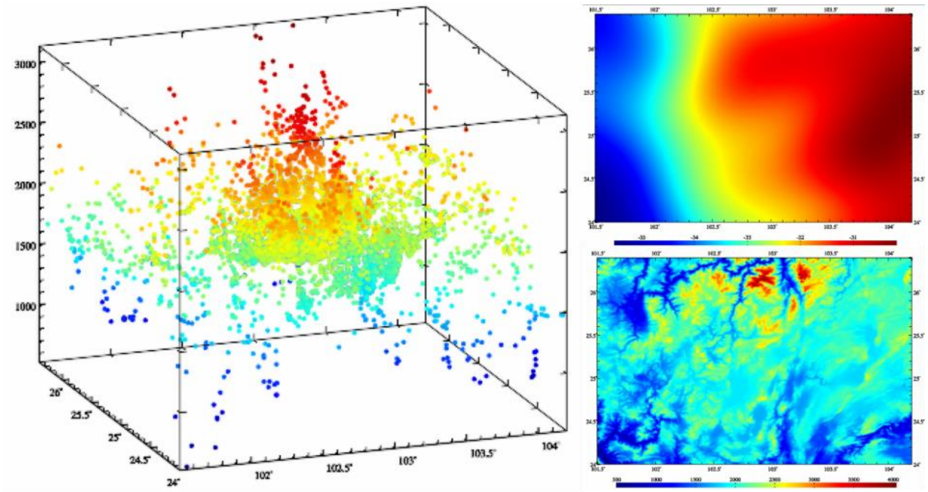
The observed terrestrial, marine and airborne gravity disturbance file obsdistgrav.txt. The file record format: ID, longitude (degree decimal), latitude, ellipsoidal height (m), observed gravity disturbance (mGal), …

The observed GNSS-leveling geoidal height file obsGNSSlgeoid.txt in orthometric height system. The file record format: ID, longitude (degree decimal), latitude, observed geoidal height (m), …

In the example, the observed gravity disturbance and GNSS-leveling geoidal heights are simulated from the EGM2008 model (the 2~1800th degree) in advance.



The observed gravity disturbances (mGal) and observed GNSS-levelling geoidal heights (m)



The distribution of gravity points, 2~180th degree model geoidal height and ellipsoidal height of the terrain surface

It should be noted that since the observed geoidal height by GNSS-leveling is essentially the height anomaly on the geoid in orthometric height system, the geoidal height at GNSS-leveling sites must be the geoidal height, which can be employed by the observed GNSS-leveling geoidal height or the model geoidal height from the EGM2008 model (the 2~180th degree).

● **The ellipsoidal height grid of calculation surface:**

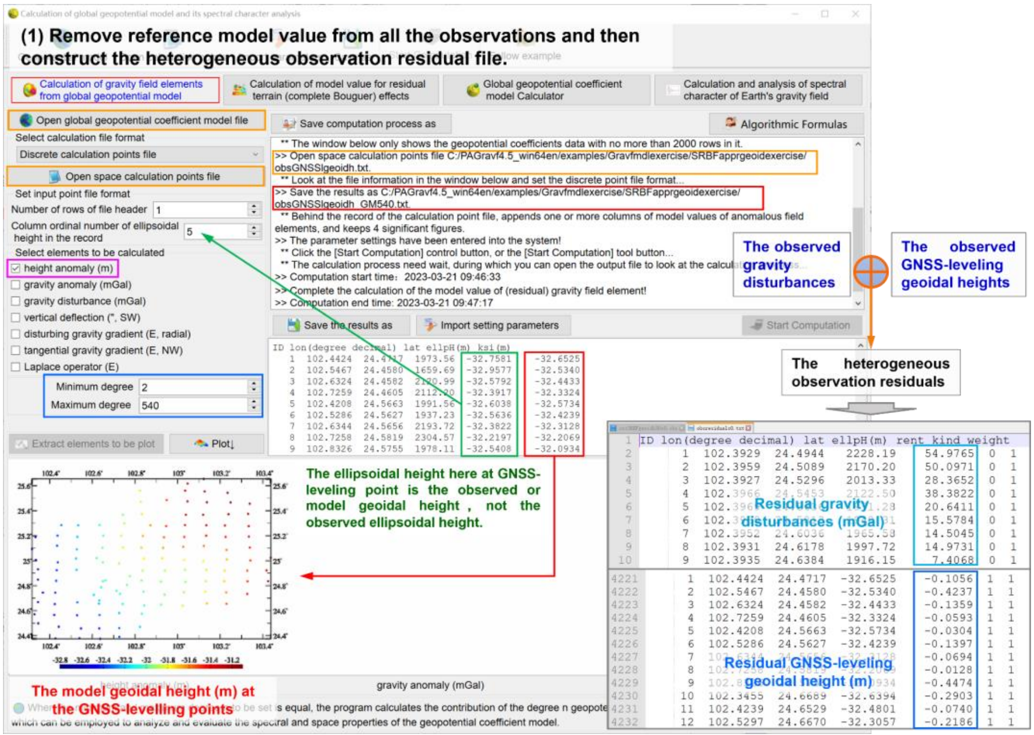
The model geoidal height grid file mdlgeoidh30s.dat calculated from the 2~180th degree geopotential model, which is employed for modelling on gravity field on geoid.

The ellipsoidal height grid file surfhgt30s.dat of the land/sea surface equal to the sum of the digital elevation model grid DEM30s.dat and the model geoidal height grid mdlgeoidh30s.dat, which is employed for modelling on ground gravity field.

Here, it is required that the grid range of the calculation surface is larger than the range of the target area to absorb edge effects.

(1) Remove reference model value from all the observations and then construct the heterogeneous observation residual file.

Call the function [Calculation of gravity field elements from global geopotential model], let the minimum degree 2 and maximum degree 540, and input the file EGM2008.gfc, observed gravity disturbance file obsdistgrav.txt and observed GNSS-levelling geoidal height file obsGNSSlgeoid.txt, calculate and remove the 2~540th degree model value of these observations to generate the heterogeneous observation file obsresiduals0.txt according to the agreed format.



The agreed format of the heterogeneous observation file record: ID (point no / name), longitude (degree decimal), latitude, ellipsoidal height (m), observation, ..., observation type (0 ~ 5), weight, ... The order of the first five attributes is fixed by convention.

The observation types and units: 0 - residual gravity disturbance (mGal), 1 - residual geoidal height (m).

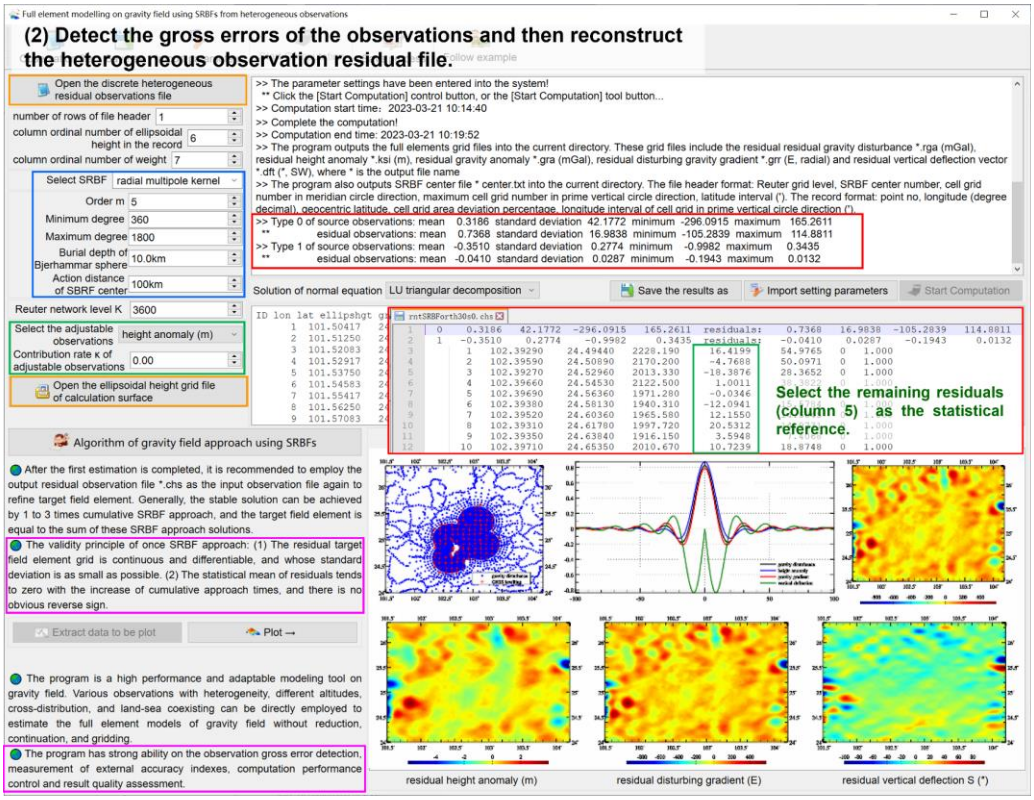
It should be noted that the ellipsoidal height of GNSS-leveling must be the geoidal height and not the ellipsoidal height at GNSS-levelling site.

(2) Detect the gross errors of the observations and then reconstruct the

heterogeneous observation residual file.

Call the program [Full element modelling on gravity field using SRBFs from heterogeneous observations], select the height anomaly as the adjustable observation, let the contribution rate $\kappa = 0$, and input the heterogeneous residual file obsresiduals0.txt and model geoidal height grid file mdlgeoidh30s.dat to estimate the residual gravity field grid rntSRBFgeoidh30s0.xxx on geoid, and get the remaining residual file rntSRBFgeoidh30s0.chs.

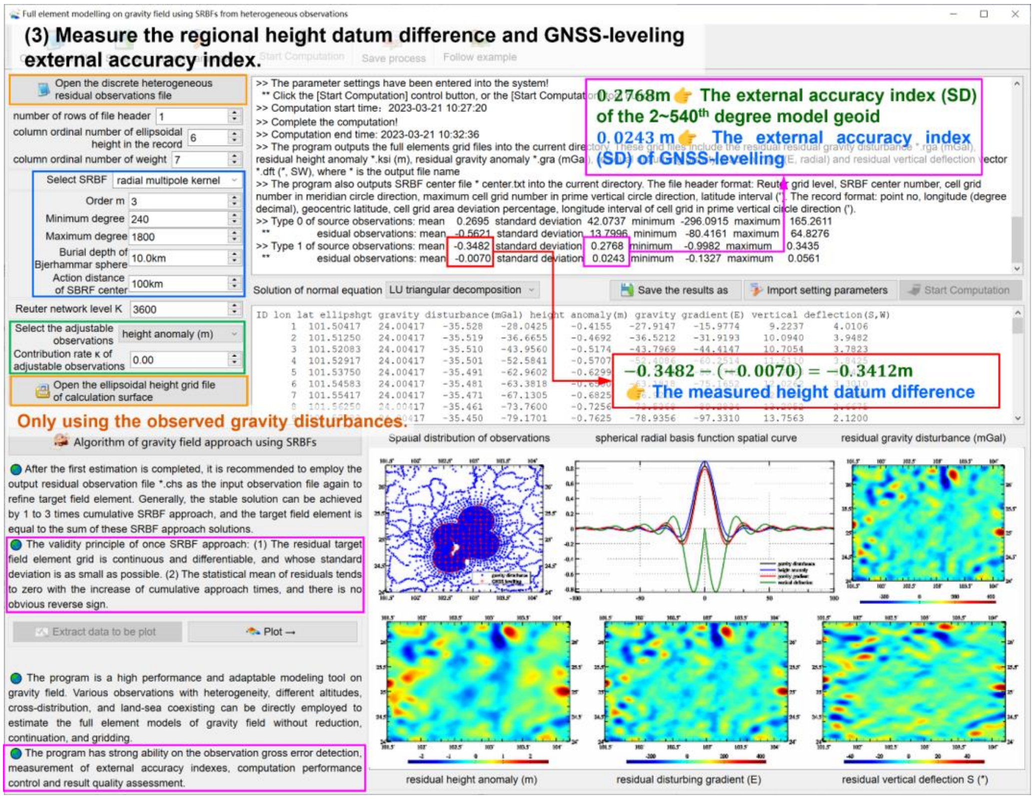
Where, xx=ksi stands for residual geoidal height (m), xxx=rga stands for residual gravity disturbance (mGal), xxx=gra stands for residual gravity anomaly (mGal), xxx=grr stands for residual disturbing gravity gradient (radial, E) and xx=dft stands for residual vertical deflection (SW, ″).



Separate the remaining residual records of the observed GNSS-leveling and observed gravity disturbance from the remaining residual file rntSRBFgeoidh30s0.chs, detect and remove the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points, and then reconstruct the new heterogeneous observation residual file obsresiduals01.txt.

(3) Measure the regional height datum difference and GNSS-leveling external accuracy index.

Replace the input file obsresiduals0.txt with the new heterogeneous observation residual file obsresiduals01.txt and repeat the step (2) to re-estimate the residual gravity field grid rntSRBFdatum30s.xxx on geoid and get the new remaining residual file rntSRBFdatum30s.chs.



Since the contribution rate of GNSS-levelling $\kappa = 0$ is set in advance, it is essentially here directly to measure the external accuracy index of the observed GNSS levelling only using the observed gravity disturbances. Before and after gross error removed, the statistical results on the observation residuals are as follows.

		number of points	mean	standard deviation	minimum	maximum
Gravity disturbance (mGal)	Original residuals	4219	0.3186	42.1772	-296.0915	165.2611
	Residuals without error	4215	0.2695	42.0737	-296.0915	165.2611
	Remaining residuals	4215	-0.4584	13.6071	-61.1040	64.8276
GNSS levelling geoidal height (m)	Original residuals	125	-0.3510	0.2774	-0.9982	0.3435
	Residuals without error	124	-0.3482①	0.2768	-0.9982	0.3435
	Remaining residuals	124	-0.0070②	0.0243③	-0.1328	0.0561

The statistical mean ① minus ② of the GNSS-levelling remaining residuals in the table, that is, -0.3482① - (-0.0070②) = -0.3412 m, is the difference between the regional height datum and global height datum (gravimetric geoid). Here provides the SRBF measurement method for regional height datum difference.

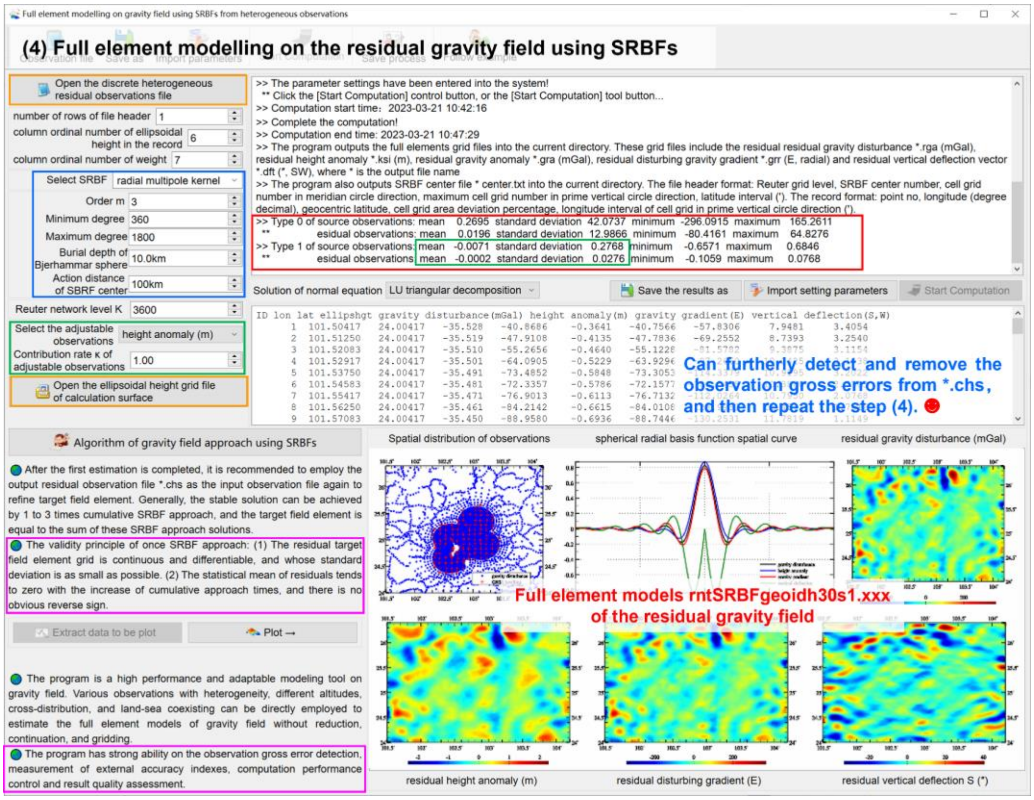
In the table, 0.0243③m is the external accuracy index of the observed GNSS-levelling expressed as standard deviation, that is, 2.43 cm. Here provides the SRBF measurement method for the external accuracy index of GNSS- leveling. The result indicates that the external accuracy of the observed GNSS-leveling is not bad than 2.43 cm (standard deviation).

In general, it is necessary to make 1 to 2 cumulative SRBF approach with *.chs as the input file to obtain the minimum of the standard deviation of the GNSS-levelling remaining residuals as the external accuracy index, and this process is omitted in this example.

After removing the regional height datum difference of -0.3411m from the GNSS-levelling residuals, the new heterogeneous observation residual file obsresiduals1.txt is reconstructed again.

(4) Full element modelling on the residual gravity field using SRBFs

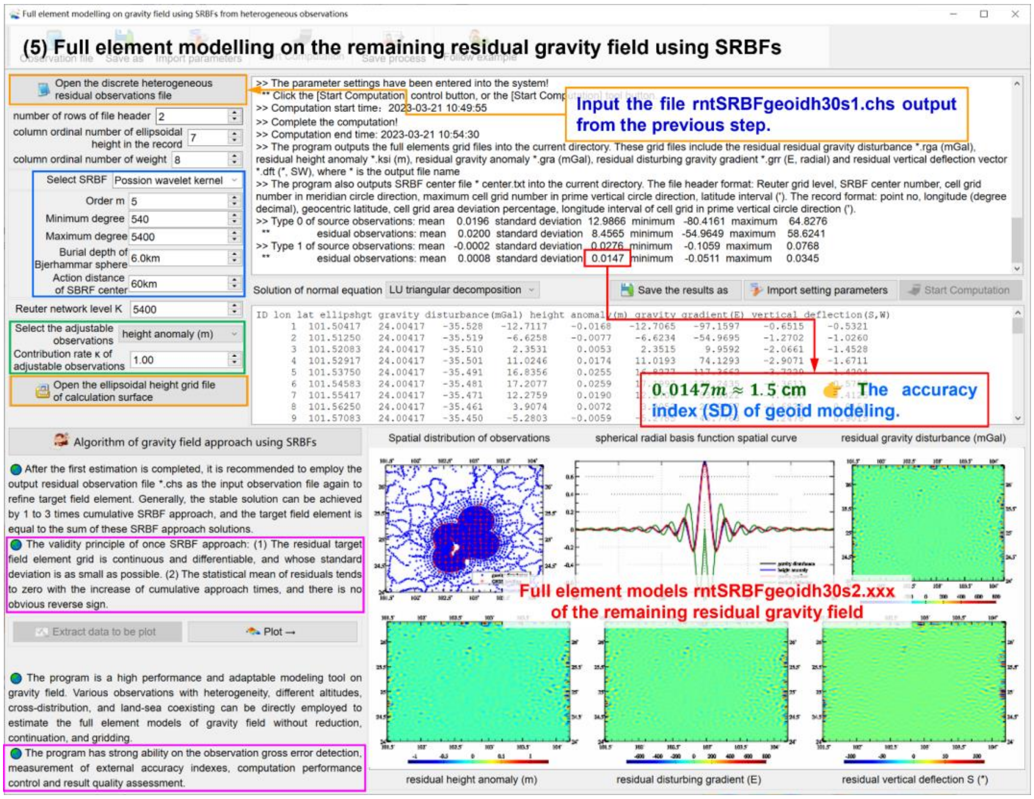
Call the program [Full element modelling on gravity field using SRBFs from heterogeneous observations], let the contribution rate $\kappa = 1$, and input the heterogeneous residual file obsresiduals1.txt and the model geoidal height grid file mdlgeoidh30s.dat to estimate the 30″ residual gravity field grid rntSRBFgeoidh30s1.xxx on geoid, and get the remaining residual file rntSRBFgeoidh30s1.chs.



[The quality control scheme] You can furtherly detect and remove the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points from the remaining residual file rntSRBFgeoidh30s1.chs, and then repeat the step (4). This process is omitted in this example.

(5) Full element modelling on the remaining residual gravity field using SRBFs

Call the program [Full element modelling on gravity field using SRBFs from heterogeneous observations], let the contribution rate $\kappa = 1$, and input the remaining residual file rntSRBFgeoidh30s1.chs and model geoidal height grid file mdlgeoidh30s.dat to estimate the 30″ remaining residual gravity field grid rntSRBFgeoidh30s1.xxx on geoid, and get the remaining residual file rntSRBFgeoidh30s1.chs.



		mean	standard deviation	minimum	maximum
Residual gravity disturbance (mGal)	Residuals	0.3523	42.1561	-296.0915	165.2611
	First SRBF	0.0196	12.9866	-80.4161	64.8276
	Second SRBF	0.0200	8.4565	-54.9649	58.6241
Residual GNSS-levelling geoidal height (m)	Residuals	-0.0071	0.2768	-0.6571	0.6846
	First SRBF	-0.0002	0.0276	-0.1059	0.0768
	Second SRBF	0.0008	0.0147④	-0.0511	0.0345

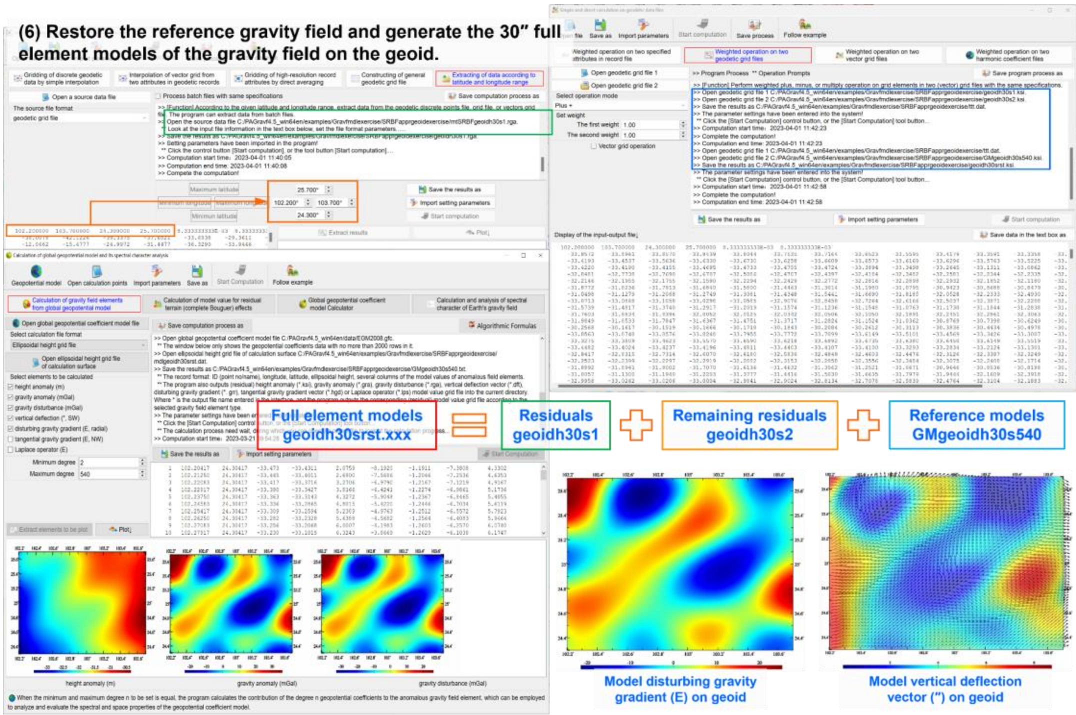
In the table, 0.0147④m = 1.5cm can be considered as the accuracy index of geoid modelling.

[The quality control scheme] You can furtherly detect and remove again the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points from the remaining residual file SRBFgeoidheight30s2.chs, and then repeat from step (4). This process is omitted in this example.

You can also do further cumulative SRBF approach to improve the results. This example omits this process.

(6) Restore the reference gravity field and generate the 30″ full element models of the gravity field on the geoid.

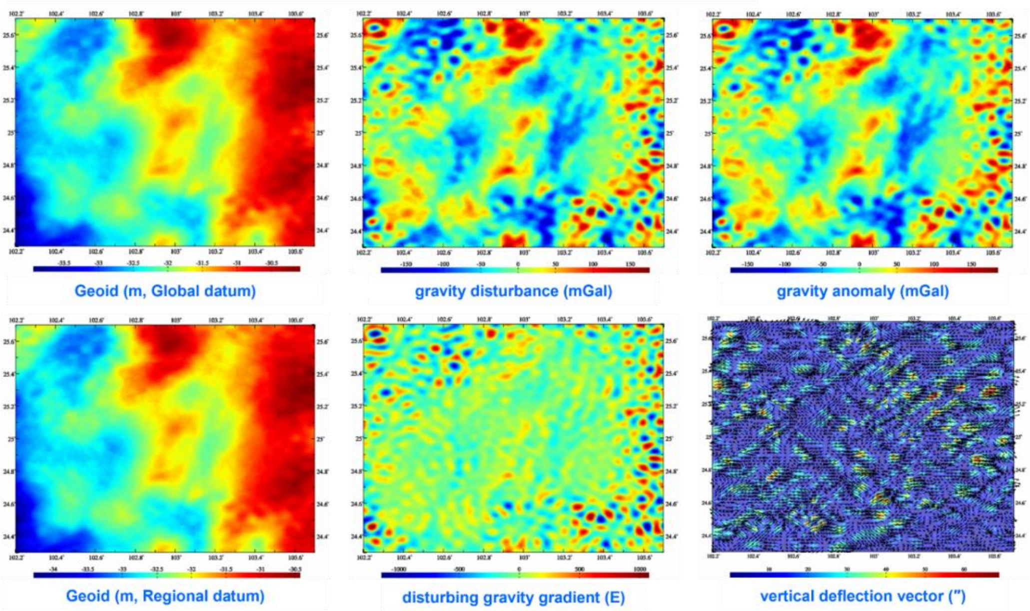
Call the function [Calculation of gravity field elements from global geopotential model], let the minimum degree 2 and maximum degree 540, input the file EGM2008.gfc, and the model geoidal height grid file mdlgeoidh30srst.dat (from mdlgeoidh30s.dat with grid edge removed), to calculate the full element grid GMgeoidh30s540.xxx of the reference gravity field on geoid.



Add the residual gravity field grid geoidh30s1.xxx (from SRBFgeoidheight30s1.xxx with grid edge removed) and remaining residual gravity field grid geoidh30s2.xxx (from SRBFgeoidheight30s2.xxx with grid edge removed) to the reference gravity field grid GMgeoidh30s540.xxx, the 30″ full element gravity field models geoidh30srst.xxx on the geoid are obtained, which include the 30″ gravimetric geoidal height grid (geoidh30srst.ksi, m), gravity disturbance grid (geoidh30srst.rga, mGal), gravity anomaly grid (geoidh30srst.gra, mGal), disturbing gravity gradient grid (geoidh30srst.grr, radial, E) and vertical deflection vector grid (geoidh30srst.dft, SW, ″).

Add the regional height datum difference -0.3411m to the 30″ gravimetric geoidal height grid geoidh30srst.ksi in global height datum, the 30″ gravimetric geoidal height grid geoidh30srgn.ksi in regional height datum can be obtained.

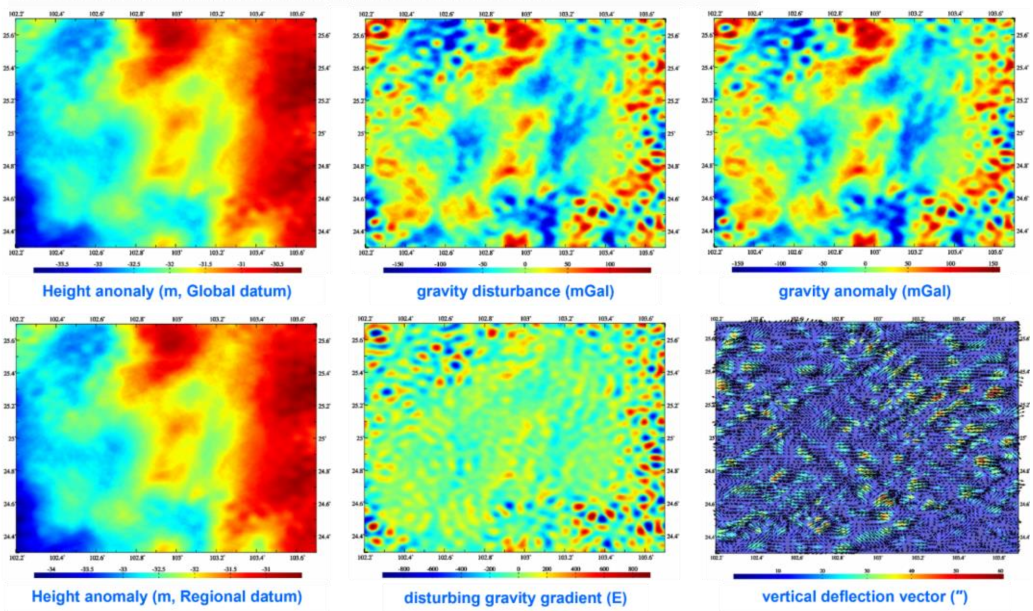
30″×30″ full element models of gravity field on geoid



So far, the full element modelling on gravity field on the geoid have been completed.

○ **Let the terrain surface as the calculation surface, and directly generate the 30″ full element models of the gravity field on the terrain surface.**

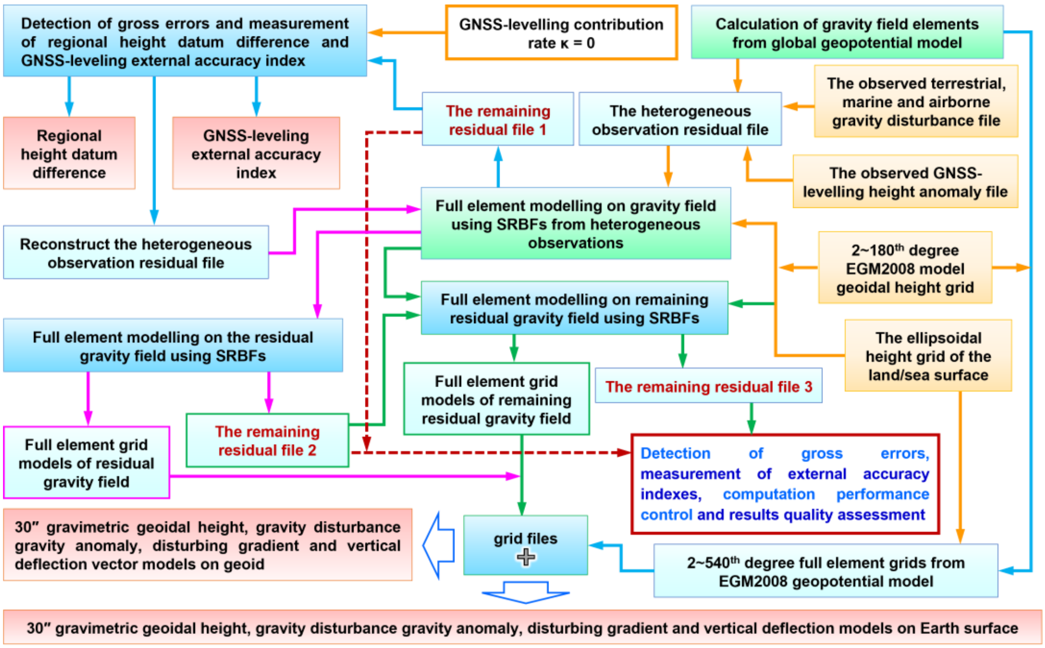
30″×30″ full element models of gravity field on terrain surface



In step (3) to step (6) above, the input data file and all the parameter settings are kept the same, and only the calculation surface is changed to the terrain surface surfhgt30s.dat. Using the same computation process, you can synchronously obtain the 30″ full element models surfhgt30srst.xxx of the gravity field on the terrain surface, which include the 30″ gravimetric ground height anomaly grid (surfhgt30srst.ksi, m, in global height datum), ground gravity disturbance grid (surfhgt30srst.rga, mGal), ground gravity anomaly grid (surfhgt30srst.gra, mGal), ground disturbing gravity gradient grid (surfhgt30srst.grr, radial, E), ground vertical deflection vector grid (surfhgt30srst.dft, SW, ″) and ground height anomaly grid (surfhgt30srgn.ksi, m, in regional height datum).

4.8.3 Simple process demo of full element modelling on gravity field using SRBFs in normal height system

Exercise purpose: From the observed terrestrial, marine and airborne gravity disturbances and GNSS-leveling height anomalies in normal height system, make the full element models on gravity field using spherical radial basis functions (SRBFs) in six steps, in which all the terrain effects are not processed, to quickly master the essentials in observation analysis, computation performance control and full element modelling on regional gravity field.



Simple process demo of full element modelling on gravity filed using SRBFs in normal height system

In this section, the observed GNSS-levelling height anomaly in the normal height system is employed to replace the observed GNSS-levelling geoidal height in orthometric height system in the 4.8.2, and the simple process of full element modelling on gravity filed using SRBFs is introduced. In the both cases, there is only a slight difference in the processing of the observed GNSS-levelling data, and the other modelling processes are the same. For the convenience, here gives the complete quick process.

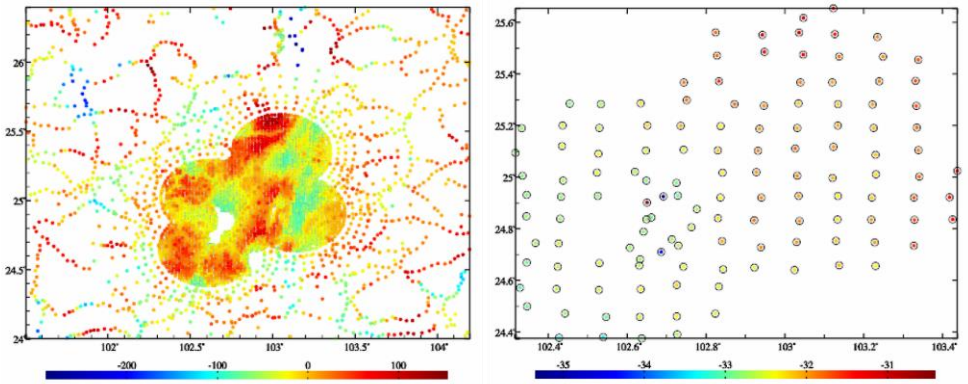
After the terrain effect processing omitted, SRBF approach process of gravity field is very simple because there is no need for additional continuation reduction, gridding and GNSS-leveling fusion process.

● **The observed gravity disturbance and GNSS-levelling data**

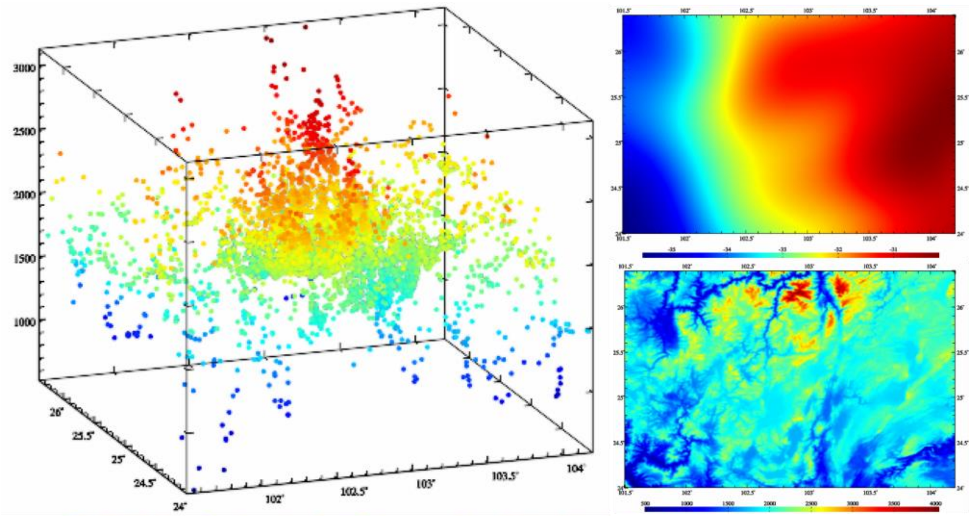
The observed terrestrial, marine and airborne gravity disturbance file obsdistgrav.txt. The file record format: ID, longitude (degree decimal), latitude, ellipsoidal height (m), observed gravity disturbance (mGal), …

The observed GNSS-leveling height anomaly file obsGNSSlksi.txt in normal height system. The file record format: ID, longitude (degree decimal), latitude, ellipsoidal height (m), observed height anomaly (m), …

In the example, the observed gravity disturbances and GNSS-leveling anomalies are simulated from the EGM2008 model (the 2~1800th degree) in advance.



The observed gravity disturbances (mGal) and observed GNSS-levelling height anomalies (m)



The distribution of gravity points, 2~180th degree model geoidal height and ellipsoidal height of the terrain surface

● **The ellipsoidal height grid of calculation surface:**

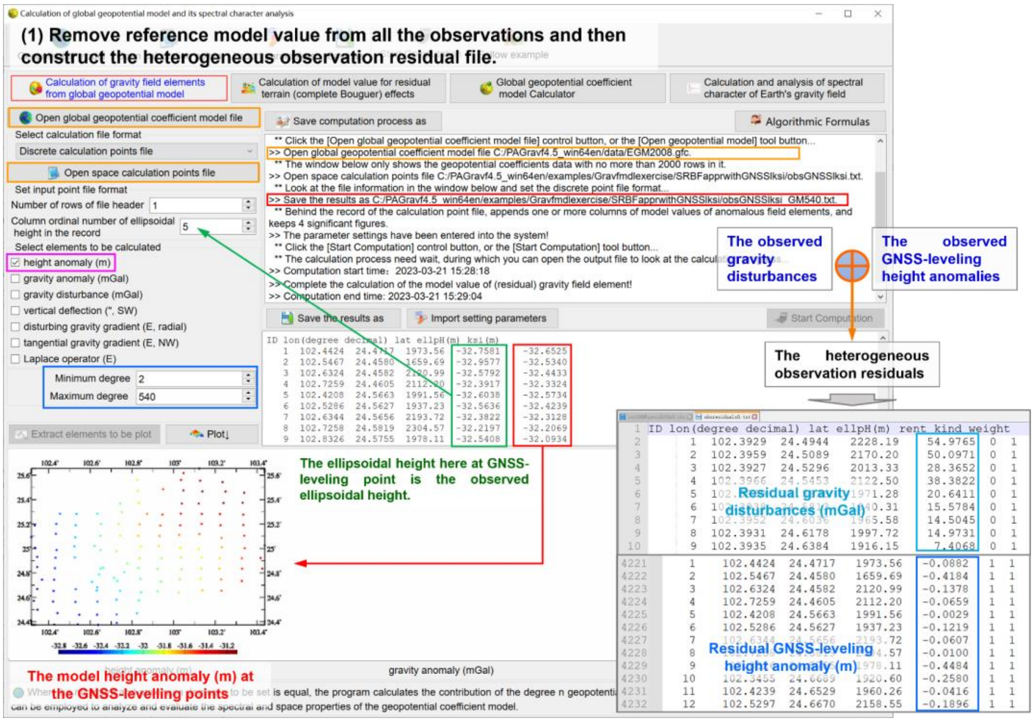
The model geoidal height grid file mdlgeoidh30s.dat calculated from the 2~180th degree geopotential model, which is employed for modelling on gravity field on geoid.

The ellipsoidal height grid file surfhgt30s.dat of the land/sea surface equal to the sum of the digital elevation model grid DEM30s.dat and model geoidal height grid mdlgeoidh30s.dat, which is employed for modelling on ground gravity field.

Here, it is required that the grid range of the calculation surface is larger than the range of the target area to absorb edge effects.

(1) Remove reference model value from all the observations and then construct the heterogeneous observation residual file.

Call the function [Calculation of gravity field elements from global geopotential model], let the minimum degree 2 and maximum degree 540, and input the file EGM2008.gfc, observed gravity disturbance file obsdistgrav.txt and observed GNSS-levelling height anomaly file obsGNSSlksi.txt, calculate and remove the 2~540th degree model value of these observations to generate the heterogeneous observation file obsresiduals0.txt according to the agreed format.



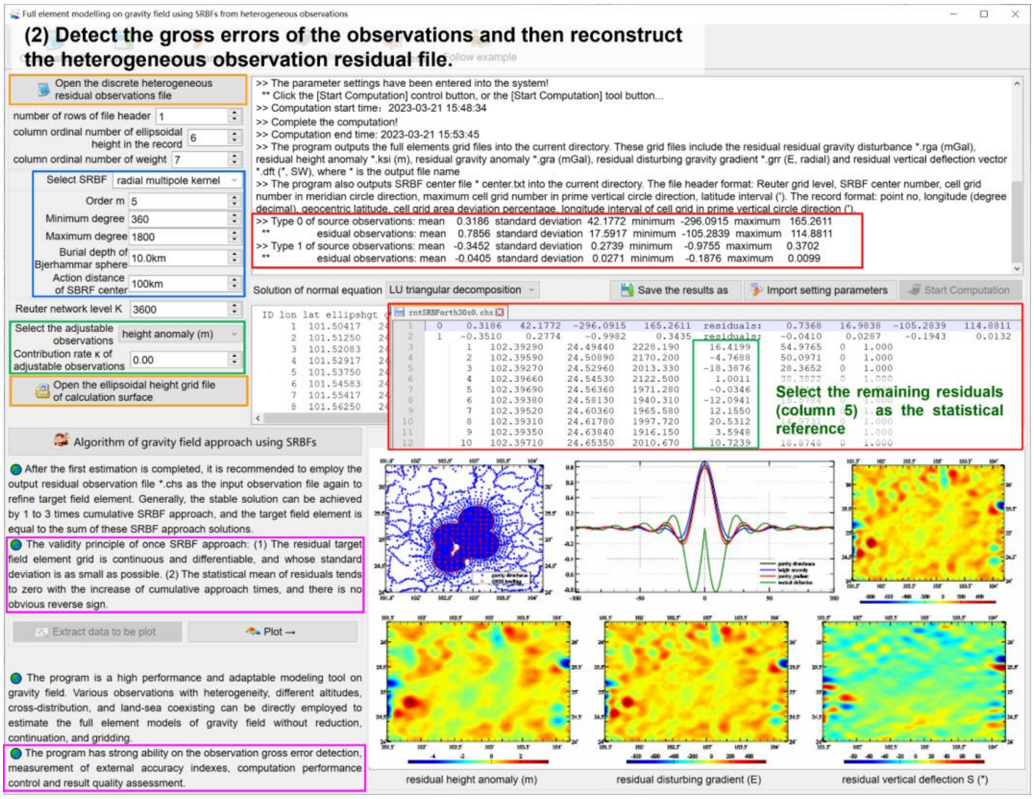
The agreed format of the heterogeneous observation file record: ID (point no/name), longitude (degree decimal), latitude, ellipsoidal height (m), observation, ..., observation type (0 ~ 5), weight, ... The order of the first five attributes is fixed by convention.

The observation types and units: 0 - residual gravity disturbance (mGal), 1 - residual height anomaly (m).

(2) Detect the gross errors of the observations and then reconstruct the heterogeneous observation residual file.

Call the program [Full element modelling on gravity field using SRBFs from heterogeneous observations], select the height anomaly as the adjustable observation, let the contribution rate $\kappa = 0$, and input the heterogeneous residual file obsresiduals0.txt and terrain surface ellipsoidal height grid file surfhgt30s.dat to estimate the residual gravity field grid SRBFsurfhgt30s0.xxx on geoid, and get the remaining residual file SRBFsurfhgt30s0.chs.

Where, xx=ksi stands for residual height anomaly (m), xxx=rga stands for residual gravity disturbance (mGal), xxx=gra stands for residual gravity anomaly (mGal), xxx=grr stands for residual disturbing gravity gradient (radial, E) and xx=dft stands for residual vertical deflection (SW, ″).

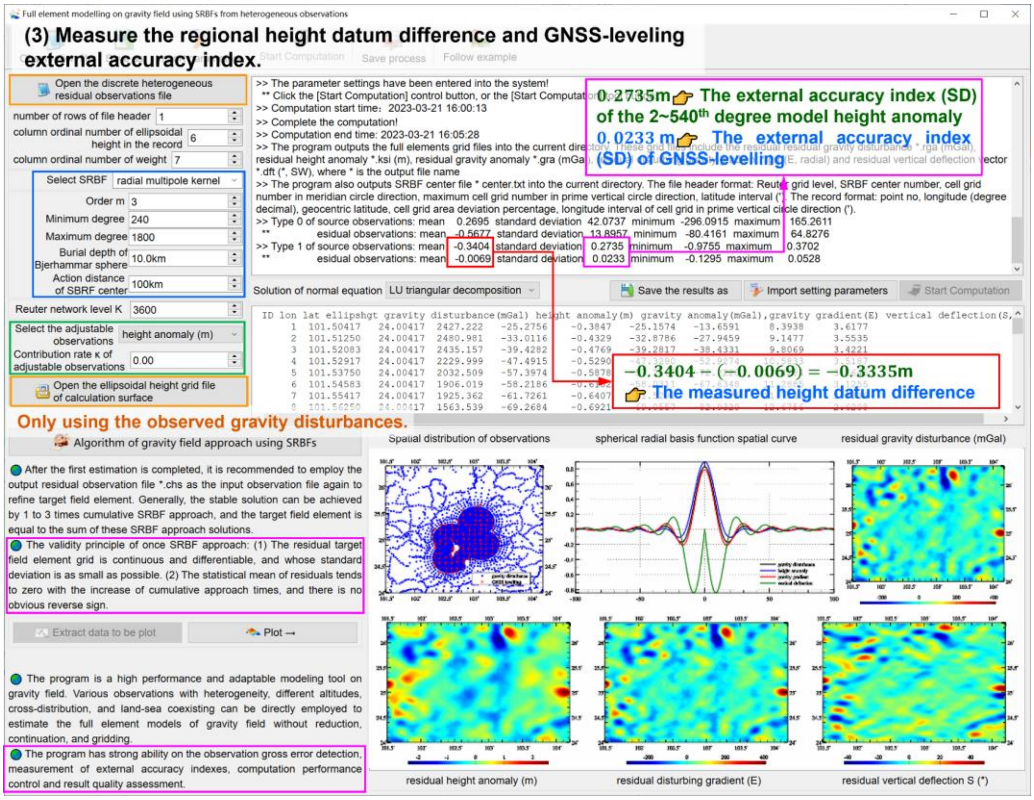


Separate the remaining residual records of the observed GNSS-leveling and observed gravity disturbance from the remaining residual file SRBFsurfhgt30s0.chs, detect and remove the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points, and then reconstruct the new heterogeneous observation residual file obsresiduals01.txt.

(3) Measure the regional height datum difference and GNSS-leveling external accuracy index.

Replace the input file obsresiduals0.txt with the new heterogeneous observation residual file obsresiduals01.txt and repeat the step (2) to re-estimate the residual gravity field grid rntSRBFdatum30s.xxx on terrain surface and get the new remaining residual file rntSRBFdatum30s.chs.

Since the contribution rate of GNSS-levelling $\kappa = 0$ is set in advance, it is essentially here directly to measure the external accuracy index of the observed GNSS levelling only using the observed gravity disturbances.



Before and after gross error removed, the statistical results on the observation residuals are as follows.

		number of points	mean	standard deviation	minimum	maximum
Gravity disturbance (mGal)	Original residuals	4219	0.3186	42.1772	-296.0915	165.2611
	Residuals without error	4215	0.2695	42.0737	-296.0915	165.2611
	Remaining residuals	4215	-0.5677	13.8957	-80.4161	64.8276
GNSS levelling height anomaly (m)	Original residuals	125	-0.3452	0.2739	-0.9755	0.3702
	Residuals without error	123	-0.3404①	0.2735	-0.9755	0.3702
	Remaining residuals	123	-0.0069②	0.0233③	-0.1295	0.0528

The statistical mean ① minus ② of the GNSS-levelling remaining residuals in the table, that is, -0.3404① - (-0.0069②) = -0.3335m, is the difference between the regional height datum and the global height datum (gravimetric geoid). Here provides the SRBF measurement method for regional height datum difference.

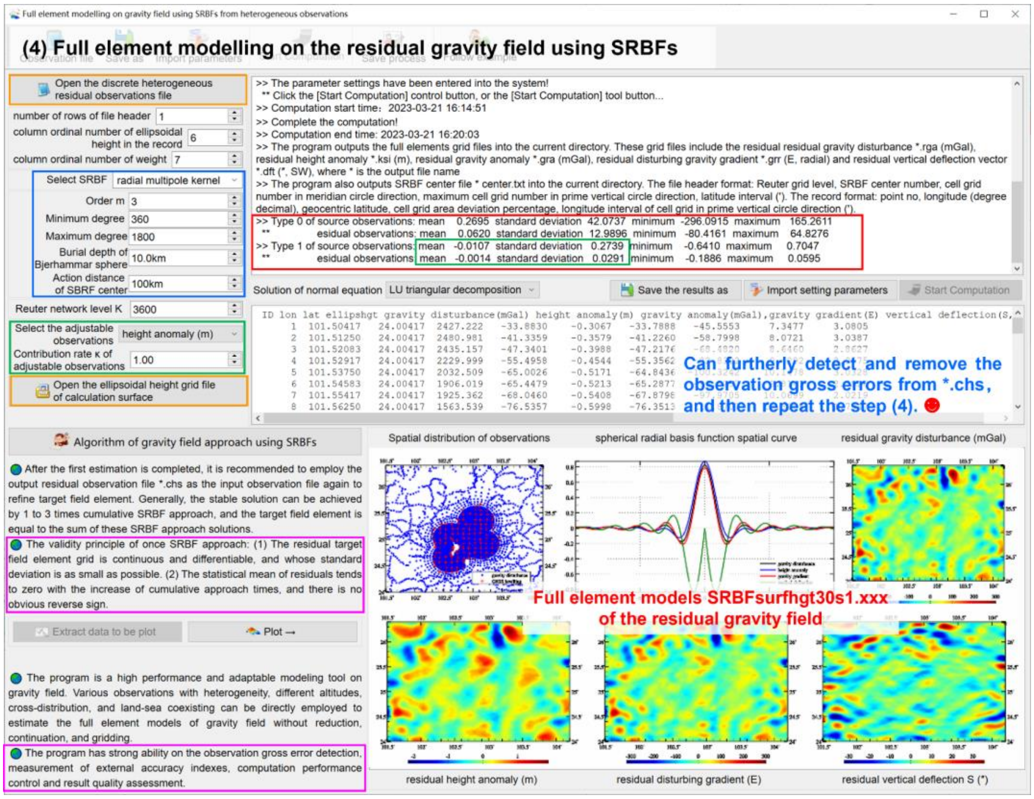
In the table, 0.0233③m is the external accuracy index of the observed GNSS-levelling expressed as standard deviation, that is, 2.33cm. Here provides the SRBF measurement method for the external accuracy index of GNSS- leveling. The result indicates that the external accuracy of GNSS-leveling is not bad than 2.33 cm (SD).

In general, it is necessary to make 1 to 2 cumulative SRBF approach with *.chs as the input file to obtain the minimum of standard deviation of GNSS-levelling remaining residuals as the external accuracy index, and this process is omitted in this example.

After removing the regional height datum difference of -0.3345m from GNSS-levelling residuals, the new heterogeneous observation residual file obsresiduals1.txt is reconstructed again.

(4) Full element modelling on the residual gravity field using SRBFs

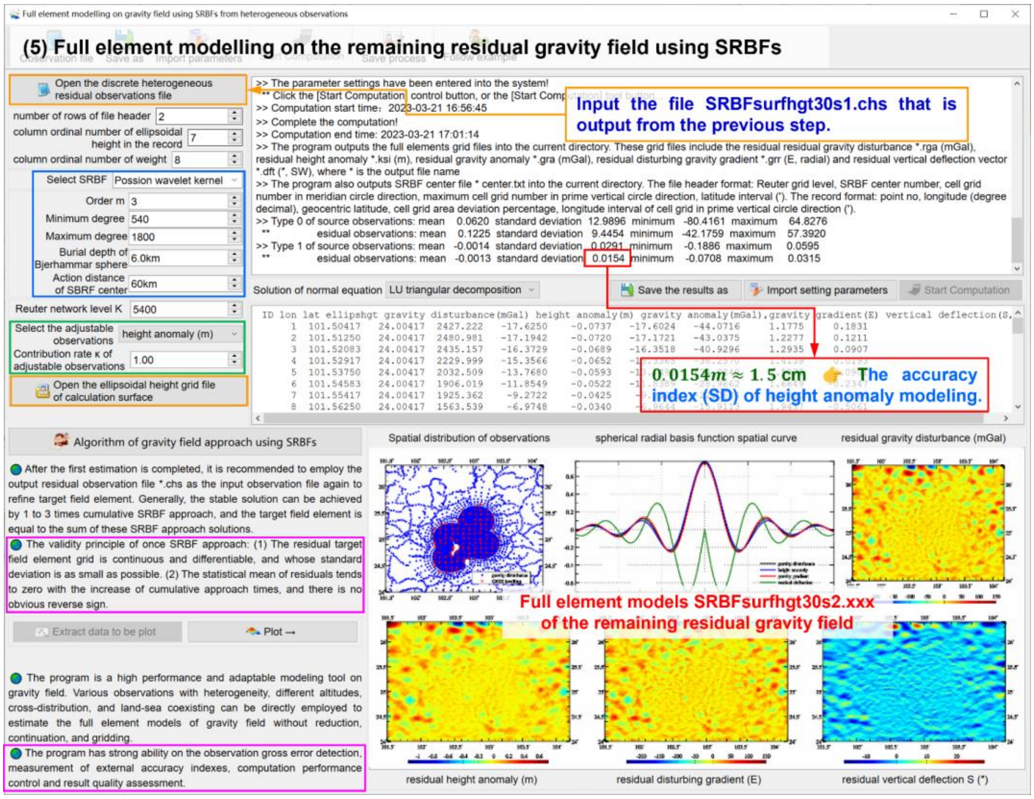
Call the program [Full element modelling on gravity field using SRBFs from heterogeneous observations], let the contribution rate $\kappa = 1$, and input the heterogeneous residual file obsresiduals1.txt and terrain surface ellipsoidal height grid file surfhgt30s.dat to estimate the 30″ residual gravity field grid SRBFsurfhgt30s1.xxx on terrain surface, and get the remaining residual file SRBFsurfhgt30s1.chs.



[The quality control scheme] You can furtherly detect and remove the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points from the remaining residual file SRBFsurfhgt30s1.chs, and then repeat the step (4). This process is omitted in this example.

(5) Full element modelling on the remaining residual gravity field using SRBFs

Call the program [Full element modelling on gravity field using SRBFs from heterogeneous observations], let the contribution rate $\kappa = 1$, and input the remaining residual file SRBFsurfhgt30s1.chs and terrain surface ellipsoidal height grid file surfhgt30s.dat to estimate the 30″ remaining residual field grid SRBFsurfhgt30s2.xxx on the terrain surface, and get the remaining residual file SRBFsurfhgt30s2.chs.



In the table below, 0.0154④m = 1.5cm can be considered as the accuracy index of ground height anomaly (quasigeoid) modelling.

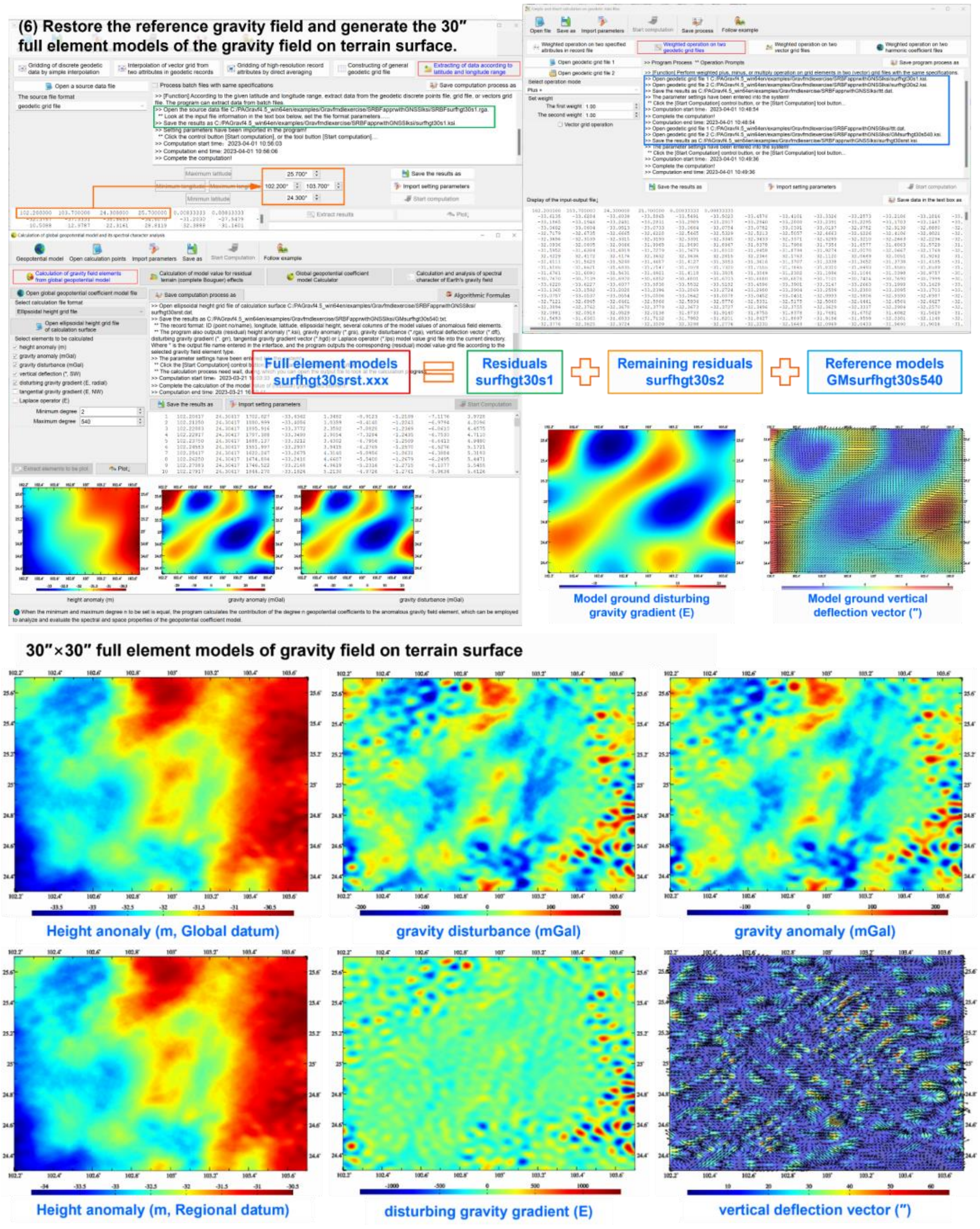
		mean	standard deviation	minimum	maximum
Residual gravity disturbance (mGal)	Residuals	0.2695	42.0737	-296.0915	165.2611
	First SRBF	0.0620	12.9866	-80.4161	64.8276
	Second SRBF	0.1309	8.5135	-50.6030	57.3920
Residual GNSS-levelling height anomaly (m)	Residuals	-0.0071	0.2768	-0.6571	0.6846
	First SRBF	-0.0014	0.0291	-0.1886	0.0595
	Second SRBF	-0.0013	0.0154④	-0.0708	0.0315

[The quality control scheme] You can furtherly detect and remove again the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points from the remaining residual file SRBFsurfhgt30s2.chs, and then repeat from step (4). This process is omitted in this example.

You can also do further cumulative SRBF approach to improve the results. This example omits this process.

(6) Restore the reference gravity field and generate the 30″ full element models of the gravity field on the terrain surface.

Call the function [Calculation of gravity field elements from global geopotential model], let the minimum degree 2 and maximum degree 540, input the file EGM2008.gfc, and the terrain surface ellipsoidal height grid file surfhgt30srst.dat (from surfhgt30s.dat with grid edge removed), to calculate the full element grid GMsurfhgt30s540.xxx of the reference gravity field on the terrain surface.



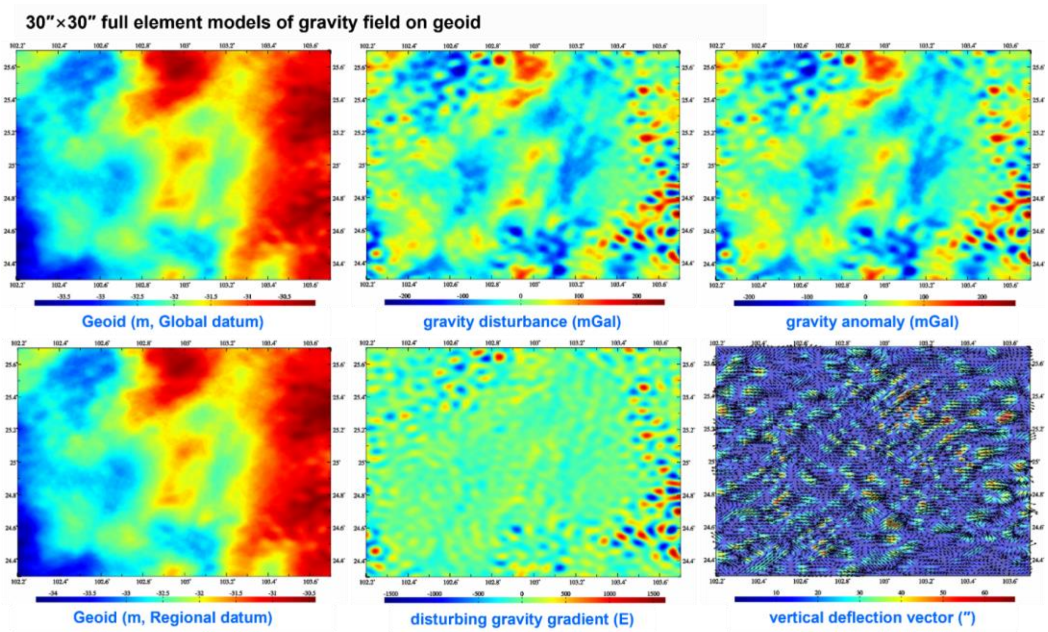
Add the residual gravity field grid surfhgt30s1.xxx (from SRBFsurfhgt30s0.xxx with grid edge removed) and remaining residual gravity field grid surfhgt30s2.xxx (from SRBFsurfhgt30s1.xxx with grid edge removed) to the reference gravity field grid GMsurfhgt30s540.xxx, the 30″ full element gravity field models surfhgt30srst.xxx on the terrain surface are obtained, which include the 30″ gravimetric ground height anomaly grid (surfhgt30srst.ksi, m), ground gravity disturbance grid (surfhgt30srst.rga, mGal), ground gravity anomaly grid (surfhgt30srst.gra, mGal), ground disturbing gravity gradient grid (surfhgt30srst.grr, radial, E) and ground vertical deflection vector grid (surfhgt30srst.dft, SW, ″).

Add the regional height datum difference -0.3411m to the 30″ gravimetric height anomaly grid surfhgt30srst.ksi in global height datum, the 30″ gravimetric height anomaly grid surfhgt30srgn.ksi in regional height datum can be obtained.

So far, the full element modelling on gravity field on the terrain surface have been completed.

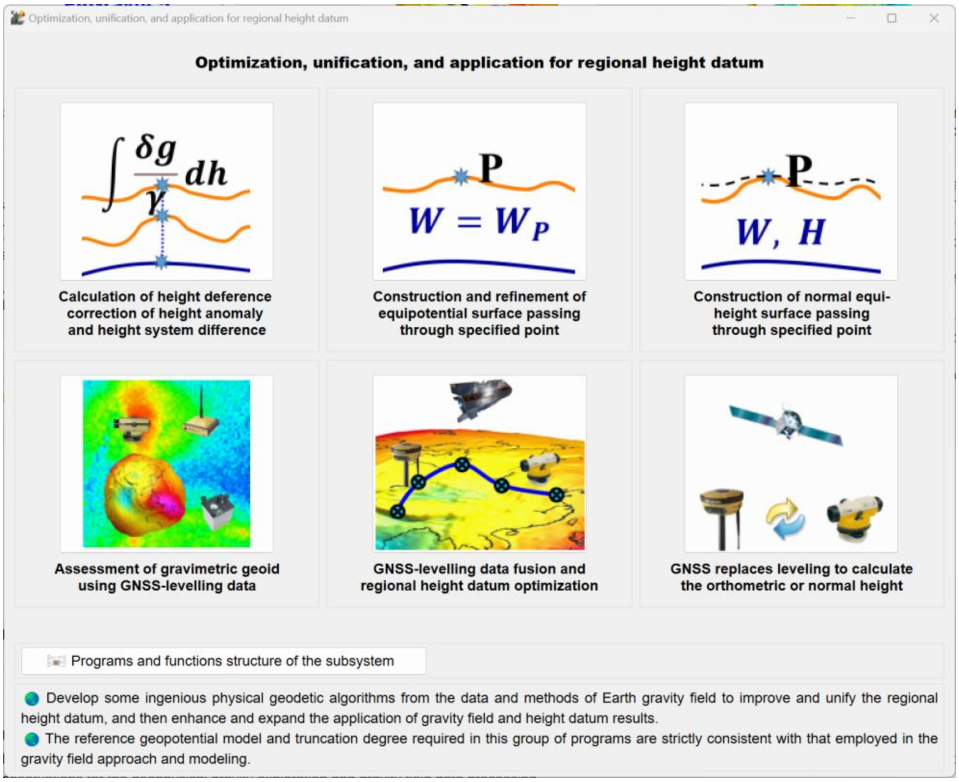
○ **Let the geoid as the calculation surface, and directly generate the 30″ full element models of the gravity field on the geoid.**

In step (3) to step (6) above, the input data file and all the parameter settings are kept same, and only the calculation surface is changed to the geoid. Using the same process, you can synchronously obtain the 30″ full element models geoidh30srst.xxx of the gravity field on the geoid, which include the 30″ gravimetric geoidal height grid (geoidh30srst.ksi, m, in global height datum), gravity disturbance grid (geoidh30srst.rga, mGal), gravity anomaly grid (surfhgt30srst.gra, mGal), disturbing gravity gradient grid (geoidh30srst.grr, radial, E), vertical deflection vector grid (geoidh30srst.dft, SW, ″) and geoidal height grid (geoidh30srgn.ksi, m) in regional height datum.



5 Optimization, unification, and application for regional height datum

Develop some ingenious physical geodetic algorithms from the data and methods of Earth gravity field to improve and unify the regional height datum, and then enhance and expand the application of gravity field and height datum results.



The reference geopotential model and truncation degree in this group of programs should be strictly consistent with that employed in gravity field approach and modelling.

5.1 Calculation of height deference correction of height anomaly and height system difference

[Calculation scheme] (a) Firstly, calculate the model height deference correction of height anomaly from the reference geopotential model. (b) Secondly, from the regional gravity field data, refine the residual height deference correction of height anomaly by the remove-restore scheme. (c) Finally, calculate the measured adjustment of height deference correction furtherly from the measured gravity at the measurement point. You can take one of the following three as the calculation result: (a), (a)+(b) or (a)+(b)+(c).

5.1.1 Calculation of height deference correction of height anomaly from the geopotential model

[Function] From the ellipsoidal height of the calculation point and target point in near-Earth space, calculate the model value of radial gradient (cm/km) and height deference correction (m) of height anomaly using the reference geopotential model.

[Input files] The calculation point position file and the global geopotential coefficient model file.

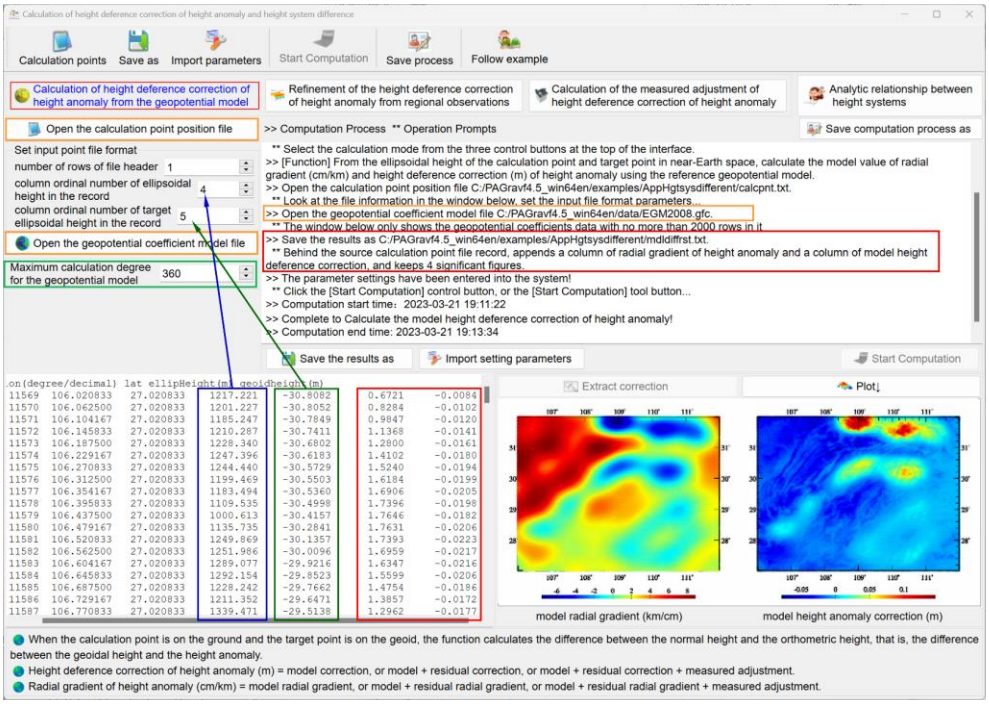
The record format of the calculation point file: ID (point no / point name), longitude (decimal degrees), latitude (decimal degrees), ...

The first row of the geopotential model file is agreed to be the scale parameters of the geopotential coefficient model: the geocentric gravitational constant of the Earth GM ($10^{14}m^3/s^2$) and semi-major axis of the Earth a(m).

[Parameter settings] Set number of rows of calculation file header, enter column ordinal number of ellipsoidal heights of current point and target point in the file record, and input the maximum calculation degree for the geopotential model.

When the calculation point is on the ground and the target point is on the geoid, the program calculates the difference between the normal height and orthometric height, that is, the difference between the geoidal height and the height anomaly.

The program selects the minimum of the maximum degree of the global geopotential model and the input maximum degree as the calculation degree.



[Output file] The model height deference correction file.

Behind the source calculation point file record, appends a column of radial gradient of height anomaly and a column of model height deference correction.

5.1.2 Refinement of the height deference correction of height anomaly from regional observations

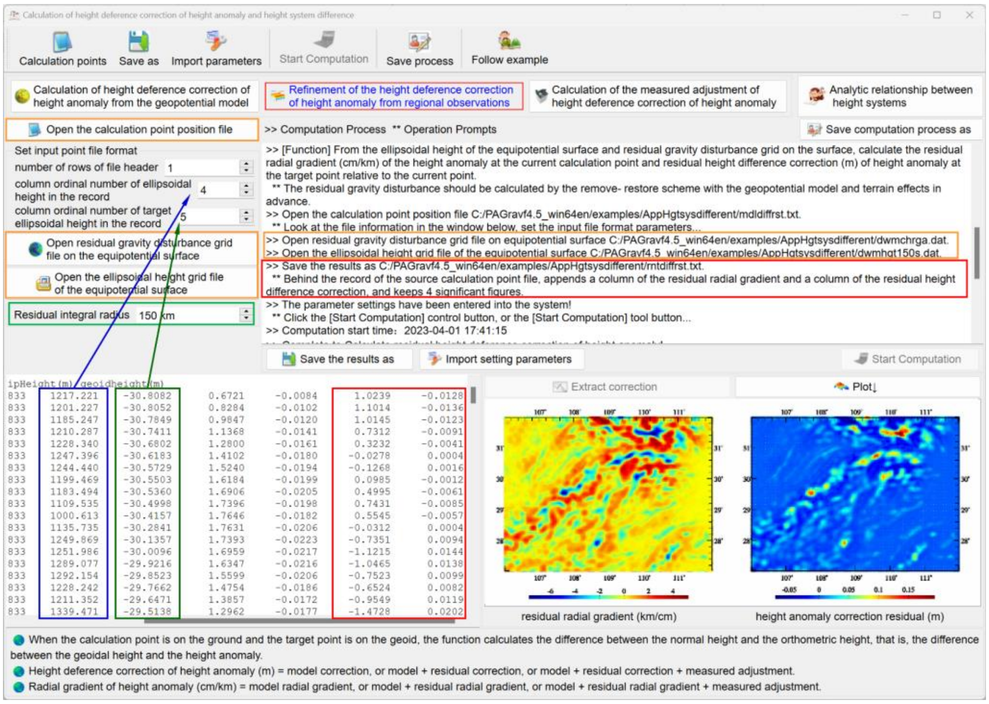
[Function] From the ellipsoidal height of the equipotential surface and residual gravity disturbance grid on the surface, calculate the residual radial gradient (cm/km) of the height anomaly at the current calculation point and residual height difference correction (m) of height anomaly at the target point relative to the current point.

[Input files] The calculation point position file, ellipsoidal height grid file of the equipotential surface and residual gravity disturbance grid file on the surface with the same grid specifications.

The record format of the calculation point file: ID (point no / point name), longitude (decimal degrees), latitude (decimal degrees), ...

The residual gravity disturbance should be calculated by the remove- restore scheme with the geopotential model and terrain effects in advance.

[Parameter settings] Set number of rows of the calculation point file header, enter column ordinal number of ellipsoidal heights of current point and target point in the file record, and input the residual integral radius.



[Output file] The residual height deference correction file.

Behind the record of the source calculation point file, appends a column of the residual radial gradient and a column of the residual height difference correction, and keeps 4 significant figures.

5.1.3 Calculation of the measured adjustment of height deference correction of height anomaly

[Function] Call the two upper left functions in turn, remove the reference model value and the refined residual value from the measured gravity disturbance at the calculation point, respectively, to obtain the remaining residual measured gravity disturbance, and then calculate the measured adjustment for radial gradient correction (cm/km) at the calculation point, and measured adjustment for the height difference correction (m) at the target point relative to the current calculation point.

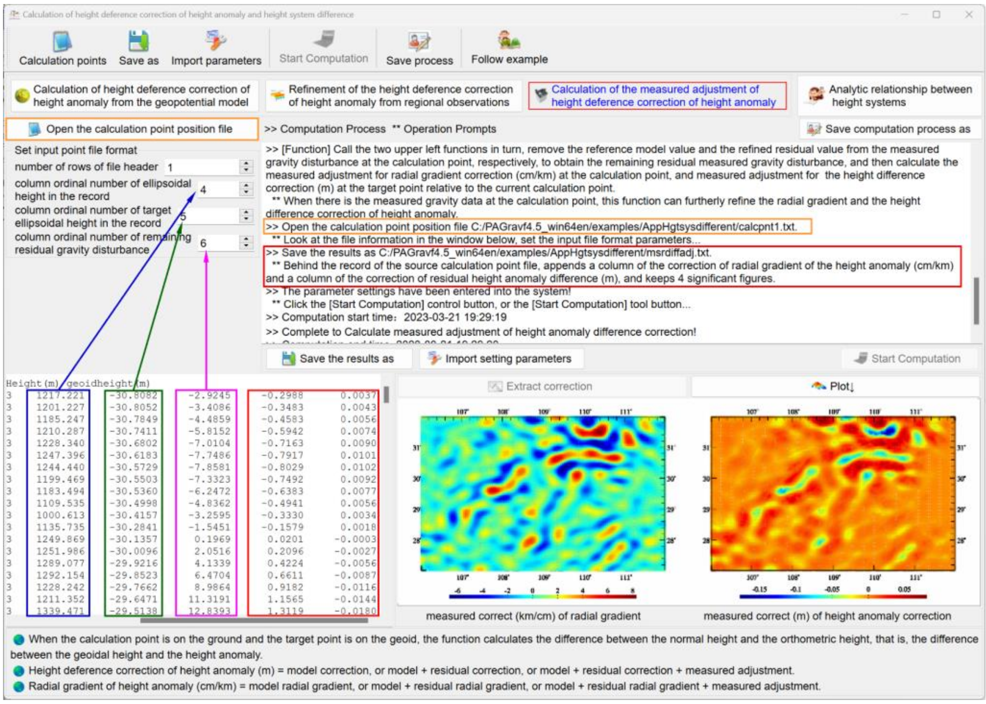
When there is the measured gravity data at the calculation point, this function can furtherly refine the radial gradient and the height difference correction of height anomaly.

[Input file] The calculation point position file with the remaining residual gravity disturbance attribute in the record.

The record format of the calculation point file: ID (point no / point name), longitude (decimal degrees), latitude (decimal degrees), ..., remaining residual gravity disturbance, …

The remaining residual gravity disturbance = the measured disturbance gravity – model gravity disturbance - residual gravity disturbance.

[Parameter settings] Set number of rows of calculation file header, enter column ordinal number of ellipsoidal heights of current point and target point in the file record, and column ordinal number of remaining residual measured gravity disturbance.



[Output file] The measured adjustment file of residual height deference.

Behind the record of the source calculation point file, appends a column of the correction of radial gradient of the height anomaly (cm/km) and a column of the correction of residual height anomaly difference (m), and keeps 4 significant figures.

Height deference correction of height anomaly (m) = model correction, or model + residual correction, or model + residual correction + measured adjustment.

Radial gradient of height anomaly (cm/km) = model radial gradient, or model + residual radial gradient, or model + residual radial gradient + measured adjustment.

5.2 Construction and refinement of equipotential surface passing through specified point

[Purpose] Firstly, calculate the model ellipsoidal height value of the gravity equipotential surface from the reference geopotential model, and then furtherly refine the ellipsoidal height grid of the equipotential surface from the anomalous field element grid by the remove-restore scheme.

5.2.1 Construction of the gravity equipotential surface from global geopotential model

[Function] From the geopotential coefficient model, calculate the model gravity (mGal) and model ellipsoidal height (m) grid of the gravity equipotential surface passing through the specified points (B, L, H).

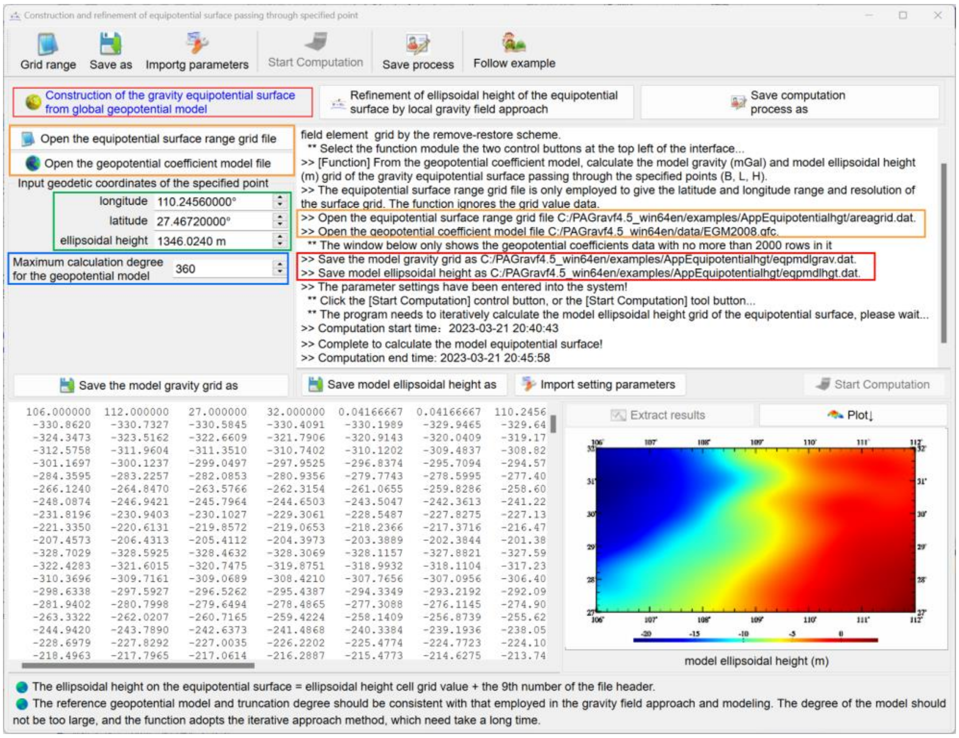
[Input files] The equipotential surface range grid file and global geopotential coefficient model file.

The equipotential surface range grid file is only employed to give the latitude and longitude range and resolution of the surface grid.

The first row of the geopotential model file is agreed to be the scale parameters of the geopotential coefficient model: the geocentric gravitational constant of the Earth GM ($10^{14}m^3/s^2$) and semi-major axis of the Earth a(m).

[Parameter settings] Input geodetic coordinates of the specified point and the maximum calculation degree for the geopotential model.

The program requires the specified point is within the range of the equipotential surface grid.



The program selects the minimum of the maximum degree of the global geopotential model and the input maximum degree as the calculation degree. The degree of the model should not be too large, and the program adopts the iterative approach method, which need take a long time.

The reference geopotential model and truncation degree should be consistent with that employed in the gravity field approach and modelling.

[Output file] The model ellipsoidal height grid file and model gravity grid file of model equipotential surface.

The ellipsoidal height on the equipotential surface = ellipsoidal height cell grid value + the 9th number of the file header.

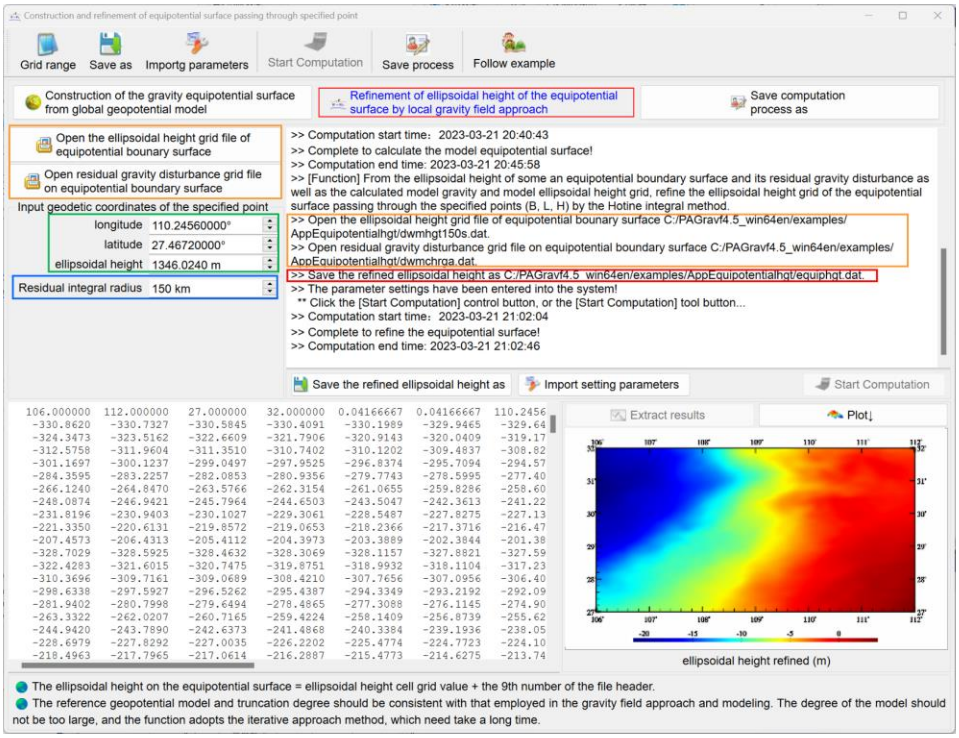
5.2.2 Refinement of ellipsoidal height of the equipotential surface by local gravity field approach

[Function] From the ellipsoidal height of some an equipotential boundary surface and residual gravity disturbance as well as the calculated model gravity and model ellipsoidal height grid, refine the ellipsoidal height grid of the equipotential surface passing through the specified points (B, L, H) by the Hotine integral method.

[Input files] The ellipsoidal height grid file of the equipotential boundary surface and residual gravity disturbance grid file on the surface, the model ellipsoidal height grid file and model gravity grid file on the model equipotential surface.

The equipotential boundary surface here refers specifically to the Stokes boundary equipotential surface where the residual gravity disturbance is located, different with the equipotential surface to be determined.

[Parameter settings] Input geodetic coordinates of the specified point and the residual integral radius.



[Output file] The refined ellipsoidal height grid file of the equipotential surface passing through the specified points (B, L, H)

The ellipsoidal height on the equipotential surface = ellipsoidal height cell grid value + the 9th number of the file header.

5.3 Construction of terrain equiheight surface passing through specified point

[Purpose] Firstly, calculate the model geopotential of the normal or orthometric equiheight surface from the reference gravity field model, and then furtherly refine the geopotential from the anomalous field element grid by the remove-restore scheme.

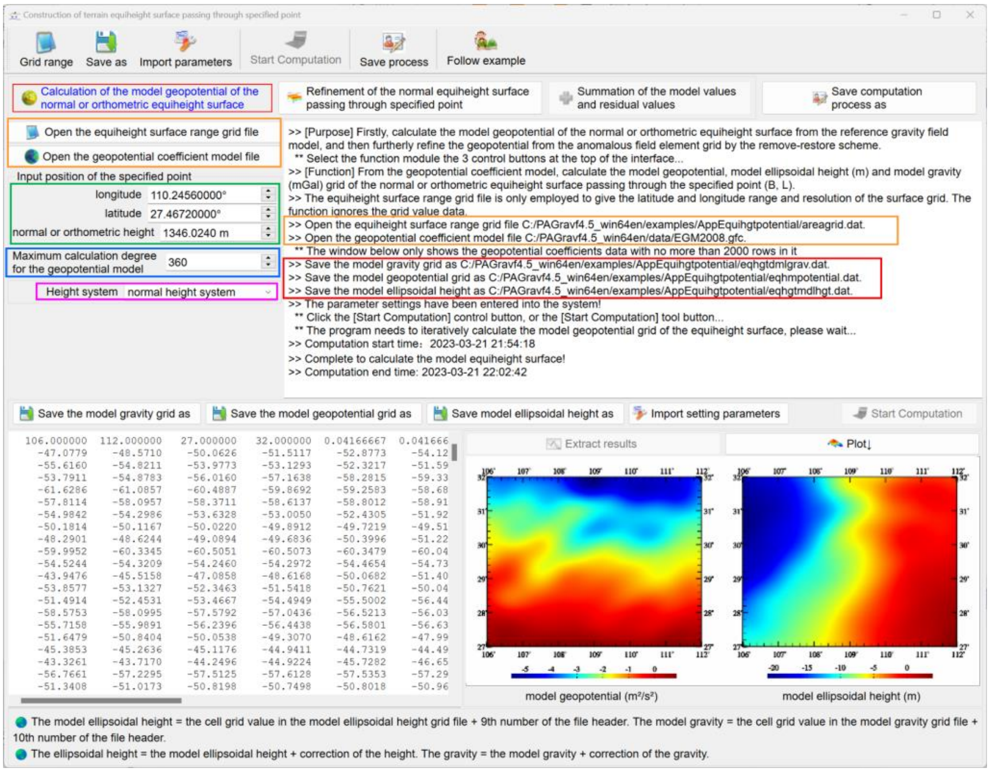
5.3.1 Calculation of the model geopotential of the normal or orthometric equiheight surface

[Function] From the geopotential coefficient model, calculate the model geopotential, model ellipsoidal height (m) and model gravity (mGal) grid of the normal or orthometric equiheight surface passing through the specified point (B, L).

[Input files] The equiheight surface range grid file and global geopotential coefficient model file.

The equiheight surface range grid file is only employed to give the latitude and longitude range and resolution of the surface grid.

The first row of the geopotential model file is agreed to be the scale parameters of the geopotential coefficient model: the geocentric gravitational constant of the Earth GM ($10^{14}m^3/s^2$) and semi-major axis of the Earth a(m).



[Parameter settings] Input geodetic coordinates of the specified point and the maximum calculation degree for the geopotential coefficient model and select the type of height system.

The program requires the specified point is within the range of the equiheight surface grid.

The program selects the minimum of the maximum degree of the global geopotential model and the input maximum degree as the calculation degree. The degree of the model should not be too large, and the program adopts the iterative approach method, which need take a long time.

The reference geopotential model and truncation degree should be consistent with that employed in the gravity field approach and modelling.

[Output files] The model geopotential grid file, model ellipsoidal height grid file and model gravity grid file of model equiheight surface.

The model ellipsoidal height = the cell grid value in the model ellipsoidal height grid file + 9th number of the file header.

The model gravity = the cell grid value in the model gravity grid file + 10th number of the file header.

5.3.2 Refinement of the normal equiheight surface passing through specified point

[Function] From the ellipsoidal height of some an equipotential boundary surface and residual gravity disturbance on the surface as well as the calculated model gravity and model ellipsoidal height grid, refine the ellipsoidal height grid of the normal or orthometric equiheight surface passing through the specified point (B, L) by the Hotine integral method.

[Input files] The ellipsoidal height grid file of the equipotential boundary surface and residual gravity disturbance grid file on the surface, the model ellipsoidal height grid file and model gravity grid file on the model equipotential surface.

The residual geopotential correction and residual ellipsoidal height correction of the equiheight surface are calculated from the residual gravity disturbance after the reference model values removed.

[Parameter settings] Input geodetic coordinates of the specified point and the residual integral radius and select the type of height system.

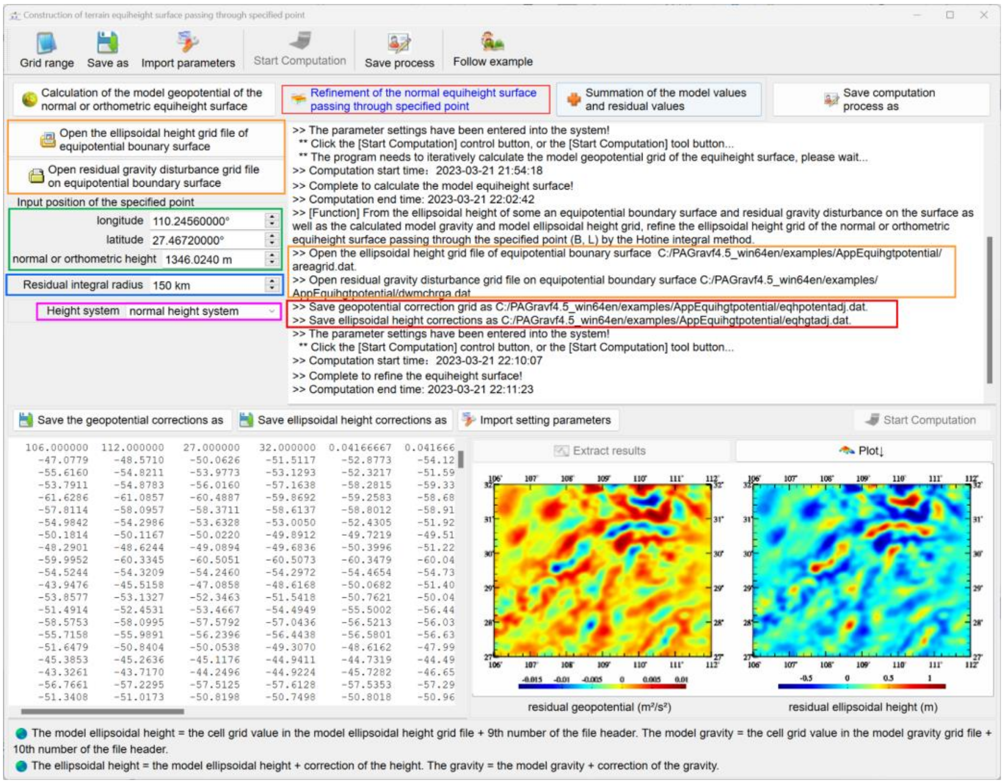
[Output files] The geopotential correction grid file and ellipsoidal height correction grid file of the equiheight surface.

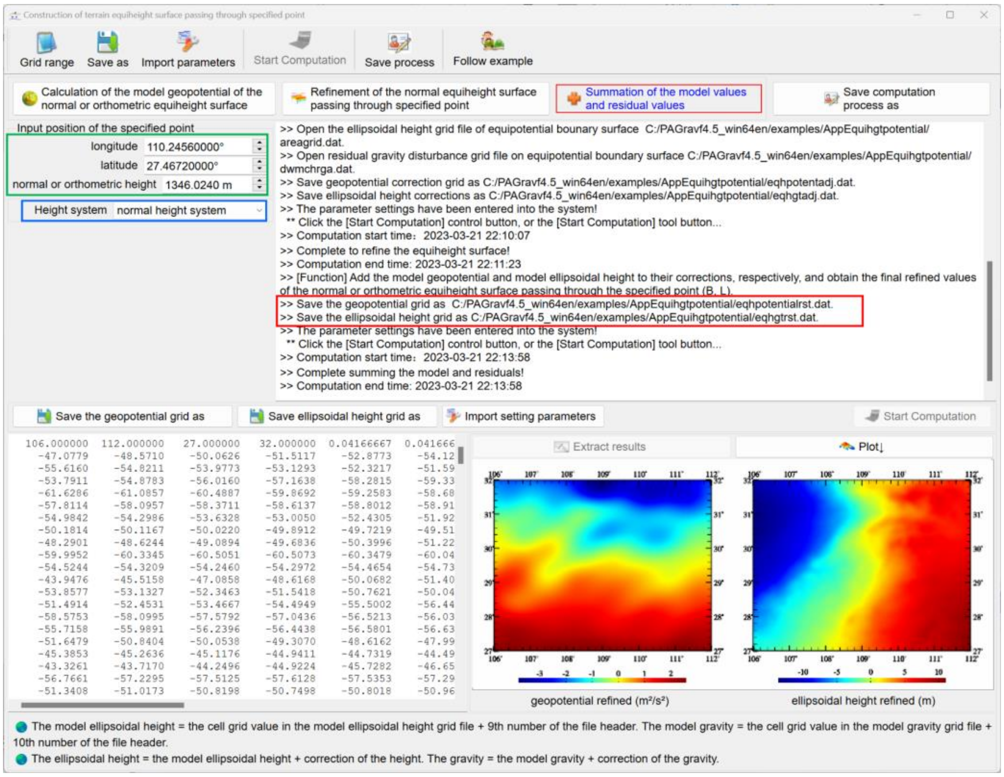
5.3.3 Summation of the model values and residual values

[Function] Add the model geopotential and model ellipsoidal height to their corrections, respectively, and obtain the final refined values of the normal or orthometric equiheight surface passing through the specified point (B, L).

[Output files] The geopotential grid file and ellipsoidal height grid file of the normal or orthometric equiheight surface passing through the specified point (B, L).

The ellipsoidal height = the model ellipsoidal height + correction of the height.
The gravity = the model gravity + correction of the gravity.





5.4 Assessment of gravimetric geoid using GNSS-levelling data

[Function] According to the spectral domain error characteristics of GNSS-leveling height anomaly and gravity field, statistically analyze the GNSS-leveling residual height anomaly (m), and then estimate the error (cm) of gravimetric height anomaly, inner coincidence error of hybrid height anomaly, error of hybrid height anomaly difference and error of GNSS-leveling height anomaly difference.

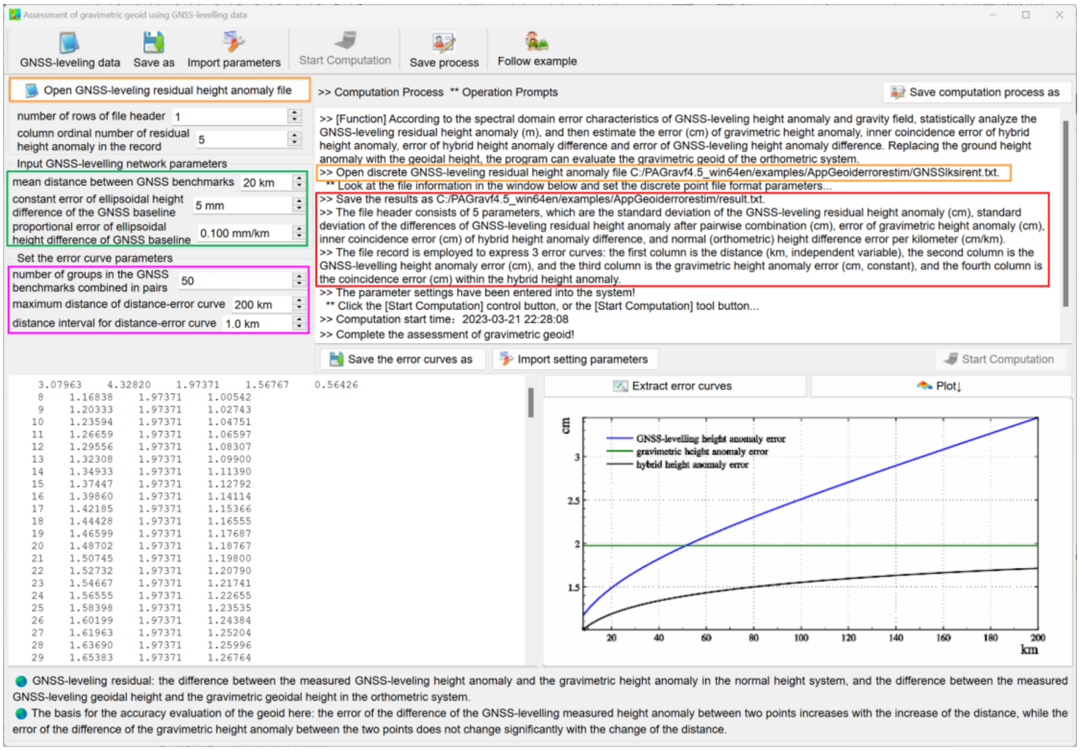
Replacing the ground height anomaly with the geoidal height, the program can evaluate the gravimetric geoid of the orthometric system.

[Input file] The discrete GNSS-leveling residual height anomaly file.

[Parameter settings] Set the input file format parameters, enter the GNSS-levelling network parameters, mean distance between GNSS benchmarks, the constant and proportional error of ellipsoidal height difference of the GNSS baseline, set the number of groups in the GNSS benchmarks combined in pairs, the maximum distance and distance interval for the distance-error curve.

The program combines each of the n GNSS benchmarks to form n(n-1)/2 sides, and after sorting them according to the side length, the GNSS-leveling residual height anomaly difference is grouped and then counted.

[Output file] The error curve file of gravimetric geoid.



The file header consists of 5 parameters, which are the standard deviation of the GNSS-leveling residual height anomaly (cm), standard deviation of the differences of GNSS-leveling residual height anomaly after pairwise combination (cm), error of gravimetric height anomaly (cm), inner coincidence error (cm) of hybrid height anomaly difference, and normal (orthometric) height difference error per kilometer (cm/km).

The file record is employed to express 3 error curves: the first column is the distance (km, independent variable), the second column is the GNSS-levelling height anomaly error (cm), and the third column is the gravimetric height anomaly error (cm, constant), and the fourth column is the coincidence error (cm) within the hybrid height anomaly.

GNSS-leveling residual: the difference between the measured GNSS-leveling height anomaly and the gravimetric height anomaly in the normal height system, and the difference between the measured GNSS-leveling geoidal height and the gravimetric geoidal height in the orthometric system.

The basis for the accuracy evaluation of the geoid here: the error of the difference of the GNSS-levelling measured height anomaly between two points increases with the increase of the distance, while the error of the difference of the gravimetric height anomaly between the two points does not change significantly with the change of the distance.

5.5 GNSS-levelling data fusion and regional height datum optimization

[Purpose] From the GNSS-leveling residual geoidal height or residual height anomaly, estimate the geopotential difference of regional height datum and its zero-point parameters, calculate the correction of hybrid geoid, and optimize the regional height datum (GNSS-levelling network).

5.5.1 Calculation of geopotential of the zero-height surface for regional height datum

[Function] From the GNSS-leveling measured geoidal heights (height anomalies, m) and residual geoidal heights (height anomalies, m), estimate the zero-height surface geopotential W_r of the regional height datum. Then with the given geopotential W_o of the global geoid, obtain the zero-height surface geopotential difference W_r -W_o of the regional height datum relative to the global geoid.

[Input file] The discrete GNSS-leveling residual file.

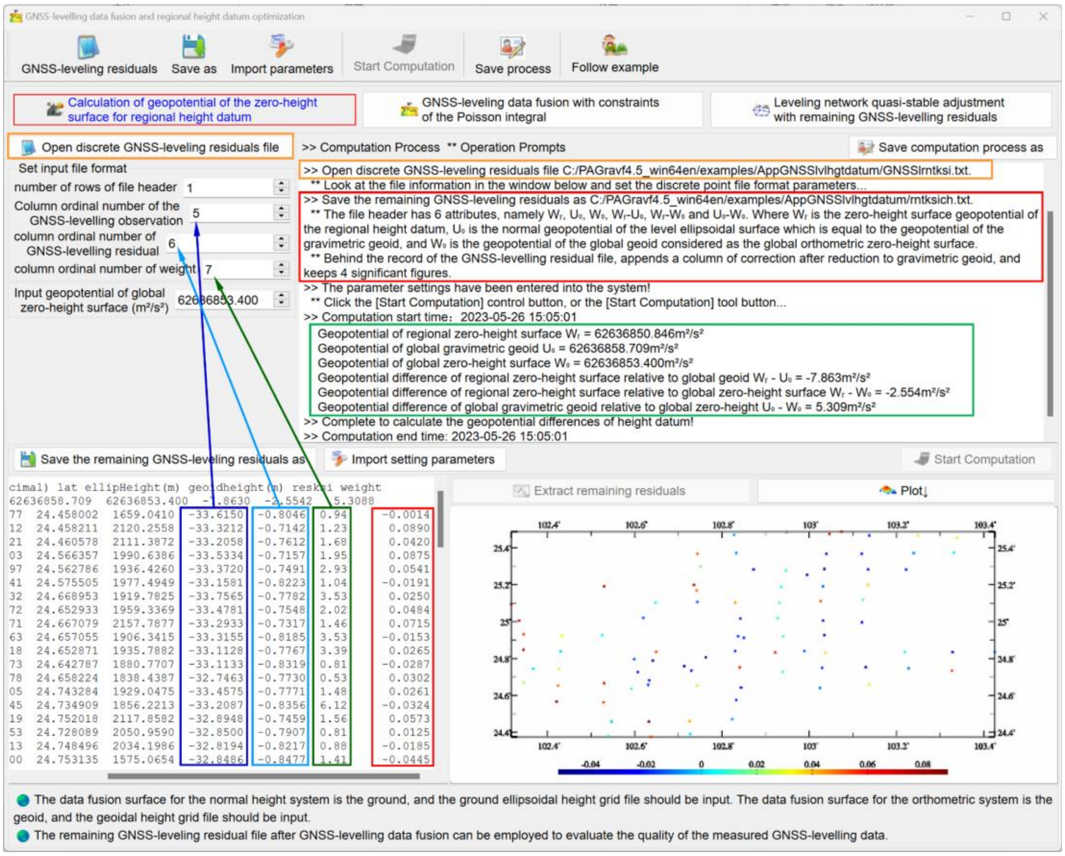
GNSS-leveling residual: the difference between the measured GNSS-leveling height anomaly and the gravimetric height anomaly in the normal height system, or the difference between the measured GNSS-leveling geoidal height and the gravimetric geoidal height in the orthometric system.

[Parameter settings] Set the input file format parameters, enter the geopotential W_o of global zero-height surface.

[Output file] The remaining GNSS-leveling residual file.

The file header has 6 attributes, namely W_r, U_0, W_0, W_r-U_0, W_r-W_0 and U_0-W_0, where W_r is the zero-height surface geopotential of the regional height datum, U_0 is the normal geopotential of the level ellipsoidal surface which is equal to the geopotential of the gravimetric geoid, and W_0 is the geopotential of the global geoid considered as the global orthometric zero-height surface.

Behind the record of the GNSS-levelling residual file, appends a column of correction after reduction to gravimetric geoid, and keeps 4 significant figures.



The normal zero-height surface and the orthometric zero-height surface always coincide everywhere and are the same, and there is no need to distinguish them.

PAGravf4.5 recommends that the geoidal geopotential W_G or U_0 should replace the empirical appoint W_0 in the IERS numerical standard. The latter is calculated from the global geopotential model and satellite altimetry data according to the Gaussian geoid definition.

5.5.2 GNSS-leveling data fusion with constraints of the Poisson integral

[Function] From the GNSS-leveling residual and the ellipsoidal height grid of the data fusion surface, estimate the correction of gravimetric geoidal height/ground height anomaly with constraints of the Poisson integral, to realize the analytical fusion of GNSS-leveling data and gravimetric geoidal height.

[Input files] The discrete (remaining) GNSS-leveling residual file, the data fusion range grid file.

The data fusion surface for the normal height system is the ground, and the ground ellipsoidal height grid file should be input. The data fusion surface for the orthometric system is the geoid, and the geoidal height grid file should be input.

[Parameter settings] Set the number of rows of the GNSS-leveling file header, the column ordinal number of the ellipsoidal height, GNSS-levelling residual, and weight, enter the iterative calculation times, residual integral radius, Laplace operator parameter and edge effect suppressing parameter.

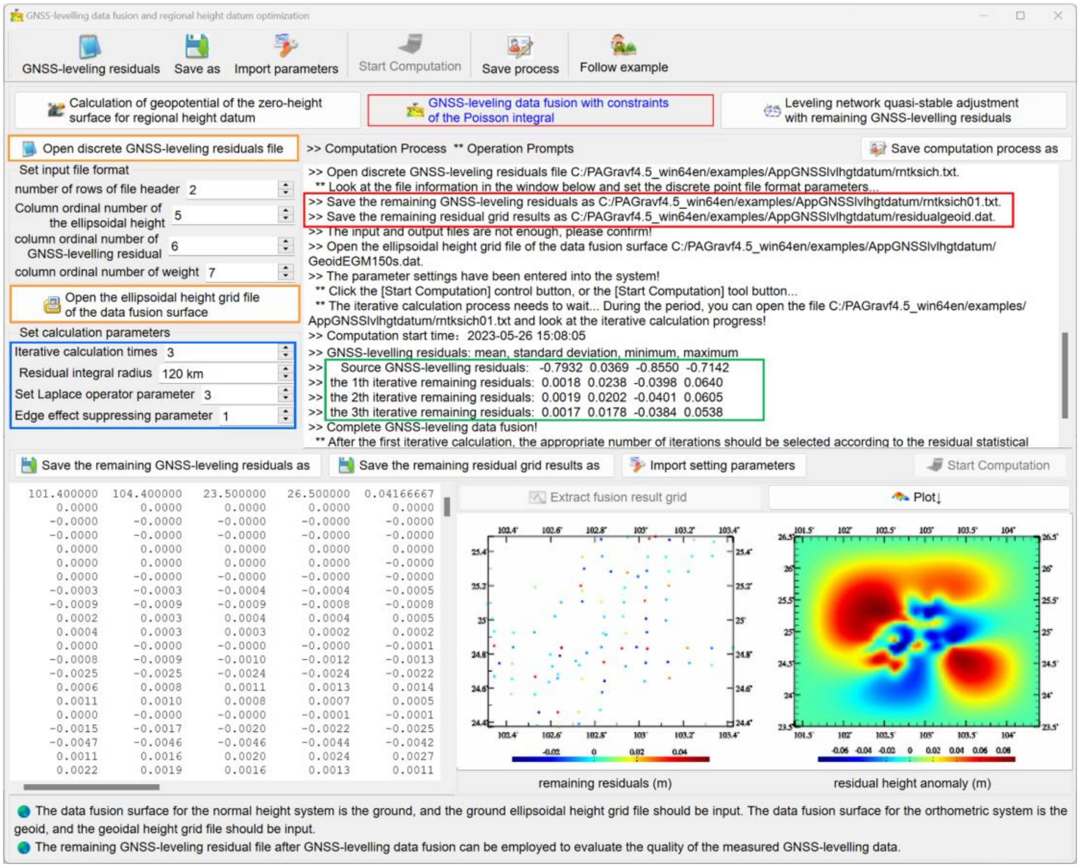
Column ordinal number of the ellipsoidal height. The geodetic height is consistent with the geodetic height of the fusion calculation surface, that is, the orthometric system is the geoidal height, and the normal height system is the ellipsoidal height at the GNSS benchmark.

The integral radius. The smaller the integral radius, the faster the calculation.

Set Laplace operator parameter. The larger the smoothing parameter, the stronger the filtering.

The edge effect suppression parameter n. The program suppresses edge and far-zone effects with the residual of the cell grid at the area margins equal to zero as the observation equation.

The local gravity field approach method does not have the capacity to deal with the problem of systematic deviation. The program automatically removes the statistical mean of GNSS-leveling residuals.



[Output file] The remaining residual geoidal height/height anomaly grid file.

The interface displays the statistical properties of the remaining GNSS-leveling residuals during the iterative calculation process.

After the first iterative calculation, the appropriate number of iterations should be selected according to the residual statistical properties of the iterative process, and the calculation should be performed once again!

The remaining GNSS-leveling residual file after GNSS-levelling data fusion can be employed to evaluate the quality of the measured GNSS-levelling data.

5.5.3 Leveling network quasi-stable adjustment with remaining GNSS-levelling residuals

[Function] Taking all GNSS-levelling points as quasi-stable benchmarks, from the remaining GNSS-levelling residuals and leveling routes file in GNSS-levelling network, the indirect least squares adjustment method with quasi-stable benchmark constraints is employed to estimate the normal (orthometric) height corrections of leveling benchmarks and the height anomaly corrections of GNSS benchmarks.

[Input files] The discrete (remaining) GNSS-leveling residual file, the leveling routines file in GNSS-levelling network.

The leveling routines file adopts the agreed format, please refer to the main interface [Format convention for geodetic data file].

[Parameter settings] Set the number of rows of the GNSS-leveling file header, the column ordinal number of the ellipsoidal height, GNSS-levelling residual, and weight.

[Output files] The normal (orthometric) height correction file of leveling benchmarks and the height anomaly correction file of GNSS benchmarks.

5.6 GNSS replaces leveling to calculate the orthometric or normal height

[Function] From the results of the regional geoid, calculate the orthometric (normal) height of the GNSS positioning point.

Please select height system firstly...

When the normal height system selected, input the ground height anomaly grid file, ground ellipsoidal height grid file, and ground gravity disturbance grid file.

When the orthometric height system selected, input the geoidal height grid file.

The ground ellipsoidal height grid is employed to stand for the location of the ground height anomaly.

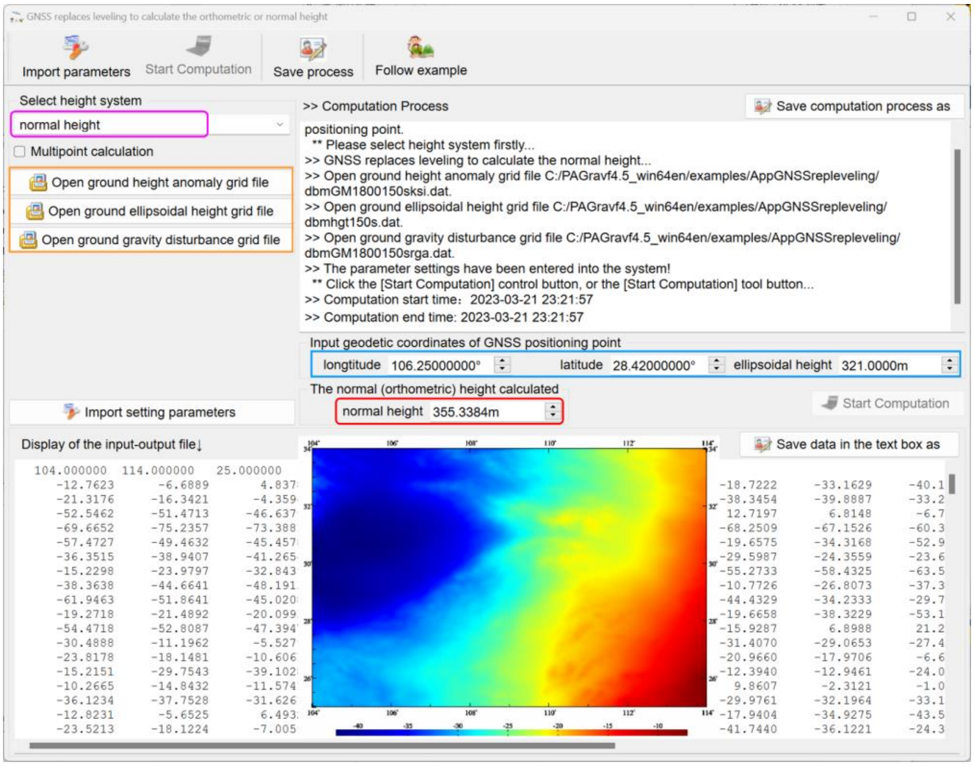
The ground gravity disturbance grid is used to compute the correction of height anomaly at GNSS point with height change.

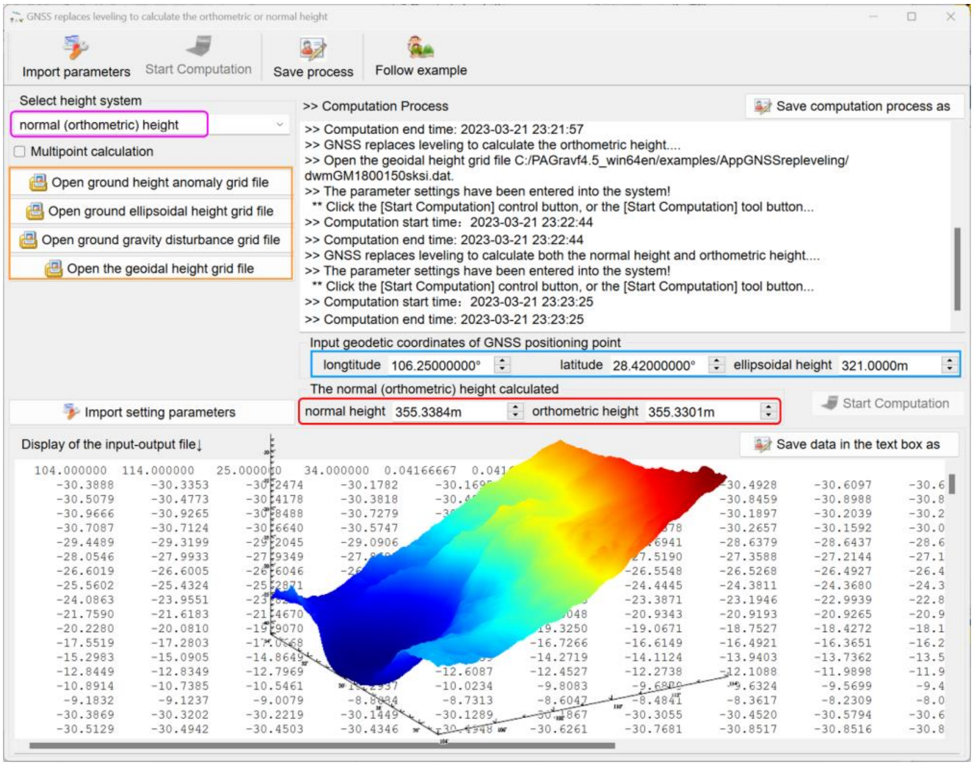
The geodetic coordinates of the calculation point can be input repeatedly, and the orthometric (normal) height at GNSS point can be calculated and displayed in time.

GNSS positioning point should be within the latitude and longitude range of the geoid model.

The ground gravity disturbances can be synchronously calculated while the gravimetric ground height anomalies are refined.

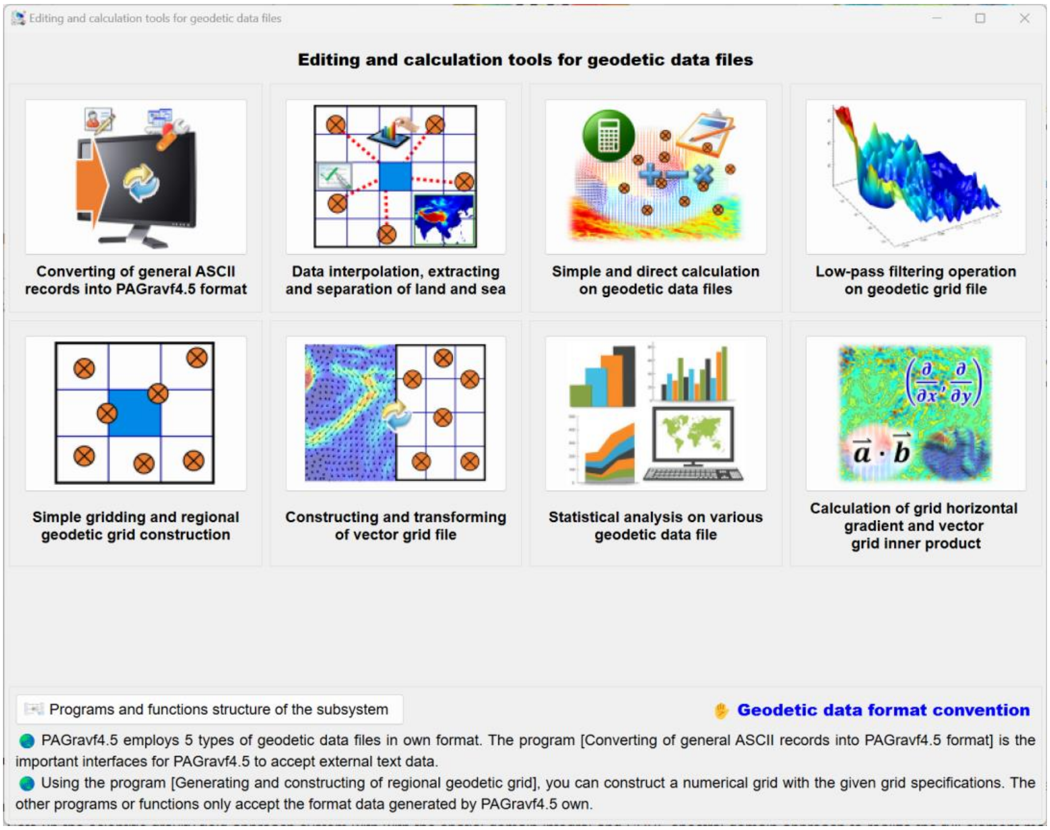
When the discrete GNSS position point file input, the program can calculate the orthometric (normal) heights for batch GNSS positioning points.


GNSS replaces leveling to calculate the orthometric or normal height
Import parameters
Start Computation
Save process
Follow example
Select height system
normal height
Multipoint calculation
Open ground height anomaly grid file
Open ground ellipsoidal height grid file
Open ground gravity disturbance grid file
>> Computation Process
Save computation process as
positioning point.
** Please select height system firstly...
>> GNSS replaces leveling to calculate the normal height...
>> Open ground height anomaly grid file C:/PAGravf4.5_win64en/examples/AppGNSSrepleveling/dbmGM1800150sksi.dat.
>> Open ground ellipsoidal height grid file C:/PAGravf4.5_win64en/examples/AppGNSSrepleveling/dbmhgt150s.dat.
>> Open ground gravity disturbance grid file C:/PAGravf4.5_win64en/examples/AppGNSSrepleveling/dbmGM1800150srga.dat.
>> The parameter settings have been entered into the system!
** Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Computation start time：2023-03-21 23:21:57
>> Computation end time: 2023-03-21 23:21:57
Input geodetic coordinates of GNSS positioning point
longtitude 106.25000000°
latitude 28.42000000°
ellipsoidal height 321.0000m
The normal (orthometric) height calculated
normal height 355.3384m
Import setting parameters
Start Computation
Display of the input-output file↓
Save data in the text box as


GNSS replaces leveling to calculate the orthometric or normal height
Import parameters
Start Computation
Save process
Follow example
Select height system
normal (orthometric) height
Multipoint calculation
Open ground height anomaly grid file
Open ground ellipsoidal height grid file
Open ground gravity disturbance grid file
Open the geoidal height grid file
>> Computation Process
Save computation process as
>> Computation end time: 2023-03-21 23:21:57
>> GNSS replaces leveling to calculate the orthometric height....
>> Open the geoidal height grid file C:/PAGravf4.5_win64en/examples/AppGnssrepleveling/dwmGM1800150sksi.dat.
>> The parameter settings have been entered into the system!
** Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Computation start time：2023-03-21 23:22:44
>> Computation end time: 2023-03-21 23:22:44
>> GNSS replaces leveling to calculate both the normal height and orthometric height....
>> The parameter settings have been entered into the system!
** Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Computation start time：2023-03-21 23:23:25
>> Computation end time: 2023-03-21 23:23:25
Input geodetic coordinates of GNSS positioning point
longtitude 106.25000000°
latitude 28.42000000°
ellipsoidal height 321.0000m
The normal (orthometric) height calculated
normal height 355.3384m
orthometric height 355.3301m
Import setting parameters
Start Computation
Display of the input-output file↓
Save data in the text box as

6 Editing, calculation, and visualization tools for geodetic data files

The geodetic data file editing and calculation assembly are mainly used for data file format conversion, interpolation and gridding, data extraction, separation and merging, vector grid and numerical grid data processing, basic operations on multiple sets of data, and other data preprocessing etc.



PAGravf4.5 employs 5 types of geodetic data files in own format. The program [Converting of general ASCII records into PAGravf4.5 format] is the important interfaces for PAGravf4.5 to accept external text data. Using the program [Generating and constructing of regional geodetic grid], you can construct a numerical grid with the given grid specifications. The other programs or functions only accept the format data generated by PAGravf4.5 own.

6.1 Converting of general ASCII records into PAGravf4.5 format

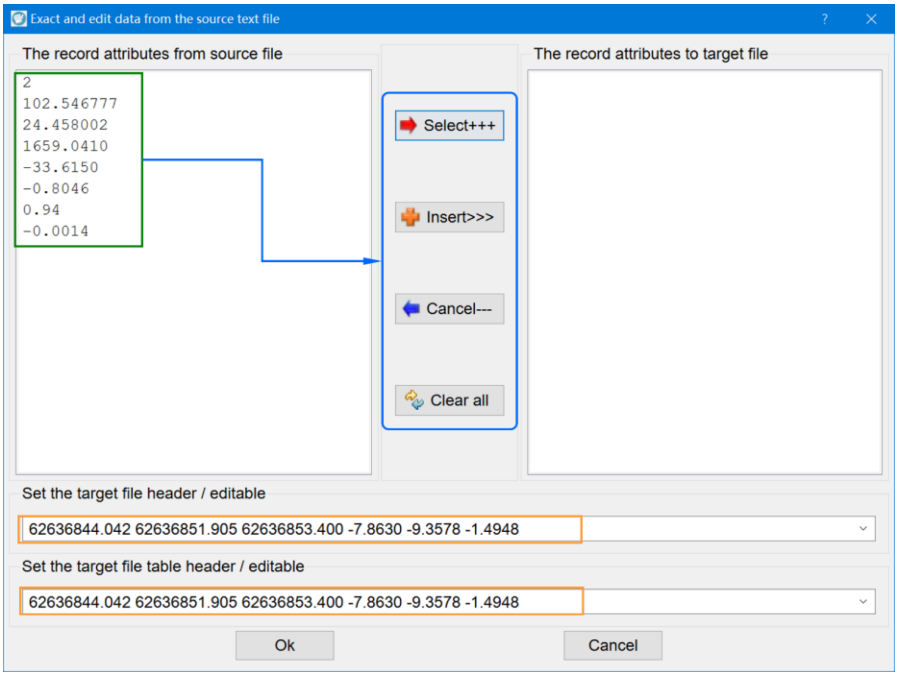
[Function] Convert the general ASCII data record file from different sources and non-standard format into the discrete geodetic record file in PAGravf4.5 format.

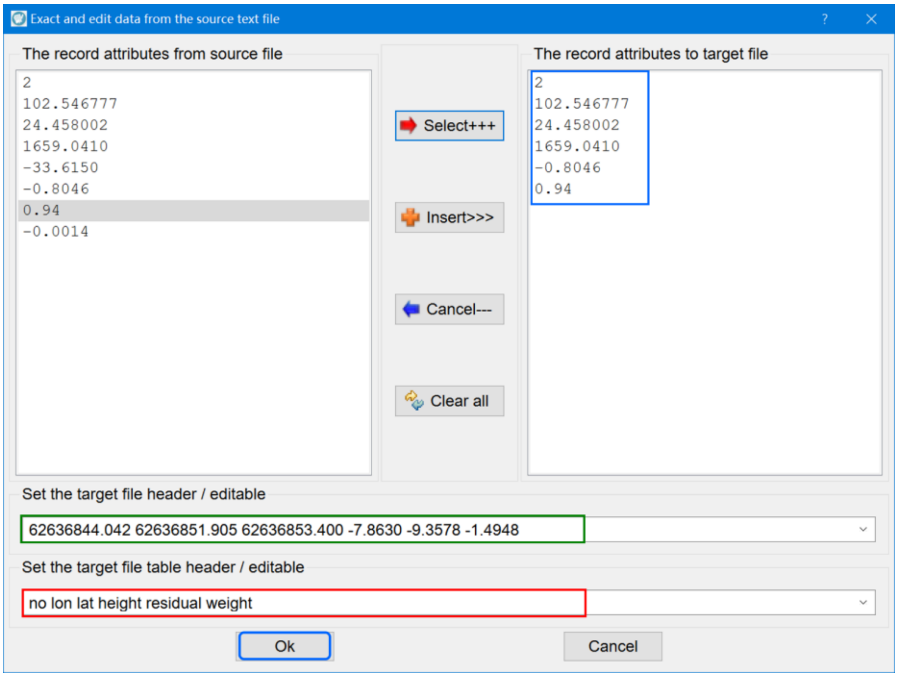
Please click the [Open the source ASCII record file] control button to open the general text record file to be standardized...

[Input file] The general ASCII data record file.

After entering the number of rows of the input file header, click the control button [Exact and edit data] to open the dialog [Exact and edit data from the source text file].

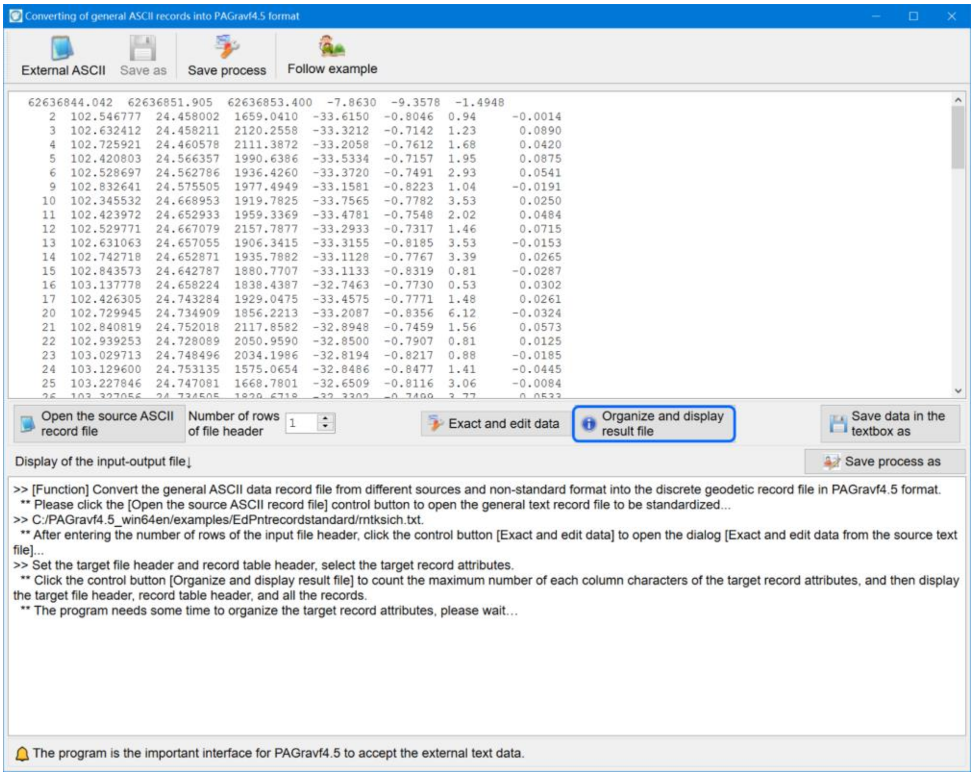
Set the target file header and record table header, select the target record attributes. If the target file does not need the record table header, please clear the text corresponding to the input text box.

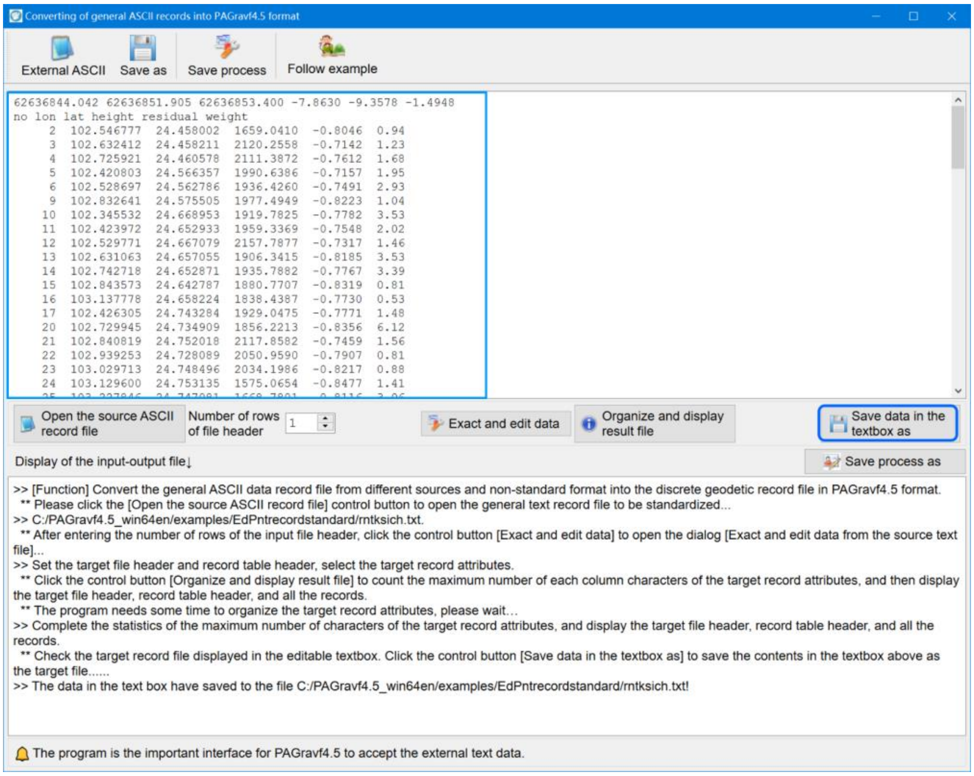




Click the button [Ok] to close the dialog. Click the control button [Organize and display result file] to count the maximum number of each column characters of the target record attributes, and then display the target file header, record table header, and all the records.

The program needs some time to organize the target record attributes, please wait…





[Output file] The discrete geodetic record file in PAGravf4.5 format.

Check the target record file displayed in the editable textbox. Click the control button [Save data in the textbox as] to save the contents in the textbox above as the target file...

The program is the important interface for PAGravf4.5 to accept the external text data.

6.2 Data interpolation, extracting and separation of land and sea

6.2.1 Changing of grid resolution by interpolation

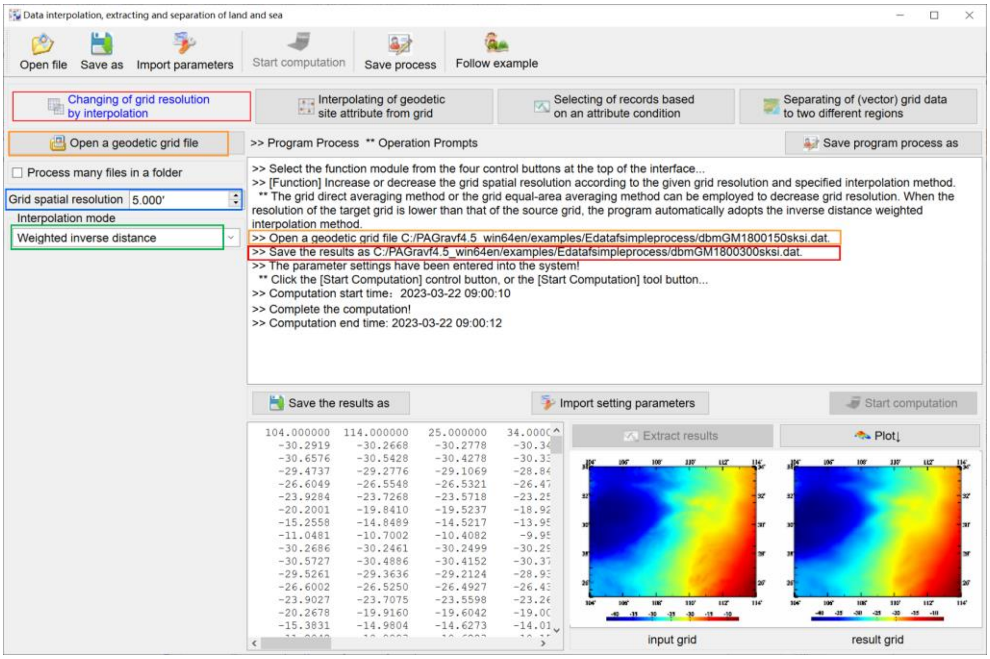
[Function] Increase or decrease the grid spatial resolution according to the given grid resolution and specified interpolation method.

[Input file] The geodetic numerical grid file.

[Parameter settings] Enter the spatial resolution for target grid and select the interpolation mode.

[Output file] The target geodetic numerical grid file.

The grid direct averaging method is that sums up all the effective source grid element values within the target grid element, and then divided the sum by the number of the effective source elements. The grid equal-area averaging method is that sums up all the effective source grid element values within the target element, and then divided the sum by the total number of source elements.



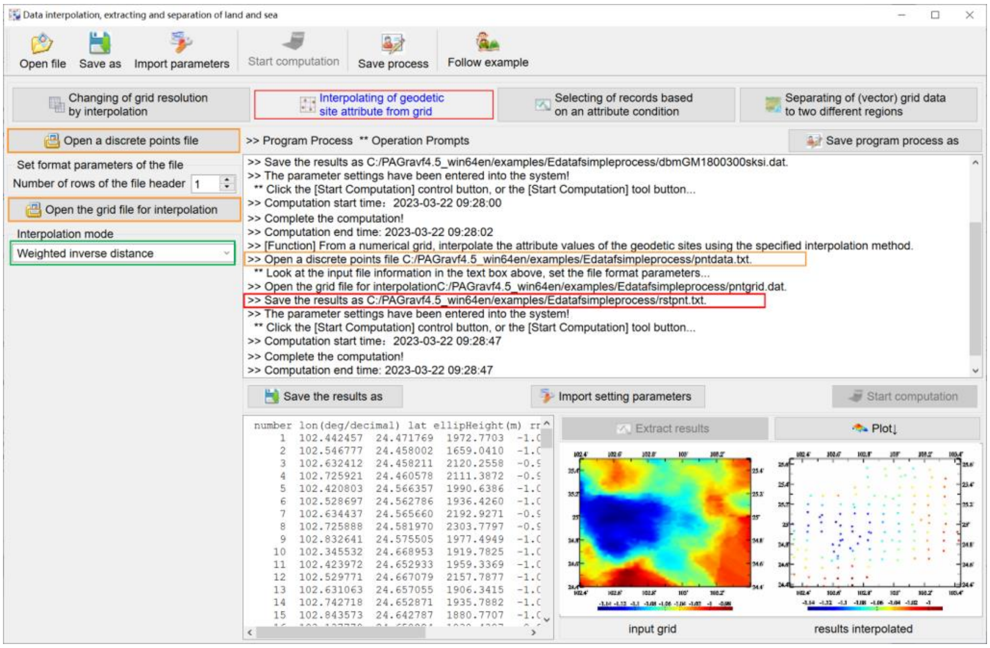
The grid direct averaging method or the grid equal-area averaging method can be employed to decrease grid resolution. When the resolution of the target grid is lower than that of the source grid, the program automatically adopts the inverse distance weighted interpolation method.

6.2.2 Interpolating of geodetic site attribute from grid

[Function] From a numerical grid, interpolate the attribute values of the geodetic sites using the specified interpolation method.

[Input files] The discrete geodetic point file to be interpolated. The geodetic numerical grid file for interpolation.

[Parameter settings] Enter number of rows of the discrete geodetic point file header and select the interpolation mode.



[Output file] The interpolated discrete geodetic point file.

The file format is the same as the input discrete geodetic point file. Behind the input file record, add one column of the interpolated value as the output file record.

6.2.3 Selecting of records based on an attribute condition

[Function] Select the geodetic records from a geodetic record file according to the maximum and minimum range of the specified attribute.

[Parameter settings] Enter number of rows of the input file header, column ordinal number of the condition attribute in the file record, and minimum and maximum of the attribute.

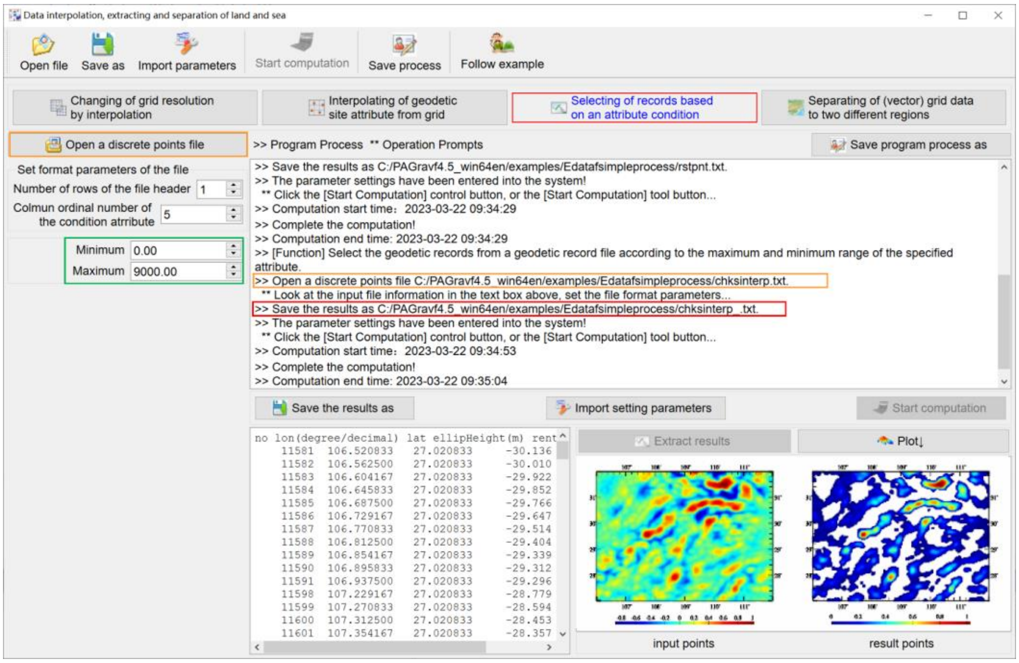
6.2.4 Separating of (vector) grid data to two different regions

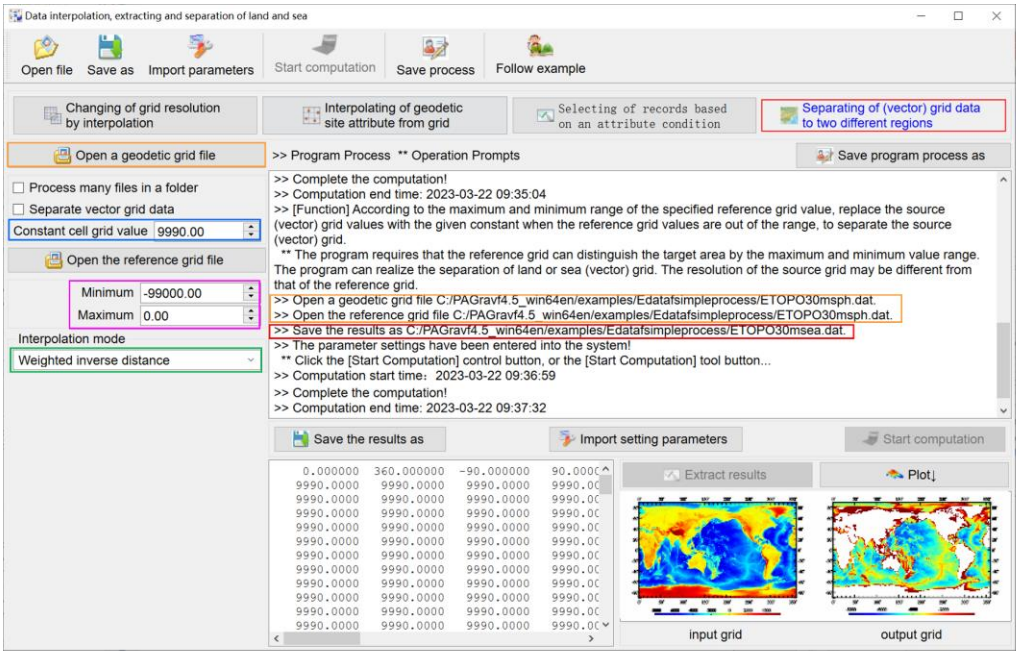
[Function] According to the maximum and minimum range of the specified reference grid value, replace the source (vector) grid values with the given constant when the reference grid values are out of the range, to separate the source (vector) grid.

The program requires that the reference grid can distinguish the target area by the maximum and minimum value range.

The program can realize the separation of land or sea (vector) grid. The resolution of the source grid may be different from that of the reference grid.

[Input files] The source geodetic (vector) grid file. The reference grid file whose grid range and resolution should not be smaller than that of the source grid file.





6.3 Simple and direct calculation on geodetic data files

6.3.1 Weighted operation on two specified attributes in record file

[Function] Perform weighted plus, minus, or multiply operation on two specified attributes in the discrete points file.

[Input file] The discrete geodetic points file.

[Parameter settings] Enter number of rows of the discrete geodetic point file header, column ordinal number of the attribute 1 and its weight, and column ordinal number of the attribute 2 and its weight. Select operation mode.

[Output file] The operated discrete geodetic point file.

The file format is the same as the input discrete geodetic points file. Behind the input file record, add 1 column of the calculation result as the output file record.

6.3.2 Weighted operation on two geodetic grid files

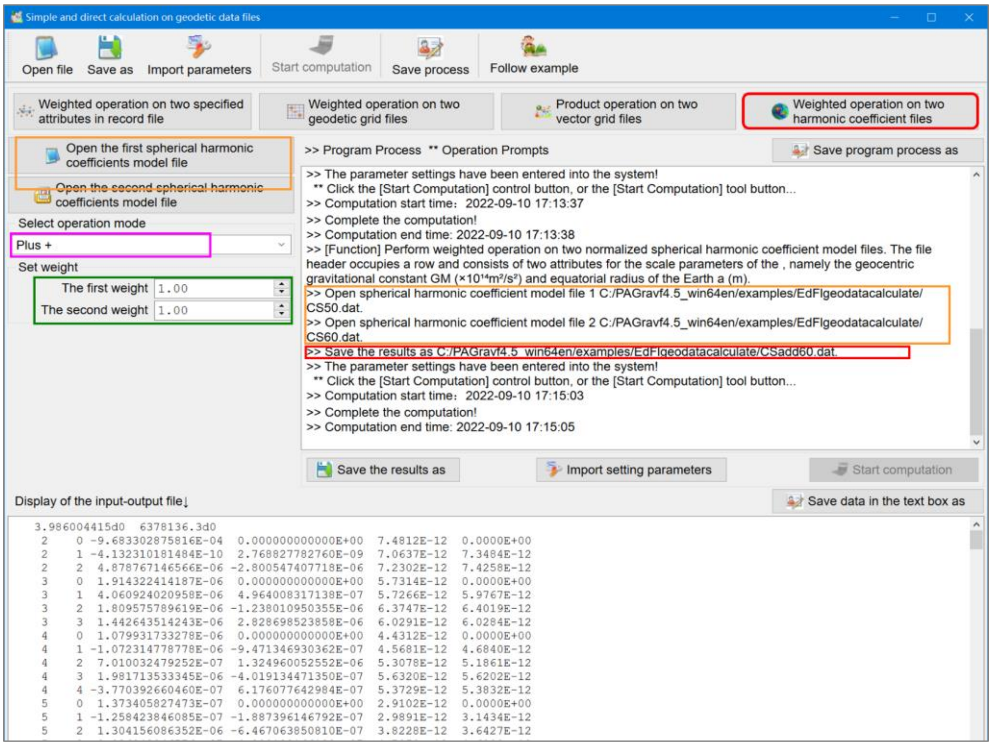
[Function] Perform weighted plus, minus, or multiply operation on grid elements in two (vector) grid files with the same specifications.

6.3.3 Product operation on two vector grid files

[Function] Perform outer product or inner product operations on vector grid elements in two vector grid files with the same specifications.

6.3.4 Weighted operation on two harmonic coefficient files

[Function] Perform weighted operation on two normalized spherical harmonic coefficient model files.



The file header occupies a row and consists of two attributes for the scale parameters of the spherical harmonic coefficients model, namely the geocentric gravitational constant GM ($\times 10^{14}\mathrm{m}^2/\mathrm{s}^2$) and equatorial radius of the Earth a (m).

6.4 Low-pass filtering operation on geodetic grid file

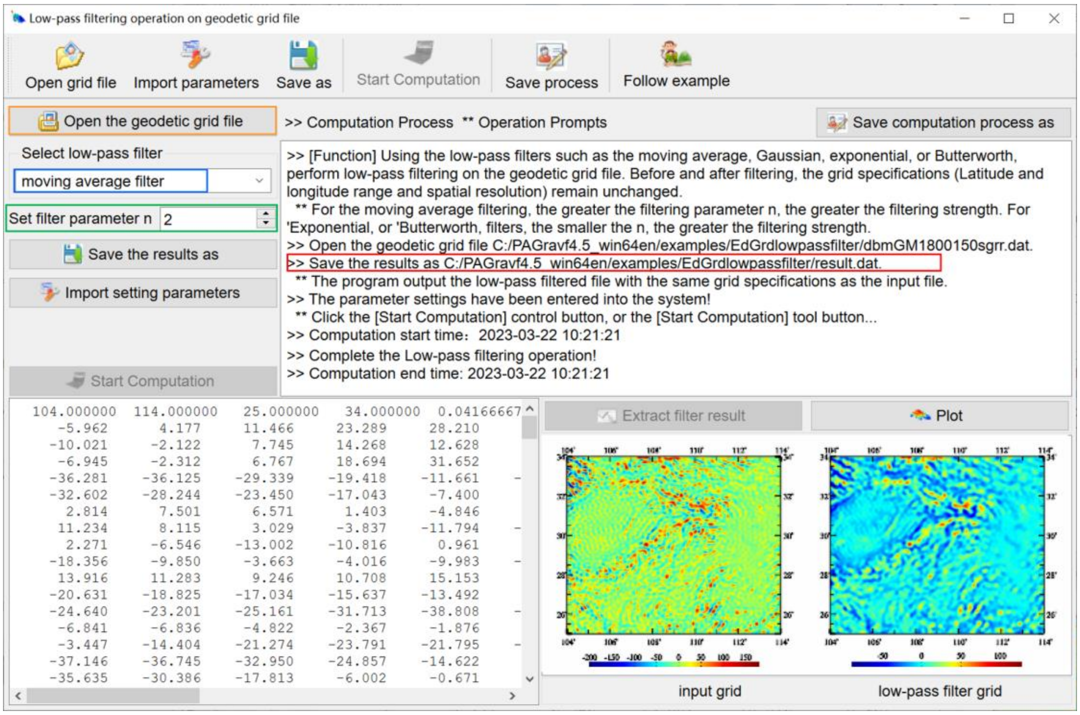
[Function] Using the low-pass filters such as the moving average, Gaussian, exponential, or Butterworth, perform low-pass filtering on the geodetic grid file. Before and after filtering, the grid specifications (Latitude and longitude range and spatial resolution) remain unchanged.

[Input file] The geodetic grid file.

[Parameter settings] Select low-pass filter, set filter parameter n.

For the moving average filtering, the greater the filtering parameter n, the greater the filtering strength. For 'Exponential' or 'Butterworth' filters, the smaller the n, the greater the filtering strength.

[Output file] The filtered geodetic grid file.



6.5 Simple gridding and regional geodetic grid construction

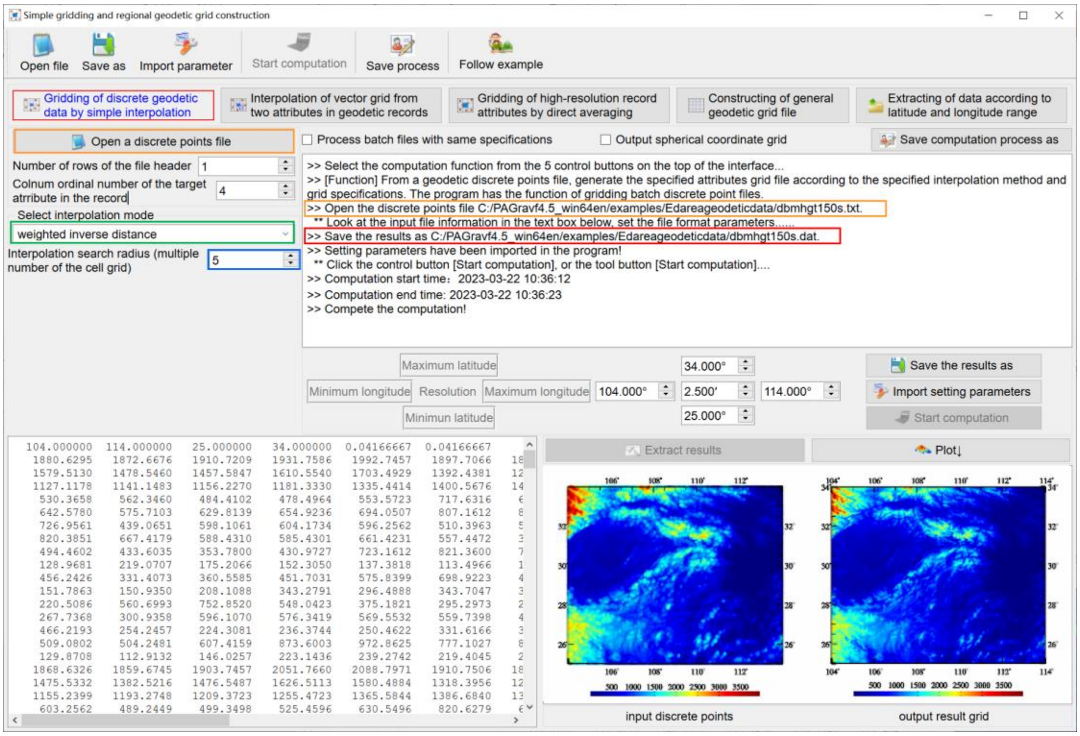
6.5.1 Gridding of discrete geodetic data by simple interpolation

[Function] From a geodetic discrete points file, generate the specified attribute grid file according to the specified interpolation method and grid specifications. The program has the function of gridding batch discrete point files.

[Input files] The discrete geodetic points file to be interpolated. The geodetic numerical grid file for interpolation.

[Parameter settings] Enter number of rows of the discrete points file header, column ordinal number of the target attribute in the file record, interpolation search radius (multiple of the grid element) and grid specifications parameters. Select the interpolation mode.

[Output file] The interpolated geodetic grid file.



6.5.2 Interpolation of vector grid from two attributes in geodetic records

[Function] From a geodetic discrete points file, generate the vector grid file according to the two specified component attributes, specified interpolation method, and given grid specifications.

6.5.3 Gridding of high-resolution record attributes by direct averaging

[Function] Using the direct averaging method, grid the high-resolution discrete observations.

6.5.4 Constructing of general geodetic grid file

[Function] According to the given latitude and longitude range and spatial resolution, generate the constant values, random number, 2D array index values, or Gaussian surface grid file.

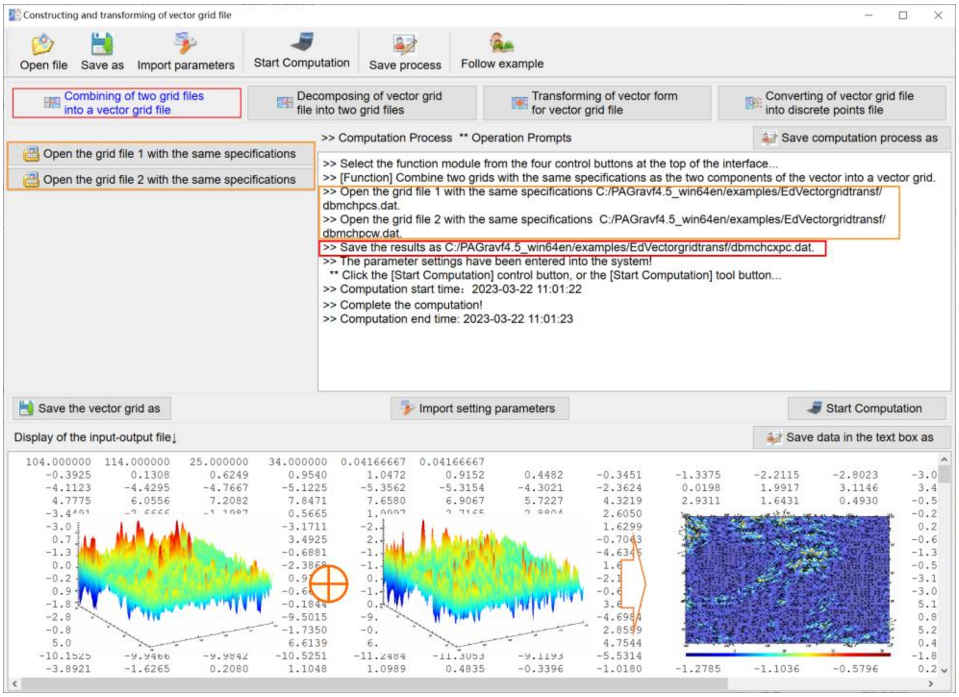
6.5.5 Extracting of data according to latitude and longitude range

[Function] According to the given latitude and longitude range, extract data from the geodetic discrete points file, grid file, or vector grid file. The program can extract data from batch files.

6.6 Constructing and transforming of vector grid file

6.6.1 Combining of two grid files into a vector grid file

[Function] Combine two grids with the same specifications as the two components of the vector into a vector grid.

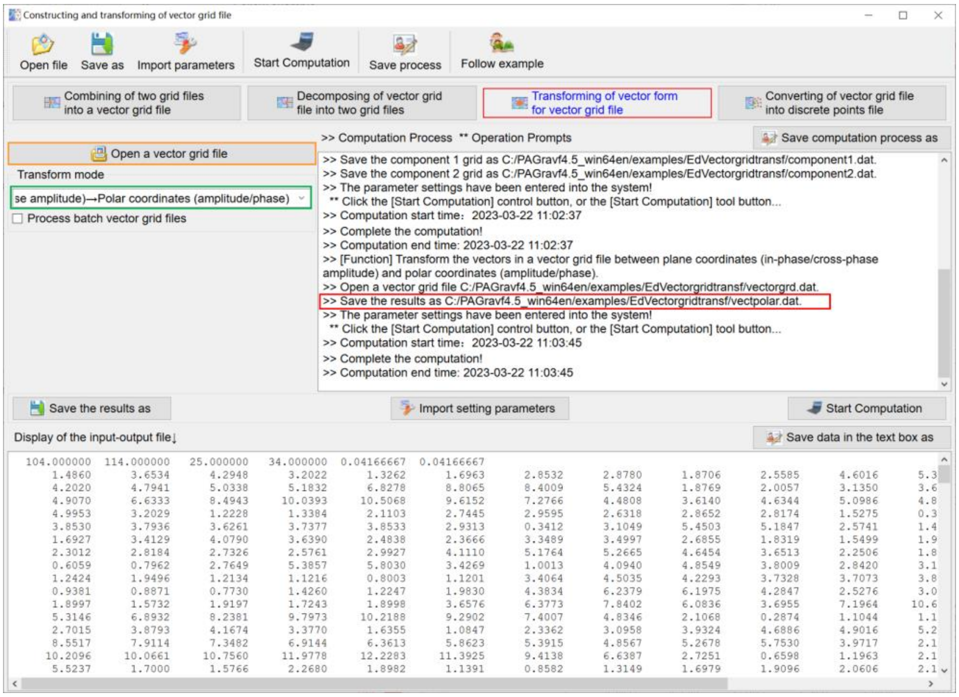


6.6.2 Decomposing of vector grid file into two grid files

[Function] Decompose a vector grid file into two component grid files.

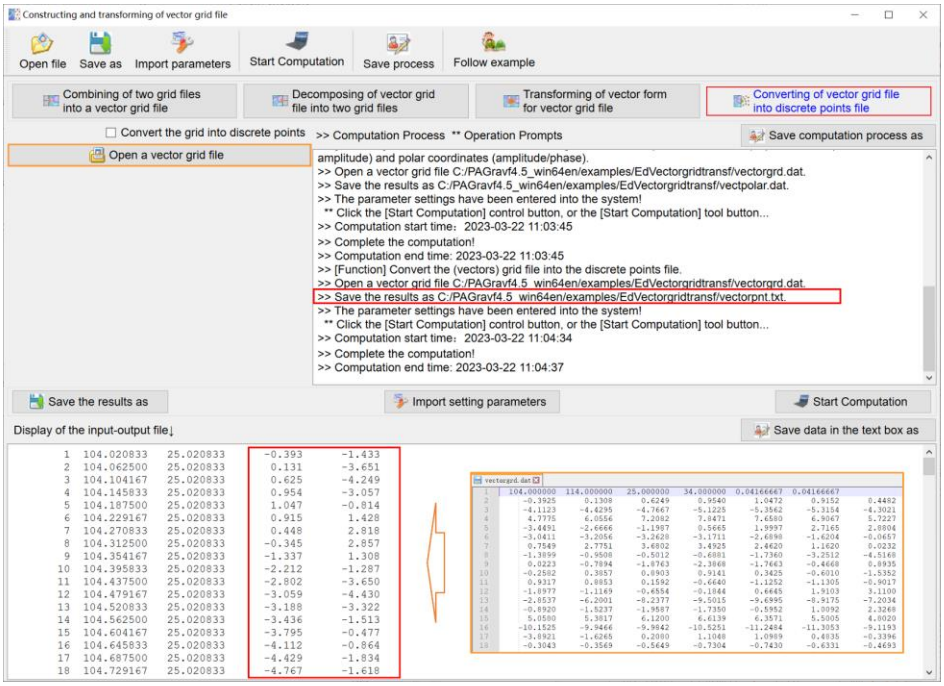
6.6.3 Transforming of vector form for vector grid file

[Function] Transform the vectors in a vector grid file between plane coordinates (in-phase/cross-phase amplitude) and polar coordinates (amplitude/phase).



6.6.4 Converting of vector grid file into discrete points file

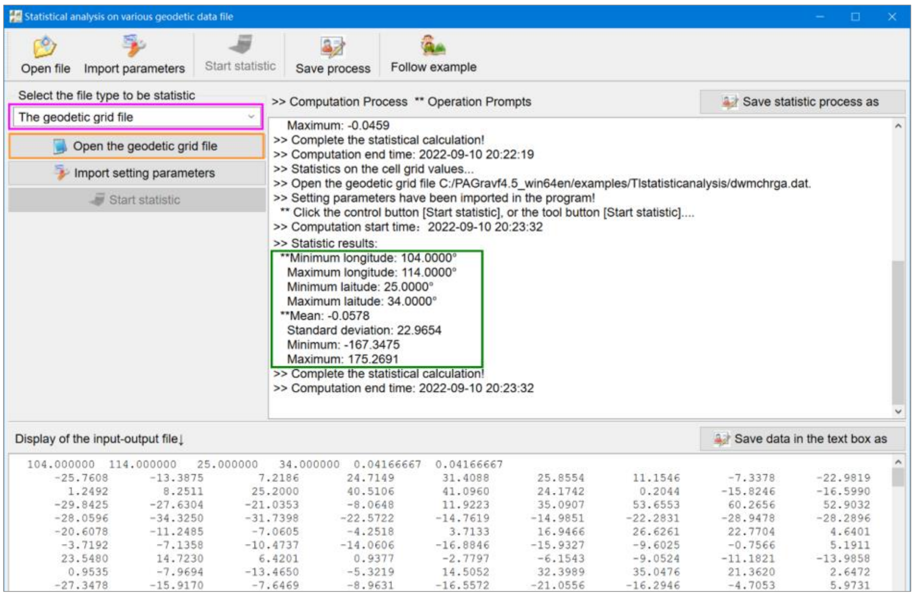
[Function] Convert the (vectors) grid file into the discrete points file.

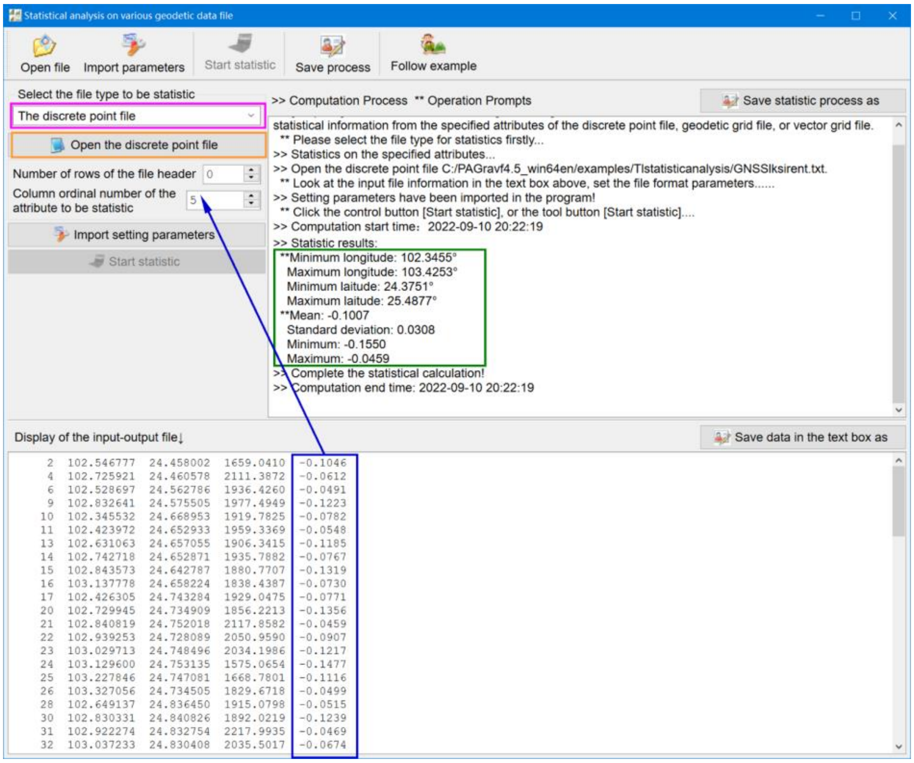


The latitude and longitude of discrete point is the latitude and longitude of the center point of the cell grid.

6.7 Statistical analysis on various geodetic data file

[Purpose] Extract the latitude and longitude range, mean, standard deviation, minimum, maximum, and other statistical information from the specified attributes of the discrete point file, geodetic grid file, or vector grid file.





6.8 Calculation of grid horizontal gradient and vector grid inner product

[Purpose] Calculate the first and second order horizontal gradient of the geodetic grid, perform an inner product operation on the two vector grids.

6.8.1 Horizontal gradient calculation on geodetic grid

[Function] Using the least squares method, estimate the first-order horizontal gradient vector (/km) or the second-order horizontal gradient vector (/km²) of the parameters of the geodetic grid. The horizontal gradient vector can be output in the form of polar coordinates or EN horizontal coordinates.

[Input files] The geodetic parameter grid file, the ellipsoidal height grid file of the surface where the parameter located.

6.8.2 Inner product operation on two vector grids

[Function] Perform an inner product operation on two vector grids with the same grid specification.

6.9 Visualization plot tools for various geodetic data files

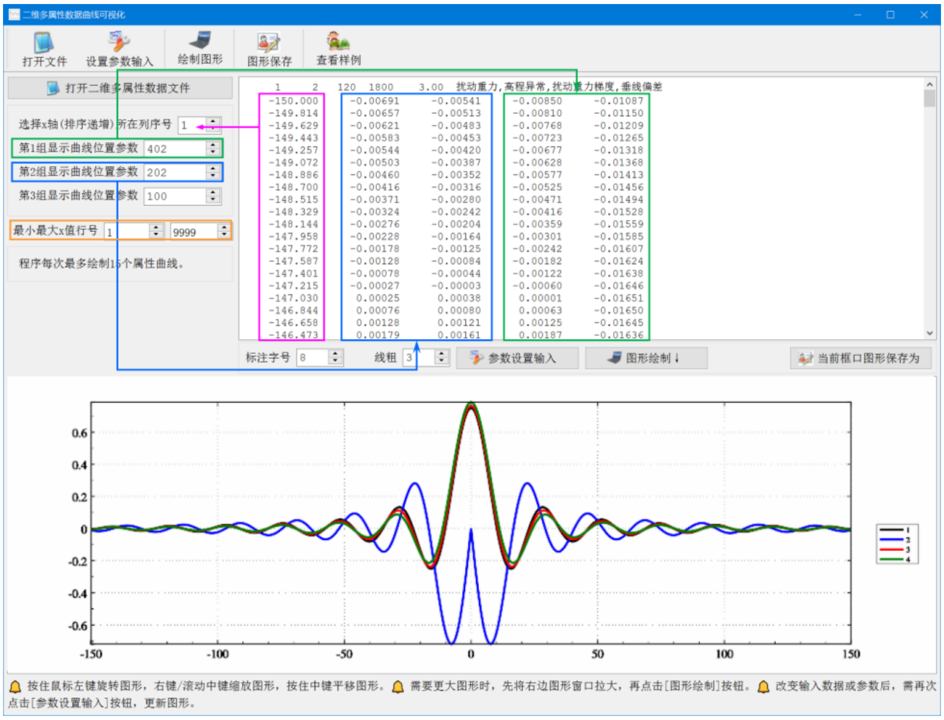
6.9.1 Visualization of multi-attribute curves from 2D geodetic data

[Function] Plot multi- attributes curves stored in a 2D geodetic data file.

The program requires the file header to occupy a row, and the x-axis column values are strictly sorted in an incremental manner.

The program can plot less than 15 curves each time.

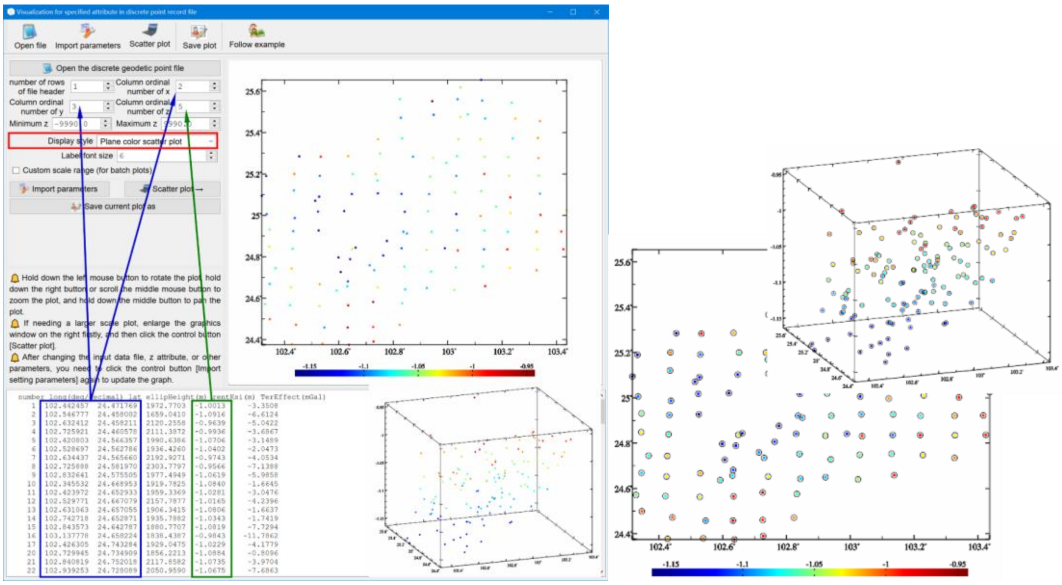
Hold down the left mouse button to rotate the plot, hold down the right button or scroll the middle mouse button to zoom the plot, and hold down the middle button to pan the plot.



If needing a larger scale plot, enlarge the graphics window on the right firstly, and then click the control button [Chart plot].

6.9.2 Visualization for specified attribute in discrete point record file

[Function] Displays the point locations and their specified attributes in a geodetic discrete point file.



If needing a larger scale plot, enlarge the graphics window on the right firstly, and then click the control button [Scatter plot].

After the input data file, z attribute, or other parameters changed, you need to click the control button [Import setting parameters] again to update the graph.

You can unify the scales by fixing the scale range for batch plots. Adjust the size of the plot window on the right and the plot requirements to an appropriate state before drawing batch plots. During plot period, the parameters and the size of the plot window are kept unchanged, and no mouse operation is performed on the plot.

6.9.3 Visualization for the geodetic grid file

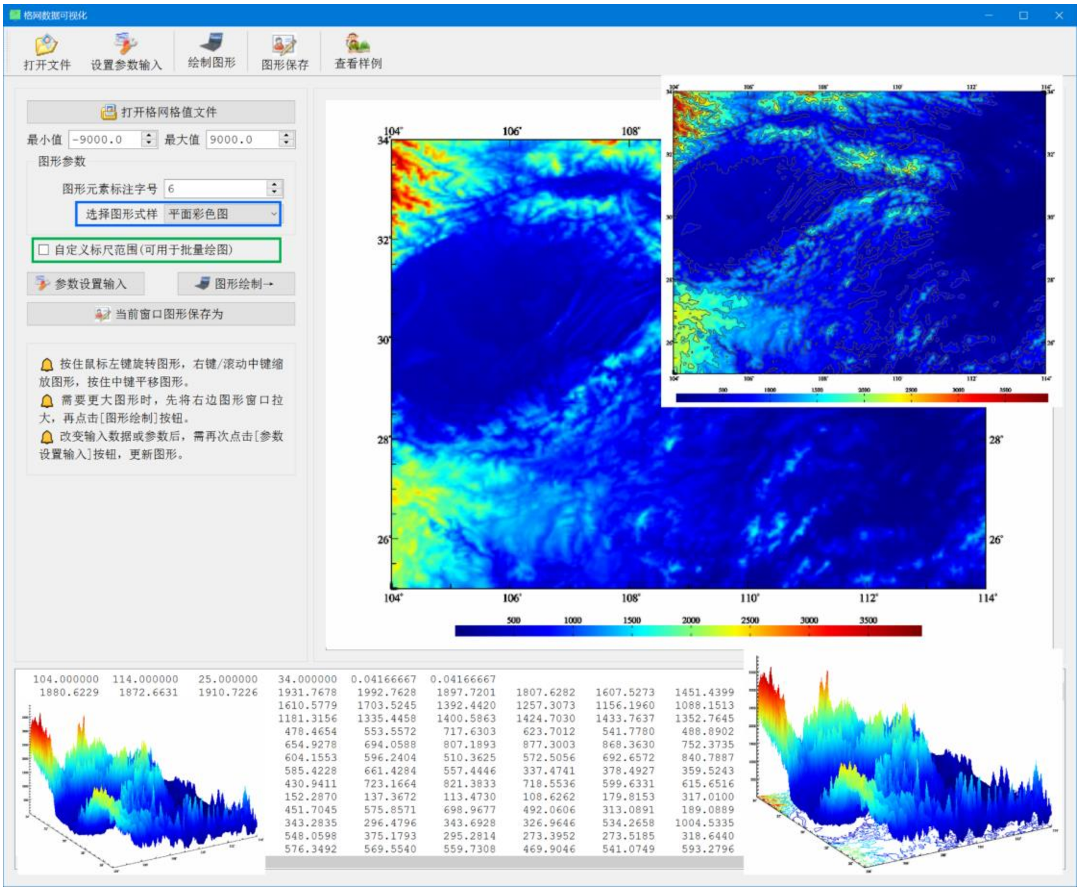
[Function] Plot the geodetic grid data from the geodetic grid file.

[Parameter settings] Select display style and set the checkbox [Custom scale range].

Allows the first component of a vector grid to be displayed as grid data.

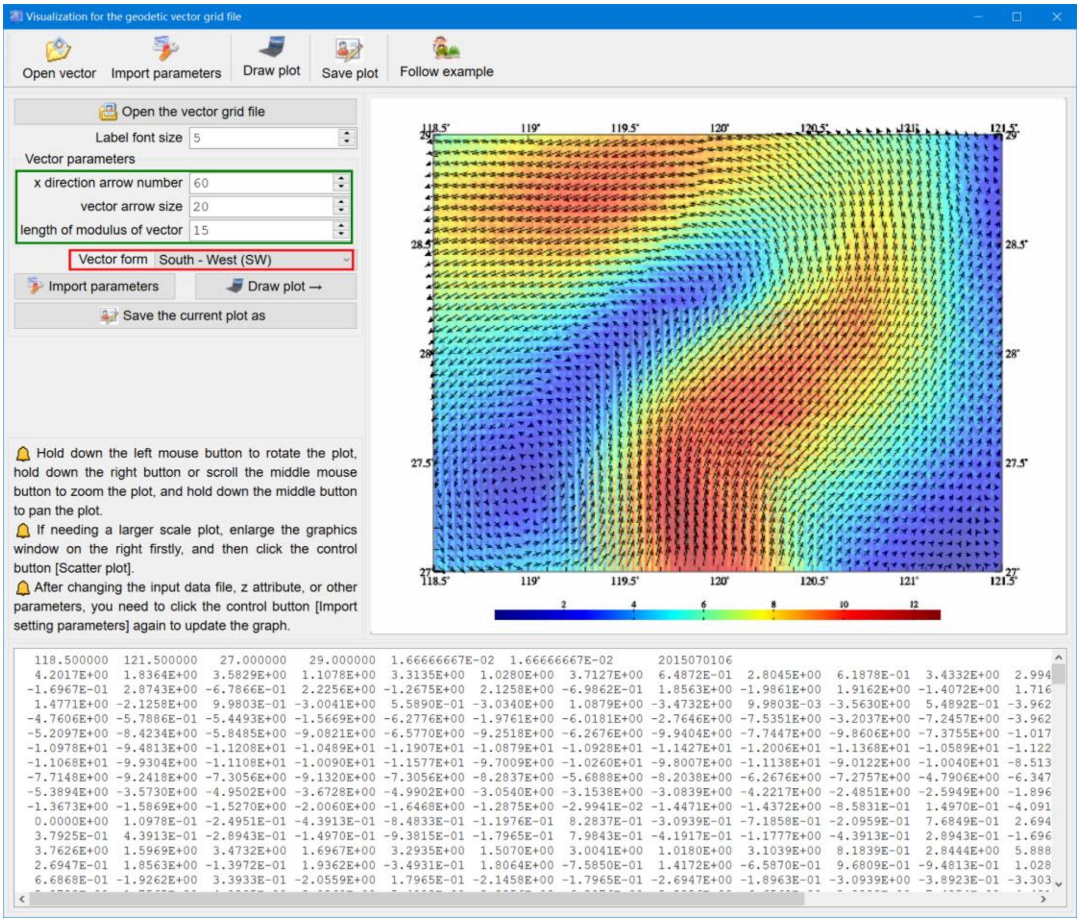
You can unify the scales by fixing the scale range for batch plots.

Adjust the size of the plot window on the right and the plot requirements to an appropriate state before drawing batch plots. During plot period, the parameters and the size of the plot window are kept unchanged, and no mouse operation is performed on the plot.



6.9.4 Visualization for the geodetic vector grid file

The X-axis and Y-axis of the plot coordinate system respectively point east and north (EN), which is the same with horizontal displacement vector.



Vector form: East-North (EN, e.g., horizontal displacement vector), South-West (SW, e.g., vertical deflection vector), North - West (NW, e.g., Tangential gravity gradient vector).

7 Featured algorithms and formulas in PAGravf4.5

More than 200 algorithm formulas here have been derived, cross-validated, programmed and verified repeatedly. In which, some of the algorithm formulas belong to PAGravf4.5 own. The performance and reliability of all these algorithmic formulas can be verified by calling the relative programs or functions in PAGravf4.5.

7.1 Calculation formulas of normal gravity field at any point in Earth space

(1) The normal geopotential U at the Earth space point (r,θ,λ) in the spherical coordinate system can be expressed as spherical harmonic series:

$$U(r,\theta)=\frac{GM}{r}\left[1-\sum_{n=1}^{\infty}\left(\frac{a}{r}\right)^{2n}J_{2n}P_{2n}(\cos\theta)\right]+\frac{1}{2}\omega^2r^2\sin^2\theta \tag{1.1}$$

$$J_{2n}=(-1)^{n+1}\frac{3e^{2n}}{(2n+1)(2n+3)}\left(1-n+\frac{5nJ_2}{e^2}\right) \tag{1.2}$$

where r is the distance from the calculation point to the center of the level ellipsoid, λ is the longitude of the calculation point, $\theta=\pi/2-\varphi$ is the geocentric colatitude, φ is the geocentric latitude, a is the semimajor axis of the ellipsoid, J_2 is the dynamic shape factor of the Earth, GM is the geocentric gravitational constant, ω is the mean rotation rate of the Earth, e is the first eccentricity of the level ellipsoid, and $P_{2n}(cos\theta)$ is the Legendre function.

(2) Taking the partial derivative of the normal gravitational potential $U(r,\theta)$ formula (1.1) in the spherical coordinate system, the normal gravity vector at the Earth space point can be obtained:

$$\gamma(r,\theta)=\gamma_r e_r+\gamma_\theta e_\theta \tag{1.3}$$

$$\gamma_r=-\frac{GM}{r^2}\left[1-\sum_{n=1}^{\infty}(2n+1)\left(\frac{a}{r}\right)^{2n}J_{2n}P_{2n}(cos\theta)\right]+\omega^2rsin^2\theta \tag{1.4}$$

$$\gamma_\theta=\frac{\partial U}{r\partial\theta}=-\frac{GM}{r^2}\left[\sum_{n=1}^{\infty}\left(\frac{a}{r}\right)^{2n}J_{2n}\frac{\partial}{\partial\theta}P_{2n}(cos\,\theta)\right]+\omega^2r\,sin\,\theta\,cos\theta \tag{1.5}$$

Since $e_r\perp e_\theta$, the normal gravity scalar value can be obtained:

$$\gamma=\sqrt{\gamma_r^2+\gamma_\theta^2} \tag{1.6}$$

and the north declination angle of the normal gravity line direction relative to the Earth center of mass can also be obtained:

$$\vartheta_\gamma=tan^{-1}\frac{\gamma_\theta}{\gamma_r} \tag{1.7}$$

(3) Furtherly taking the partial derivative of the normal gravity vector $\gamma(r,\theta)$ of the formula (1.3), the diagonal elements of the normal gravity gradient tensor at the earth space point in the spherical coordinate system can be obtained:

$$U_{rr}=-2\frac{GM}{r^3}\left[1-\sum_{n=1}^{\infty}(n+1)(2n+1)\left(\frac{a}{r}\right)^{2n}J_{2n}P_{2n}(cos\theta)\right]+\omega^2sin^2\theta \tag{1.8}$$

$$U_{\theta\theta}=\frac{\partial^2U}{r^2\partial\theta^2}=\frac{\partial\gamma_\theta}{r\partial\theta}=-\frac{GM}{r^3}\left[\sum_{n=1}^{\infty}\left(\frac{a}{r}\right)^{2n}J_{2n}\frac{\partial^2}{\partial\theta^2}P_{2n}(cos\,\theta)\right]+\omega^2\,cos2\,\theta \tag{1.9}$$

Since $e_r \perp e_\theta$, the normal gravity gradient scalar value can be obtained:

$$U_{nn} = \sqrt{U_{rr}^2 + U_{\theta\theta}^2} \tag{1.10}$$

and the north declination angle of the normal gravity gradient direction relative to the mass center of the Earth can also be obtained:

$$\vartheta_E = tan^{-1}\frac{U_{\theta\theta}}{U_{rr}} \tag{1.11}$$

(4) Low-dgree Legendre function $P_n(t)$ and its first and second derivative algorithms with respect to θ

Let $t = cos\,\theta\,,\quad u = sin\,\theta,$ (1.12)

then $P_n(t) = \frac{2n-1}{n}tP_{n-1}(t) - \frac{n-1}{n}P_{n-2}(t)$ (1.13)

$$P_1 = t,\quad P_2 = \frac{1}{2}(3t^2 - 1) \tag{1.14}$$

$$\frac{\partial}{\partial\theta}P_n(t) = \frac{2n-1}{n}t\frac{\partial}{\partial\theta}P_{n-1}(t) - \frac{2n-1}{n}uP_{n-1}(t) - \frac{n-1}{n}\frac{\partial}{\partial\theta}P_{n-2}(t) \tag{1.15}$$

$$\frac{\partial}{\partial\theta}P_1(t) = -u,\quad \frac{\partial}{\partial\theta}P_2(t) = -3ut \tag{1.16}$$

$$\frac{\partial^2}{\partial\theta^2}P_n(t) = \frac{2n-1}{n}\left(t\frac{\partial^2}{\partial\theta^2}P_{n-1} - 2u\frac{\partial}{\partial\theta}P_{n-1} - tP_{n-1}\right) - \frac{n-1}{n}\frac{\partial^2}{\partial\theta^2}P_{n-2} \tag{1.17}$$

$$\frac{\partial^2}{\partial\theta^2}P_1(t) = -t,\quad \frac{\partial^2}{\partial\theta^2}P_2(t) = 3(1 - 2t^2) \tag{1.18}$$

7.2 Calculation formulas of Earth gravity field from geopotential coefficient model

The disturbing potential T or height anomaly ζ at the space point (r,θ,λ) outside the Earth can be expressed as the following spherical harmonic series:

$$T(r,\theta,\lambda) = \zeta\gamma = \frac{GM}{r}\sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}(\delta\bar{C}_{nm}cosm\lambda + \bar{S}_{nm}sinm\lambda)\bar{P}_{nm} \tag{2.1}$$

where $\bar{C}_{nm}, \bar{S}_{nm}$ are called the fully normalized Stokes coefficients, also known as the geopotential coefficients, $\bar{P}_{nm} = \bar{P}_{nm}(t)$ is the fully normalized associative Legendre function, n is called the degree of the geopotential coefficient, m is called order of geopotential coefficients. And

$$\delta\bar{C}_{2n,0} = \bar{C}_{2n,0} + \frac{J_{2n}}{\sqrt{4n+1}} \tag{2.2}$$

$$\delta\bar{C}_{2n,m} = \bar{C}_{2n,m}(m > 0)\quad \delta\bar{C}_{2n+1,m} = \bar{C}_{2n+1,m} \tag{2.3}$$

The spherical harmonic series of gravity anomaly Δg, gravity disturbance δg, vertical deflection (ξ,η), disturbing gravity gradient T_{rr} and tangential gravity gradient (T_{NN},T_{WW}) at the space point (r,θ,λ) outside the Earth can be respectively expressed as:

$$\Delta g = \frac{GM}{r^2}\sum_{n=2}^{\infty}(n-1)\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}(\delta\bar{C}_{nm}cosm\lambda + \bar{S}_{nm}sinm\lambda)\bar{P}_{nm} \tag{2.4}$$

$$\delta g = -T_r = \frac{GM}{r^2}\sum_{n=2}^{\infty}(n+1)\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}(\delta\bar{C}_{nm}cosm\lambda + \bar{S}_{nm}sinm\lambda)\bar{P}_{nm} \tag{2.5}$$

$$\xi = \frac{T_\theta}{\gamma r} = \frac{GM}{\gamma r^2}\sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n}(\delta\bar{C}_{nm}cosm\lambda + \bar{S}_{nm}sinm\lambda)\frac{\partial}{\partial\theta}\bar{P}_{nm} \quad (2.6)$$

$$\eta = -\frac{T_\lambda}{\gamma rsin\theta} = \frac{GM}{\gamma r^2 sin\theta}\sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n \sum_{m=1}^{n} m(\delta\bar{C}_{nm}sinm\lambda - \bar{S}_{nm}cosm\lambda)\bar{P}_{nm} \quad (2.7)$$

$$T_{rr} = \frac{GM}{r^3}\sum_{n=2}^{\infty}(n+1)(n+2)\left(\frac{a}{r}\right)^n \sum_{m=0}^{n}(\delta\bar{C}_{nm}cosm\lambda + \bar{S}_{nm}sinm\lambda)\bar{P}_{nm} \quad (2.8)$$

$$T_{NN} = \frac{1}{r}T_r + \frac{1}{r^2}T_{\theta\theta}$$
$$= -\frac{\delta g}{r} + \frac{GM}{r^3}\sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n}(\delta\bar{C}_{nm}cosm\lambda + \bar{S}_{nm}sinm\lambda)\frac{\partial^2}{\partial\theta^2}\bar{P}_{nm} \quad (2.9)$$

$$T_{WW} = \frac{1}{r}T_r + \frac{1}{r^2}T_\theta ctg\theta + \frac{1}{r^2 sin^2\theta}T_{\lambda\lambda} = -\frac{\delta g}{r} + \frac{\xi\gamma}{r}ctg\theta$$
$$-\frac{GM}{r^3 sin^2\theta}\sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n} m^2(\delta\bar{C}_{nm}cosm\lambda + \bar{S}_{nm}sinm\lambda)\bar{P}_{nm} \quad (2.10)$$

here, $T_r = \frac{\partial}{\partial r}T(r,\theta,\lambda), \quad T_{rr} = \frac{\partial^2}{\partial r^2}T(r,\theta,\lambda)$ (2.11)

$$T_\theta = \frac{\partial}{\partial\theta}T(r,\theta,\lambda), \quad T_{\theta\theta} = \frac{\partial^2}{\partial\theta^2}T(r,\theta,\lambda) \quad (2.12)$$

$$T_\lambda = \frac{\partial}{\partial\lambda}T(r,\theta,\lambda), \quad T_{\lambda\lambda} = \frac{\partial^2}{\partial\lambda^2}T(r,\theta,\lambda) \quad (2.13)$$

$$T_{rr} + T_{NN} + T_{WW} \equiv 0, \quad T_{rr}^n + T_{NN}^n + T_{NN}^n \equiv 0, \quad T_* = \sum_{n=2}^{\infty} T_*^n \quad (2.14)$$

where T_*^n represents the degree n harmonic component of T_*. The N axis points North and the W axis points West.

Equation (2.14) is the Laplace equation, which can be employed to check the spatial and spectral domain performance of the geopotential model.

7.3 Algorithms of normalized associative Legendre function and its derivative

(1) Standard forward column recursion algorithm for $\bar{P}_{nm}(t)$ (n<1900)

$$\begin{cases} \bar{P}_{nm}(t) = a_{nm}t\bar{P}_{n-1,m}(t) - b_{nm}\bar{P}_{n-2,m}(t) \quad \forall n > 1, m < n \\ \bar{P}_{nn}(t) = u\sqrt{\frac{2n+1}{2n}}\bar{P}_{n-1,n-1} \quad n > 1 \end{cases} \quad (3.1)$$

$$a_{nm} = \sqrt{\frac{(2n-1)(2n+1)}{(n+m)(n-m)}}, \quad b_{nm} = \sqrt{\frac{(2n+1)(n+m-1)(n-m-1)}{(2n-3)(n+m)(n-m)}}$$

$$\bar{P}_{00}(t) = 1, \quad \bar{P}_{10}(t) = \sqrt{3}t, \quad \bar{P}_{11}(t) = \sqrt{3}u \quad (3.2)$$

(2) Improved Belikov recursion algorithm for $\bar{P}_{nm}(t)$ (n<64800)

When $n = 0,1$, $\bar{P}_{nm}(t)$ is calculated according to (3.2). And when n≥2, we have:

$$\bar{P}_{n0}(t) = a_n t\bar{P}_{n-1,0}(t) - b_n\frac{u}{2}\bar{P}_{n-1,1}(t), \; m = 0 \quad (3.3)$$

$$\bar{P}_{nm}(t) = c_{nm}t\bar{P}_{n-1,m}(t) - d_{nm}u\bar{P}_{n-1,m+1}(t) + e_{nm}u\bar{P}_{n-1,m-1}(t), m > 0 \quad (3.4)$$

$$a_n = \sqrt{\frac{2n+1}{2n-1}}, \quad b_n = \sqrt{\frac{2(n-1)(2n+1)}{n(2n-1)}} \quad (3.5)$$

$$c_{nm}=\frac{1}{n}\sqrt{\frac{(n+m)(n-m)(2n+1)}{2n-1}},\quad d_{nm}=\frac{1}{2n}\sqrt{\frac{(n-m)(n-m-1)(2n+1)}{2n-1}} \tag{3.6}$$

here when m>0,

$$e_{nm}=\frac{1}{2n}\sqrt{\frac{2}{2-\delta_0^{m-1}}}\sqrt{\frac{(n+m)(n+m-1)(2n+1)}{2n-1}} \tag{3.7}$$

(3) Cross degree-order recursive algorithm for $\bar{P}_{nm}(t)$ (n<20000)

When $n=0{,}1$, $\bar{P}_{nm}(t)$ is calculated according to (3.2). And when n≥2, we have:

$$\bar{P}_{nm}(t)=\alpha_{nm}\bar{P}_{n-2,m}(t)+\beta_{nm}\bar{P}_{n-2,m-2}(t)-\gamma_{nm}\bar{P}_{n,m-2}(t) \tag{3.8}$$

$$\alpha_{nm}=\sqrt{\frac{(2n+1)(n-m)(n-m-1)}{(2n-3)(n+m)(n+m-1)}}$$

$$\beta_{nm}=\sqrt{1+\delta_0^{m-2}}\sqrt{\frac{(2n+1)(n+m-2)(n+m-3)}{(2n-3)(n+m)(n+m-1)}} \tag{3.9}$$

$$\gamma_{nm}=\sqrt{1+\delta_0^{m-2}}\sqrt{\frac{(n-m+1)(n-m+2)}{(n+m)(n+m-1)}}$$

(4) Non-singular recursive algorithm for $\frac{\partial}{\partial\theta}\bar{P}_{nm}(\cos\theta)$

Considering that the first derivative of $\bar{P}_{nm}(\cos\theta)$ with respect to θ is

$$\frac{\partial}{\partial\theta}\bar{P}_{nm}(\cos\theta)=-\sin\theta\frac{\partial}{\partial t}\bar{P}_{nm}(t) \tag{3.10}$$

we have

$$\begin{cases}\frac{\partial}{\partial\theta}\bar{P}_{n0}=-\sqrt{\frac{n(n+1)}{2}}\bar{P}_{n1},\quad \frac{\partial}{\partial\theta}\bar{P}_{n1}=\sqrt{\frac{n(n+1)}{2}}\bar{P}_{n0}-\frac{\sqrt{(n-1)(n+2)}}{2}\bar{P}_{n2}\\ \frac{\partial}{\partial\theta}\bar{P}_{nm}=\frac{\sqrt{(n+m)(n-m+1)}}{2}\bar{P}_{n,m-1}-\frac{\sqrt{(n-m)(n+m+1)}}{2}\bar{P}_{n,m+1},\quad m>2\end{cases} \tag{3.11}$$

$$\frac{\partial}{\partial\theta}\bar{P}_{00}(t)=0,\quad \frac{\partial}{\partial\theta}\bar{P}_{10}(t)=-\sqrt{3}u,\quad \frac{\partial}{\partial\theta}\bar{P}_{11}(t)=\sqrt{3}t \tag{3.12}$$

(5) Non-singular recursive algorithm for $\frac{\partial^2}{\partial\theta^2}\bar{P}_{nm}(\cos\theta)$

$$\begin{cases}\frac{\partial^2}{\partial\theta^2}\bar{P}_{n0}=-\frac{n(n+1)}{2}\bar{P}_{n0}+\sqrt{\frac{n(n-1)(n+1)(n+2)}{8}}\bar{P}_{n2}\\ \frac{\partial^2}{\partial\theta^2}\bar{P}_{n1}=-\frac{2n(n+1)+(n-1)(n+2)}{4}\bar{P}_{n1}+\frac{\sqrt{(n-2)(n-1)(n+2)(n+3)}}{4}\bar{P}_{n3}\end{cases} \tag{3.13}$$

$$\begin{aligned}\frac{\partial^2}{\partial\theta^2}\bar{P}_{nm}=&\frac{\sqrt{(n-m+1)(n-m+2)(n+m-1)(n+m)}}{4}\bar{P}_{n,m-2}\\ &-\frac{(n+m)(n-m+1)+(n-m)(n+m+1)}{4}\bar{P}_{nm}\\ &-\frac{(n+m)(n-m+1)+(n-m)(n+m+1)}{4}\bar{P}_{nm}\\ &+\frac{\sqrt{(n-m-1)(n-m)(n+m+1)(n+m+2)}}{4}\bar{P}_{n,m+2},\quad m>2\end{aligned} \tag{3.14}$$

$$\frac{\partial^2}{\partial\theta^2}\bar{P}_{00}(t)=0,\quad \frac{\partial^2}{\partial\theta^2}\bar{P}_{10}(t)=-\sqrt{3}t,\quad \frac{\partial^2}{\partial\theta^2}\bar{P}_{11}(t)=-\sqrt{3}u \tag{3.15}$$

7.4 Boundary value correction for ellipsoid and spherical boundary surface

(1) Ellipsoid correction of gravity. The correction of the gravity g on an ellipsoid surface outside the Earth from the vertical direction to the normal gravity direction, also known as the vertical deflection correction of gravity.

$$\varepsilon_p = \gamma sin\theta cos\theta \left[3J_2\left(\frac{a}{r}\right)^2 + \frac{\omega^3 r^3}{GM}\right]\xi \tag{4.1}$$

(2) The correction of the gravity g from the normal gravity direction to the geocentric direction

$$\varepsilon_h = \gamma e^2 sin\theta cos\theta\xi \tag{4.2}$$

(3) The correction of the normal gravity γ from the normal gravity direction to the geocentric direction

$$\varepsilon_\gamma = 3\gamma\left[J_2\frac{a^2}{r^3}(3cos^2\theta - 1) - \frac{\omega^3 r^3}{GM}sin^2\theta\right]T \tag{4.3}$$

When the boundary surface is an ellipsoidal surface, only one ellipsoid correction is required in equation (4.1). When the boundary surface is spherical surface, it is necessary to perform three items boundary value corrections using equations (4.1) ~ (4.3).

7.5 Terrain effect algorithms on various anomalous field elements outside geoid

The theory of the Earth's gravitational field points out that any type of anomalous gravity field element outside the Earth can be expressed as a linear combination of the disturbing potential, gravity disturbance or their partial derivatives on some an equipotential surface. For example, the vertical deflection can be expressed by the local horizontal partial derivative of the disturbing potential, and the disturbing gravity gradient can be expressed by the vertical derivative of the gravity disturbance. Therefore, if we can get the terrain effects on disturbing potential and gravity disturbance, we also can get the terrain effects on other types of anomalous gravity field elements.

7.5.1 Expression of land terrain mass gravitational field - land complete Bouguer effect

The land terrain mass gravitational field is also called as the complete Bouguer effect of land terrain, which is defined as the effect of the terrain mass above the geoid on the Earth's gravity field or on its various gravity field elements.

(1) Land complete Bouguer effect on disturbing potential

Ignoring the mass effect of the Earth's atmosphere, the disturbing potential T at the calculation point outside the Earth can be expressed as the sum of the terrain mass gravitational potential T^t (land complete Bouguer effect) and the disturbing potential T^{NT} after the terrain removed:

$$T = T^{NT} + T^t = T^{NT} + T^B + T^R \tag{5.1}$$

where T^t is the gravitational potential generated by the total terrain mass at the calculation point, which is called as the terrain effect on the disturbing potential or land complete Bouguer effect. T^R is the effect of the local terrain mass on the gravitational potential at the calculation point, called the local terrain effect on the disturbing potential. T^B is the gravitational potential at the calculation point generated by the mass of the spherical shell with a thickness equal to the terrain height, which is called the spherical shell Bouguer effect on the disturbing potential.

From the harmonic properties of the disturbing potential T outside the Earth, it can be known that the land complete Bouguer effect T^t, local terrain effect T^R and spherical shell Bouguer effect T^B on the disturbing potential outside the earth are all harmonic.

Under the spherical approximation, the complete Bouguer effect on disturbing potential in the near-Earth space outside the Earth ($r \geq R + h$, R is the mean radius of the Earth) can be expressed as:

$$T^t = T^B + T^R = 4\pi G\rho_0 \frac{R^2 h}{r}\left(1 + \frac{h}{R} + \frac{h^2}{3R^2}\right) + T^R \tag{5.2}$$

where G is the gravitational constant, h is the terrain height directly below the calculation point, r is the geocentric distance of the calculation point, ρ_0 is the geometric mean density of the terrain from the ground to the geoid, and ρ_0 = 2.67×10³km/m³.

(2) Land complete Bouguer effect on gravity disturbance

Substituting (5.1) into the definition of gravity disturbance, we get

$$\delta g = -\frac{\partial T^{NT}}{\partial r} - \frac{\partial T^t}{\partial r} = \delta g^{NT} + \delta g^t = \delta g^{NT} + \delta g^B + \delta g^R \tag{5.3}$$

where δg^t is called the complete Bouguer effect on gravity disturbance, δg^B is the spherical shell Bouguer effect, and δg^R is called the local terrain effect on gravity disturbance.

Under spherical approximation, the complete Bouguer effect on gravity disturbance:

$$\delta g^t = \delta g^B + \delta g^R = 4\pi G\tilde{\rho} \frac{R^2 h}{r^2}\left(1 + \frac{h}{R} + \frac{h^2}{3R^2}\right) + \delta g^R \tag{5.4}$$

Equations (5.2) and (5.4) are truncated to the quadratic term of h/R, which are suitable on ground and near-Earth space (such as aviation altitude), but not suitable on satellite altitude.

7.5.2 Integral formula of local terrain effect outside the Earth

(1) The local terrain effect on disturbing potential

According to the definition, only considering the surface density ρ, the local terrain effect on disturbing potential can be expressed as:

$$T^R = \gamma\zeta^R = G\rho \iint_S \int_{R+h}^{R+h'} L^{-1}(r, \psi, r')\, dr'\, ds \tag{5.5}$$

where $ds = r'^2 cos\varphi' d\varphi' d\lambda'$ is the integral area element on the ground, and $L = \sqrt{r^2 + r'^2 - 2rr'cos\psi}$ is the space distance from the move point (namely the integral volume element $dV = dr'ds$) to the calculation point.

$$\int L^{-1}(r,\psi,r')\,dr' = ln(r' - rt + L) + C \tag{5.6}$$

where $t = cos\psi$, C is the integral constant.

When the calculation point is the same as the move point, the integral of local terrain effect on disturbing potential is singular:

$$T^R|_0 = \frac{1}{6}G\rho_0 A_0\sqrt{A_0/\pi}\left(h_{xx} + h_{yy}\right) \tag{5.7}$$

where ρ_0 is the terrain density at the calculation point, A_0 is the area of the integral area element at the calculation point, and h_{xx}, h_{yy} are the second-order horizontal partial derivatives of the terrain at the calculation point in the north direction x and the east direction y.

(2) The local terrain effect on gravity disturbance

According to the definition, the local terrain effect on gravity disturbance can be expressed as:

$$\delta g^R = -T_r^R = -\frac{\partial T^R}{\partial r} = -G\rho\iint_s \int_{R+h}^{R+h'} \frac{\partial L^{-1}(r,\psi,r')}{\partial r} dr' ds \tag{5.8}$$

where, $\int \frac{\partial L^{-1}(r,\psi,r')}{\partial r} dr' = -\int \frac{r - r't}{L^3} dr' = -\frac{r'}{rL} + C$ (5.9)

When the calculation point is the same as the move point, the integral of local terrain effect on gravity disturbance is singular:

$$\delta g^R|_0 = \frac{1}{2}G\rho_0\sqrt{\pi A_0}\left(h_x^2 + h_y^2\right) \tag{5.10}$$

where (h_x, h_y) is the terrain slope vector at the calculation point

(3) The local terrain effect on vertical deflection

Considering $\frac{\partial\psi}{\partial\varphi} = -cos\alpha$, $\frac{\partial\psi}{\partial\lambda} = -cos\varphi sin\alpha$, we have:

$$\xi^R = \frac{T_\theta^R}{\gamma r} = -\frac{\partial T^R}{\gamma r\partial\varphi} = -\frac{\partial T^R}{\gamma r\partial\psi}\frac{\partial\psi}{\partial\varphi} = \frac{\partial T^R}{\gamma r\partial\psi}cos\alpha$$

$$= \frac{G\rho}{\gamma r}\iint_s \int_{R+h}^{R+h'} \frac{\partial L^{-1}(r,\psi,r')}{\partial\psi} dr' cos\alpha ds \tag{5.11}$$

$$\eta^R = -\frac{T_\lambda^R}{\gamma r sin\theta} = -\frac{\partial T^R}{\gamma cos\varphi\partial\lambda} = -\frac{\partial T^R}{\gamma cos\varphi\partial\psi}\frac{\partial\psi}{\partial\lambda} = \frac{\partial T^R}{\gamma cos\varphi\partial\psi}cos\varphi sin\alpha$$

$$= \frac{G\rho}{\gamma r}\iint_s \int_{R+h}^{R+h'} \frac{\partial L^{-1}(r,\psi,r')}{\partial\psi} dr' sin\alpha ds \tag{5.12}$$

where $\int \frac{\partial L^{-1}(r,\psi,r')}{\partial\psi} dr' = \frac{r - r't}{Lsin\psi} + C$, (5.14)

α is the geodetic azimuth of ψ, which can be obtained from the spherical trigonometric formula:

$$sin\psi cos\alpha = cos\varphi sin\varphi' - sin\varphi cos\varphi' cos(\lambda' - \lambda) \tag{5.14}$$

$$sin\psi sin\alpha = cos\varphi' sin(\lambda' - \lambda) \tag{5.15}$$

(4) The local terrain effect on disturbing gravity gradient

$$T_{rr}^R = \frac{\partial^2}{\partial r^2}T^R = G\rho\iint_s \int_{R+h}^{R+h'} \frac{\partial^2 L^{-1}(r,\psi,r')}{\partial r^2} dr'\, ds \tag{5.16}$$

where $\int \frac{\partial^2 L^{-1}(r,\psi,r')}{\partial r^2} dr' = \int \left[-\frac{1}{L^3} + \frac{3(r-r't)^2}{L^5}\right] dr' = \frac{r'}{r^2 L} + \frac{r'(r-r't)}{rL^3} + C$ (5.17)

(5) The local terrain effect on tangential gravity gradient

$$T_{NN}^R = \frac{1}{r} T_r^R + \frac{1}{r^2} T_{\theta\theta}^R = -\frac{1}{r}\delta g^R - \frac{1}{r^2} T_{\varphi\varphi}^R \quad (5.18)$$

$$T_{ww}^R = \frac{1}{r} T_r + \frac{1}{r^2} T_\theta ctg\theta + \frac{1}{r^2 sin^2\theta} T_{\lambda\lambda} = -\frac{1}{r}\delta g^R + \frac{\gamma}{r}\xi^R ctg\theta + \frac{1}{r^2 cos^2\varphi} T_{\lambda\lambda}^R \quad (5.19)$$

$$T_{\varphi\varphi}^R = \frac{\partial^2 T^R}{\partial\psi^2}\frac{\partial^2\psi}{\partial\varphi^2}, \quad T_{\lambda\lambda}^R = \frac{\partial^2 T^R}{\partial\psi^2}\frac{\partial^2\psi}{\partial\lambda^2} \quad (5.20)$$

Taking the partial derivative with respect to φ on both sides of equation (5.14), considering $\frac{\partial\psi}{\partial\varphi} = -cos\alpha,\ \ \frac{\partial\psi}{\partial\lambda} = -cos\varphi sin\alpha$, we have:

$$sin\psi\frac{\partial^2\psi}{\partial\varphi^2} = -\sin\varphi sin\varphi' - cos\varphi cos\varphi' cos(\lambda' - \lambda) + cos\psi cos^2\alpha \quad (5.21)$$

In the same way, taking the partial derivatives of both sides of (5.15) with respect to λ, we have: $-cos\psi cos\varphi sin^2\alpha + sin\psi\frac{\partial^2\psi}{\partial\lambda^2} = -cos\varphi' sin(\lambda' - \lambda)$, so that we can get:

$$sin\psi\frac{\partial^2\psi}{\partial\lambda^2} = -cos\varphi' sin(\lambda' - \lambda) + cos\psi cos\varphi sin^2\alpha \quad (5.22)$$

Calculate the second-order partial derivative with respect to the spherical angular distance ψ on both sides of the integral of the local terrain effect on disturbing potential, we have:

$$\frac{\partial^2 T^R}{\partial\psi^2} = G\rho\iint_s \int_{R+h}^{R+h'} \frac{\partial^2}{\partial\psi^2}\frac{1}{L} dr' ds = G\rho\iint_s \int_{R+h}^{R+h'} \frac{\partial^2}{\partial\psi^2}\frac{1}{\sqrt{r^2+r'^2-2rr'cos\psi}} dr' ds \quad (5.23)$$

$$\int \frac{\partial^2}{\partial\psi^2}\frac{1}{L} dr' = \frac{r'(6r^2+4r'^2+6r^2cos2\psi - rr'cos3\psi) - rt(4r^2+11r'^2)}{4L^3 sin^2\psi} \quad (5.24)$$

7.5.3 Fast FFT algorithms of the integral of local terrain effect on various field elements

(1) Fast algorithm of the integral of local terrain effect on disturbing potential

Using the local spherical polar coordinate system, let the z-axis be the radial direction from the Earth center of mass, that is, the zenith direction, the $z = 0$ is the terrain surface, and $\tilde{h}$ is the height of the calculation point relative to the terrain surface. In this case, $dz = dr'$, $d\tilde{h} = dr$, then the local terrain effect on disturbing potential (5.5) is equivalent to:

$$T^R = G\rho\iint_s \int_0^{\Delta h} \frac{dz}{L} ds = G\rho\iint_s \int_0^{\Delta h} \frac{dz}{\sqrt{(\tilde{h}-z)^2 + l^2}} ds$$

$$= G\rho\iint_s \left[ln\frac{\sqrt{(\tilde{h}-\Delta h)^2+l^2} - \tilde{h} + \Delta h}{\sqrt{(\tilde{h}-\Delta h)^2+l^2} + \tilde{h} - \Delta h} - ln\frac{\sqrt{\tilde{h}^2+l^2} - H}{\sqrt{\tilde{h}^2+l^2} + H}\right] ds \quad (5.25)$$

where Δh is the height difference of the integral move area element ds on the surface relative to the terrain surface directly below the calculation point, $l = 2r_0 sin(\psi/2)$ is the spherical distance from the move area element to the calculation point, and r_0 is the geocentric distance of the surface directly below the calculation point, and ψ is the spherical angular distance from the move point to the calculation point.

Expand the integrand in Eq. (5.25) to order 3 near $z=0$, we have:

$$T^R = G\rho \iint_s \left[\frac{1}{\mathcal{L}}\Delta h + \frac{\tilde{h}}{2\mathcal{L}^3}\Delta h^2 + \frac{2\tilde{h}^2 - l^2}{6\mathcal{L}^5}\Delta h^3\right] ds \tag{5.26}$$

where $\mathcal{L} = \sqrt{\tilde{h}^2 + l^2}$ is the space distance from the move area element ds to the calculation point in the case of $z=0$. Here $\mathcal{L} \neq L$, and L is the space distance from the move volume element $dzds$ to the calculation point.

Considering $\Delta h^2 = h'^2 - 2h'h + h^2$, $\Delta h^3 = h'^3 - 3h'^2 h + 3h'h^2 - h^3$, each item on the right side of the formula (5.26) can be quickly calculated using the FFT algorithm. where, h is the terrain height directly below the calculation point, and h' is the terrain height of the move area element.

(2) Fast algorithm of the integral of local terrain effect on gravity disturbance

In the same way, the local terrain effect on gravity disturbance (5.8) is equivalent to:

$$\delta g^R = \frac{G\rho}{r}\iint_s \left[\frac{(r_0+z)}{\sqrt{(\tilde{h}-z)^2+l^2}}\right]_0^{\Delta h} ds = \frac{G\rho}{r}\iint_s \left[\frac{r_0+\Delta h}{\sqrt{\left(\tilde{h}-\Delta h\right)^2+l^2}} - \frac{r_0}{\mathcal{L}}\right] ds \tag{5.27}$$

Expand the integrand in Eq. (5.27) to order 4 near $z=0$, we have:

$$\delta g^R = \frac{G\rho}{r}\iint_s \left[\begin{array}{c}\frac{r\tilde{h}+\mathcal{L}^2}{\mathcal{L}^3}\Delta h + \frac{2\tilde{h}\mathcal{L}^2 + r_0\left(2\tilde{h}^2 - l^2\right)}{2\mathcal{L}^5}\Delta h^2 + \frac{2\tilde{h}^3 r + \tilde{h}^2 l^2 - 3r_0\tilde{h}l^2 - l^4}{2\mathcal{L}^7}\Delta h^3 \\ + \frac{8r\tilde{h}^4 - 4\tilde{h}^3 l^2 - 12\tilde{h}l^4 - 24r_0\tilde{h}^2 l^2 + 3l^4 r_0}{8\mathcal{L}^9}\Delta h^4\end{array}\right] ds \tag{5.28}$$

where $\Delta h^4 = h'^4 - 4h'^3 h + 6h^2 h'^2 - 4h'h^3 + h^4$。

Each item on the right side of the formula (5.28) can be quickly calculated using the FFT algorithm. In PAGravf4.5, the numerical integral of the local terrain effect on gravity disturbance is calculated using formula (5.27).

(3) Fast algorithm of the integral of local terrain effect on vertical deflection

Expand the integrand in the integral formula of the local terrain effect on vertical deflection to order 3 near $z=0$:

$$\int_{R+h}^{R+h'} \frac{\partial L^{-1}(r,\psi,r')}{\partial\psi} dr'$$
$$= -\frac{r^2 sin\psi}{\mathcal{L}^3}\Delta h - \frac{3\tilde{h}r^2 sin\psi}{2\mathcal{L}^5}\Delta h^2 - \left[\frac{r^2 sin\psi}{3\mathcal{L}^5} + \frac{5r^2 sin\psi(2\tilde{h}^2 - l^2)}{6\mathcal{L}^7}\right]\Delta h^3 \tag{5.29}$$

Substitute (5.29) into (5.11) and (5.12) formulas, and the FFT algorithm can be used to quickly calculate the local terrain effect on vertical deflection.

(4) Fast algorithm of the integral of local terrain effect on disturbing gravity gradient

The local terrain effect on disturbing gravity gradient (5.16) is equivalent to:

$$T_{rr}^R = G\rho \iint_s \left[\frac{\tilde{h}-\Delta h}{((\tilde{h}-\Delta h)^2+l^2)^{3/2}} - \frac{\tilde{h}}{\mathcal{L}^3}\right] ds \tag{5.30}$$

Expand the integrand in Eq. (5.30) to order 3 near $z=0$, we have:

$$T_{rr}^{R}=G\rho\iint_{s}\left[\begin{array}{c}\frac{2\tilde{h}^{2}-l^{2}}{\mathcal{L}^{5}}\Delta h-\frac{3\tilde{h}(2\tilde{h}^{2}-3l^{2})}{2\mathcal{L}^{7}}\Delta h^{2}+\\ \frac{4\tilde{h}^{4}+6r^{4}-12\tilde{h}^{2}l^{2}-(6r^{4}+3r^{2}l^{2})t}{\mathcal{L}^{9}}\Delta h^{3}\end{array}\right]ds \tag{5.31}$$

(5) Fast algorithm of the integral of local terrain effect on tangential gravity gradient

The integrand function in Equation (5.23) is equivalent to:

$$\int_{R+h}^{R+h'}\frac{\partial^{2}}{\partial\psi^{2}}\frac{1}{L}dr'=\int_{0}^{\Delta h}\frac{\partial^{2}}{\partial\psi^{2}}\frac{1}{\sqrt{(\tilde{h}-z)^{2}+4r_{0}^{2}sin^{2}(\psi/2)}}dz$$

$$=\frac{1}{8sin^{2}\frac{\psi}{2}}\left[\frac{\tilde{h}(2\mathcal{L}^{2}+r_{0}^{2}sin^{2}\psi)}{\mathcal{L}^{3}}-\frac{(\tilde{h}-\Delta h)(2\mathcal{L}^{2}+r_{0}^{2}sin^{2}\psi-4\tilde{h}\Delta h+2\Delta h^{2})}{(\mathcal{L}^{2}-2\tilde{h}\Delta h+\Delta h^{2})^{3/2}}\right] \tag{5.32}$$

Expand it to order 3 near z=0, we have:

$$\int_{R+h}^{R+h'}\frac{\partial^{2}}{\partial\psi^{2}}\frac{1}{L}dr'=-\frac{2(\tilde{h}^{2}+2r_{0}^{2})cos\psi+r_{0}^{2}(-5+cos2\psi)}{2L^{5}}r_{0}^{2}\Delta h$$
$$+\frac{6(\tilde{h}^{2}+2r_{0}^{2})cos\psi+3r_{0}^{2}(-7+3cos2\psi)}{4\mathcal{L}^{7}}\tilde{h}r_{0}^{2}\Delta h^{2}$$
$$+\frac{(8\tilde{h}^{4}+12\tilde{h}^{2}r_{0}^{2}-19r_{0}^{4})cos\psi-r_{0}^{2}(36\tilde{h}^{2}-18r_{0}^{2}-(24\tilde{h}^{2}-2r_{0}^{2})cos2\psi+3r_{0}^{2}cos3\psi)}{4\mathcal{L}^{9}}r_{0}^{2}\Delta h^{3} \tag{5.33}$$

If the calculation point is also on the terrain surface, there are $\tilde{h}=0$, $\mathcal{L}=l$, formulas (5.25) ~ (5.33) can be greatly simplified.

7.6 Seawater Bouguer effect and land-sea residual terrain effect

7.6.1 Marine terrain gravitational field - seawater complete Bouguer effect

The marine terrain gravitational field is usually represented by the seawater complete Bouguer effect. The seawater complete Bouguer effect is defined as the effect on the Earth's gravity field (various gravity field elements) because of the density of seawater compensated to the density of land terrain.

The seawater complete Bouguer effect on disturbing potential can be directly expressed by the integral formula as:

$$T^{o}=G\beta\iint_{s}\int_{R+d}^{R}L^{-1}(r,\psi,r')\,dr'\,ds \tag{6.1}$$

where $d<0$ is the seafloor depth, β is the seawater compensation density, equal to the difference between the terrain density and the seawater density, $\beta=\rho_{0}-\rho_{w}$ =1.64×10³km/m³, and L is the space distance of the move volume element of the water body ($dV'=dr'ds$) to the calculation point.

Using the local spherical polar coordinate system, let the z-axis be the radial direction from the Earth center of mass, that is, the zenith direction, $z=0$ represents the sea level, then equation (6.1) is equivalent to:

$$T^{o}=G\beta\iint_{s}\int_{d}^{0}\frac{dz}{L}ds=G\beta\iint_{s}\int_{d}^{0}\frac{dz}{\sqrt{(\tilde{h}-z)^{2}+l^{2}}}ds$$

$$=G\beta\iint_{s}\left[ln\frac{\sqrt{\tilde{h}^{2}+l^{2}}-\tilde{h}}{\sqrt{\tilde{h}^{2}+l^{2}}+\tilde{h}}-ln\frac{\sqrt{(\tilde{h}-d)^{2}+l^{2}}-\tilde{h}+d}{\sqrt{(\tilde{h}-d)^{2}+l^{2}}+\tilde{h}-d}\right]ds \tag{6.2}$$

where, $ds=r'^{2}d\sigma=r'^{2}cos\varphi'd\varphi'd\lambda'$ is the area element on the sea surface, $\mathcal{L}=$

$\sqrt{\tilde{h}^2+l^2}$ is the space distance between the move aera element ds on the sea surface and the calculation point ($\mathcal{L} \neq L$), and $\tilde{h}$ is the altitude of the calculation point. $l = 2r_0 sin\frac{\psi}{2}$ is the spherical distance between the calculation point and the move point on the sea surface, r_0 is the mean geocentric distance of the sea surface, the mean radius of the Earth, R.

In the same way, the seawater complete Bouguer effect on gravity disturbance is:

$$\delta g^o = -\frac{\partial T^o}{\partial r} = -G\beta \iint_s \int_{R+d}^{R} \frac{\partial L^{-1}(r,\psi,r')}{\partial r} dr'\, ds \tag{6.3}$$

Equation (6.3) is equivalent to

$$\delta g^o = \frac{G}{r} \iint_s \beta \int_d^0 \frac{(r_0+z)dz}{\sqrt{(\tilde{h}-z)^2+l^2}} ds = \frac{G\beta}{r} \iint_s \left[\frac{r_0}{\mathcal{L}} - \frac{r_0+d}{\sqrt{\left(\tilde{h}-d\right)^2+l^2}}\right] ds \tag{6.4}$$

Considering $\frac{\partial\psi}{\partial\varphi} = -cos\alpha,\quad \frac{\partial\psi}{\partial\lambda} = -cos\varphi sin\alpha$, the seawater complete Bouguer effect on vertical deflection is equal to:

$$\xi^o = \frac{T_\theta^o}{\gamma r} = \frac{G\beta}{\gamma r} \iint_s \int_{R+d}^{R} \frac{\partial L^{-1}(r,\psi,r')}{\partial \psi} dr' cos\alpha ds \tag{6.5}$$

$$\eta^o = -\frac{T_\lambda^o}{\gamma r sin\theta} = \frac{G\beta}{\gamma r} \iint_s \int_{R+d}^{R} \frac{\partial L^{-1}(r,\psi,r')}{\partial \psi} dr' sin\alpha ds \tag{6.6}$$

The seawater complete Bouguer effect on disturbing gravity gradient is:

$$T_{rr}^o = \frac{\partial^2}{\partial r^2} T^o = G\beta \iint_s \int_{R+d}^{R} \frac{\partial^2 L^{-1}(r,\psi,r')}{\partial r^2} dr'\, ds \tag{6.7}$$

Equation (6.7) is equivalent to

$$T_{rr}^o = G\beta \iint_s \left[\frac{\tilde{h}-d}{((\tilde{h}-d)^2+l^2)^{3/2}} - \frac{h}{\mathcal{L}^3}\right] ds \tag{6.8}$$

Similarly, by expanding the integrand in the above integral formula near the sea level $z = 0$, the fast FFT algorithm formula can be derived.

Expand the integrand in Eq. (6.2) to order 3 near $z = 0$, we have:

$$T^o = G\beta \int_d^0 \frac{1}{L} dz ds = G\beta \iint_s \left(\frac{1}{\mathcal{L}} d + \frac{\tilde{h}}{2\mathcal{L}^3} d^2 + \frac{2\tilde{h}^2 - l^2}{6\mathcal{L}^5} d^3\right) ds \tag{6.9}$$

Expand the integrand in Eq. (6.3) to order 4 near $z = 0$, we have:

$$\delta g^0 = \frac{G}{r} \iint_s \beta \left[\begin{array}{c} \frac{r\tilde{h}+\mathcal{L}^2}{\mathcal{L}^3} d + \frac{2\tilde{h}\mathcal{L}^2 + r_0\left(\tilde{h}^2+\mathcal{L}^2\right)}{2\mathcal{L}^5} d^2 + \frac{2\tilde{h}^3 r + \tilde{h}^2 l^2 - 3r_0\tilde{h}l^2 - l^4}{2\mathcal{L}^7} d^3 \\ + \frac{8r\tilde{h}^4 - 4\tilde{h}^3 l^2 - 12\tilde{h}l^4 - 24r_0\tilde{h}^2 l^2 + 3l^4 r_0}{8\mathcal{L}^9} d^4 \end{array}\right] ds \tag{6.10}$$

Expand the integrand in the integral formula of the seawater complete Bouguer effect on vertical deflection to order 3 near $z = 0$:

$$\int_{R+d}^{R} \frac{\partial L^{-1}(r,\psi,r')}{\partial\psi} dr'$$
$$= -\frac{r^2 sin\psi}{\mathcal{L}^3} d - \frac{3\tilde{h}r^2 sin\psi}{2\mathcal{L}^5} d^2 - \left[\frac{r^2 sin\psi}{3\mathcal{L}^5} + \frac{5r^2 sin\psi(2\tilde{h}^2 - l^2)}{6\mathcal{L}^7}\right] d^3 \tag{6.11}$$

From equation (6.8), we can get the seawater complete Bouguer effect on disturbing gravity gradient:

$$T_{rr}^{o}=-\frac{\partial \delta g^{o}}{\partial r}=G\beta\iint_{s}\left[\begin{array}{c}\frac{2\tilde{h}^{2}-l^{2}}{\mathcal{L}^{5}}d+\frac{3\tilde{h}\left(2\tilde{h}^{2}-3l^{2}\right)}{2\mathcal{L}^{7}}d^{2}+\\ \frac{4\tilde{h}^{4}+6r^{4}-12\tilde{h}^{2}l^{2}-(6r^{4}+3r^{2}l^{2})cos\psi}{\mathcal{L}^{9}}d^{3}\end{array}\right]ds \quad (6.12)$$

The items on the right side of equations (6.9) to (6.12) can be quickly calculated by the FFT algorithm. If the calculation point is also on the sea surface, with $h=0$, $\mathcal{L}=l$, formulas (6.2) ~ (6.12) can be greatly simplified.

The seawater complete Bouguer effects on various gravity field elements are relatively large, and a larger integral radius should be employed for the integral calculation, such as not less than 250km.

7.6.2 Integral algorithms of residual terrain effects on various field elements outside the geoid

The land-sea residual terrain effect is defined as the short-wave and ultra-short-wave components of the land-sea complete Bouguer effect. The residual terrain effects on various types of field elements can be calculated by firstly constructing the land-sea residual terrain model and then using the integral method.

The residual terrain model (RTM) can be obtained by subtracting the low-resolution land-sea terrain model from the high-resolution land-sea terrain model with the same grid specification.

The integral formula of residual terrain effect is similar in form to the integral formula of local terrain effect/seawater complete Bouguer effect, the difference lies in the adopted the density of the move element and the radial integral domain.

(1) Numerical Integral of residual terrain effects on various field elements outside the geoid

The residual terrain effects on disturbing potential can be directly expressed as:

$$T^{rtm}=G\iint_{s}\int_{R}^{R+\delta'}\beta' L^{-1}(r,\psi,r')\,dr'\,ds \quad (6.13)$$

where, δ',β' are the residual terrain height and density at the move area element $ds=r'^{2}cos\varphi' d\varphi' d\lambda''$, respectively. When ds is located in the land area, δ' is the residual terrain height δh, and β' is the terrain density ρ_0 (= 2.67×10^{3}km/m^3), and when ds is located in the ocean area, δ' is the residual sea depth δd, β' is seawater compensation density $\rho_0-\rho_w$ (where seawater density ρ_w =1.03×10^{3}km/m^3).

It is not difficult to find that whether the area element ds is in the land area or in the sea area, the residual terrain δ' could be positive or negative.

In the same way, the residual terrain effect on gravity disturbing is equal to:

$$\delta g^{rtm}=-\frac{\partial T^{rtm}}{\partial r}=-G\iint_{s}\beta'\int_{R}^{R+\delta'}\frac{\partial L^{-1}(r,\psi,r')}{\partial r}dr'\,ds \quad (6.14)$$

The residual terrain effect on vertical deflection is equal to:

$$\xi^{rtm}=\frac{G}{\gamma r}\iint_{s}\beta'\int_{R}^{R+\delta'}\frac{\partial L^{-1}(r,\psi,r')}{\partial\psi}dr' cos\alpha ds$$

$$\eta^{rtm} = \frac{G}{\gamma r}\iint_s \beta' \int_R^{R+\delta'} \frac{\partial L^{-1}(r,\psi,r')}{\partial \psi} dr' sin\alpha ds \tag{6.15}$$

The residual terrain effect on disturbing gravity gradient is equal to:

$$T_{rr}^{rtm} = \frac{\partial^2}{\partial r^2} T^{rtm} = G\iint_s \beta' \int_R^{R+\delta'} \frac{\partial^2 L^{-1}(r,\psi,r')}{\partial r^2} dr'\, ds \tag{6.16}$$

Similarly, using a local spherical coordinate system, let the z-axis be the radial direction (zenith direction), and $z = 0$ is the terrain surface/sea surface. Let $\mathcal{L} = \sqrt{\tilde{h}^2 + l^2}$ be the three-dimensional space distance between the move area element and the calculation point, then formulas (6.13) ~ (6.16) can be rewritten as:

$$T^{rtm} = G\iint_s \beta' \int_0^{\delta'} \frac{dz}{\sqrt{(\tilde{h}-z)^2 + l^2}} ds$$

$$= G\iint_s \beta' \left[ln \frac{\sqrt{(\tilde{h}-\delta')^2 + l^2} - \tilde{h} + \delta'}{\mathcal{L} + \tilde{h} - \delta'} - ln\frac{\mathcal{L}-\tilde{h}}{\mathcal{L}+\tilde{h}} \right] ds \tag{6.17}$$

$$\delta g^{rtm} = \frac{G}{r}\iint_s \beta' \iint_0^{\Delta h} \frac{\partial}{\partial \tilde{h}} \frac{dz}{\sqrt{(\tilde{h}-z)^2 + l^2}} ds = \frac{G}{r}\iint_s \beta' \left[\frac{1}{\sqrt{(\tilde{h}-\Delta h)^2 + l^2}} - \frac{1}{\mathcal{L}} \right] ds \tag{6.18}$$

$$\int_R^{R+\delta'} \frac{\partial L^{-1}(r,\psi,r')}{\partial \psi} dr' = \frac{1}{2} ctg\frac{\psi}{2} \left[\frac{\tilde{h}-\delta'}{\sqrt{(\tilde{h}-\delta')^2 + l^2}} - \frac{\tilde{h}}{\mathcal{L}} \right] \tag{6.19}$$

$$T_{rr}^{rtm} = G\iint_s \beta' \left[\frac{\tilde{h}-\delta'}{((\tilde{h}-\delta')^2 + l^2)^{3/2}} - \frac{\tilde{h}}{\mathcal{L}^3} \right] ds \tag{6.20}$$

(2) Fast FFT algorithms of the integral of residual terrain effect on various field elements

The integrand in the above integral formula is expanded near z=0, where z=0 is the terrain surface/sea surface.

Expand the integrand in Eq. (6.17) to order 3 near $z = 0$, we have:

$$T^{rtm} = -G\iint_s \beta' \left(\frac{1}{\mathcal{L}}\delta' + \frac{\tilde{h}}{2\mathcal{L}^3}\delta'^2 + \frac{2\tilde{h}^2 - l^2}{6\mathcal{L}^5}\delta'^3 \right) ds \tag{6.21}$$

Expand the integrand in Eq. (6.18) to order 4 near $z = 0$, we have:

$$\delta g^{rtm} = \frac{G}{r}\iint_s \beta' \left[\frac{\tilde{h}}{\mathcal{L}^3}\delta' + \frac{2\tilde{h}^2 - l^2}{2\mathcal{L}^5}\delta'^2 + \frac{\tilde{h}(2\tilde{h}^2 - 3l^2)}{2\mathcal{L}^7}\delta'^3 + \frac{8\tilde{h}^4 - 24\tilde{h}^2 l^2 + 3l^4}{8\mathcal{L}^9}\delta'^4 \right] ds \tag{6.22}$$

Expand the integrand in the integral formula (6.19) of the residual terrain effect on vertical deflection to order 3 near $z = 0$:

$$\int_R^{R+\delta'} \frac{\partial L^{-1}(r,\psi,r')}{\partial \psi} dr'$$

$$= -\frac{r^2 sin\psi}{\mathcal{L}^3}\delta' - \frac{3\tilde{h} r^2 sin\psi}{2\mathcal{L}^5}\delta'^2 - \left[\frac{r^2 sin\psi}{3\mathcal{L}^5} + \frac{5r^2 sin\psi(2\tilde{h}^2 - l^2)}{6\mathcal{L}^7} \right] \delta'^3 \tag{6.23}$$

Expand the integrand in Eq. (6.20) to order 4 near z=0, we have:

$$T_{rr}^{rtm} = G\iint_s \beta' \left[\frac{2\tilde{h}^2 - l^2}{\mathcal{L}^5}\delta' + \frac{3\tilde{h}(2\tilde{h}^2 - 3l^2)}{2\mathcal{L}^7}\delta'^2 + \frac{8\tilde{h}^4 - 24\tilde{h}^2 l^2 + 3l^4}{2\mathcal{L}^9}\delta'^3 \right] ds \tag{6.24}$$

7.6.3 Spherical harmonic analysis and synthesis of land-sea terrain masses

(1) The terrain areal density $q(\varphi,\lambda)$ of any point P(R,φ,λ) on the land-sea surface can be expressed as:

$$q(\varphi,\lambda)=\beta h=R\sum_{n=1}^{\infty}\sum_{m=0}^{n}[A_{nm}cosm\lambda+B_{nm}sinm\lambda]\,\bar{P}_{nm}(sin\varphi) \qquad (6.25)$$

where R is the mean radius of the Earth (PAGravf4.5 replaces R with the semimajor axis a of the Earth to facilitate the combination with the geopotential model), and A_{nm}, B_{nm} are the degree n order m normalized terrain mass spherical harmonic coefficients.

In formula (6.25), when P is on the land terrain surface, h is the terrain height ($h>0$), β is the terrain density, $\beta=\rho_0$ =2.67×10³km/m³, and when P is on the sea surface, h is the sea depth (h<0), β is the compensation density of seawater, equal to the difference between terrain density ρ_0 and seawater density ρ_w , $\beta=\rho_0-\rho_w$ =1.64×10³km/m³.

(2) The land-sea complete Bouguer effect on the gravitational potential at the point (r,θ,λ) outside geoid in spherical coordinates can be expressed by the spherical harmonic series of global land-sea terrain masses as:

$$V^{tbg}(r,\theta,\lambda)=\frac{3GM}{r\rho_e}\sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}(A_{nm}cosm\lambda+B_{nm}sinm\lambda)\bar{P}_{nm}(cos\theta) \qquad (6.26)$$

where ρ_e =5.517×10^{3}kg/m^3 is the mean density of the Earth.

(3) The land-sea residual terrain effect on the gravitational potential at the point (r,θ,λ) outside geoid in spherical coordinates can be expressed by the spherical harmonic series of global land-sea terrain masses as:

$$V^{rtm}(r,\theta,\lambda)=\frac{3GM}{r\rho_e}\sum_{n=N}^{\infty}\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}(A_{nm}cosm\lambda+B_{nm}sinm\lambda)\bar{P}_{nm}(cos\theta) \qquad (6.27)$$

where N is the minimum degree of the residual terrain model.

(4) The relationship between the normalized terrain masses spherical harmonic coefficient and the normalized terrain Stokes coefficient:

$$\bar{C}_{nm}^{t}=\frac{3}{\rho_e}\frac{1}{2n+1}A_{nm},\quad \bar{S}_{nm}^{t}=\frac{3}{\rho_e}\frac{1}{2n+1}B_{nm} \qquad (6.28)$$

The terrain masses between the surface and the geoid, and the seawater compensation masses between the sea surface and the seabed together generate the terrain gravitational field, that is, the complete Bouguer effect. The terrain Stokes coefficients $\bar{C}_{nm}^{t}$, $\bar{S}_{nm}^{t}$ are the normalized spherical harmonic expansion coefficients of the terrain gravitational field., that is, the complete Bouguer effect on the Stokes coefficients of global geopotential.

7.7 Local terrain compensation and terrain Helmert condensation

7.7.1 Terrain Helmert condensation effect on various gravity field elements outside geoid

The Helmert condensation of terrestrial terrain involves a concept called terrain masses compensation, or terrain compensation for short. Terrain compensation effect on any type of gravity field element outside the geoid is defined as the amount of mass compensation for this type of anomalous field element to offset the change in the Earth's gravitational field caused by the removal of terrain mass.

The Helmert condensation process of the terrain masses can be decomposed into two steps: the first step is to deduct the gravitational field generated by the terrain masses, that is, to subtract the effect of the terrain, and the second step is to compensate for the change of the gravitational field caused by the deduction of the terrain masses, that is, to add the terrain compensate effect.

For any type of anomalous field element α outside the geoid, the change of the field element caused by terrain Helmert condensation is called as the terrain Helmert condensation effect on this type of field element, which can be uniformly expressed as:

$$\alpha^h = \alpha^t - \alpha^c \tag{7.1}$$

where, α^h is the terrain Helmert condensation effect on the anomalous field element α, α^t is the complete Bouguer effect on α, and α^c is the terrain compensation effect on α.

The residual terrain effect is equal to the difference between the high-resolution and low-resolution complete Bouguer effect, and similarly, the terrain Helmert condensation effect is equal to the difference between the complete Bouguer effect and the terrain compensation effect.

The space outside the geoid after terrain Helmert condensation is called as the Helmert space, and the corresponding gravitational field is called as the Helmert gravitational field, which is harmonious with one difference from the actual Earth's gravitational field due to terrain Helmert condensation.

7.7.2 Algorithm formulas of terrain compensation and Helmert condensation effect

The following presents the spherical approximation algorithms for terrain compensation effects on various gravity field elements in the near-Earth harmonic space outside the geoid.

(1) The terrain compensation effect on disturbing potential

$$T^c = T^B + T^{cR} = T^B + G \iint_s \frac{\mu' - \mu}{L} ds \tag{7.2}$$

where, T^{cR} is called the local terrain compensation effect on disturbing potential, ds is the move area element on the unit sphere, μ is called the terrain compensation density, and under the spherical approximation:

$$\mu = \rho_0 h \left(1 + \frac{h}{R} + \frac{h^2}{3R^2}\right) \tag{7.3}$$

where, h is the terrain height directly below the calculation point and ρ_0 is the terrain density.

The geocentric distance is replaced by the mean geocentric distance of the calculation surface and the terrain surface, respectively, then the local terrain compensation effect integral of the second term on the right side of (7.2) can be directly calculated by the FFT algorithm.

When the calculation point is the same as the move point, the integral of compensation effect on disturbing potential is singular:

$$T^{cR}|_0 = \frac{R^2}{6\tilde{r}^2} G A_0 \sqrt{A_0/\pi}\left(\mu_{xx} + \mu_{yy}\right) \tag{7.4}$$

where μ_{xx}, μ_{yy} are the second-order partial derivatives of the terrain compensation density at the calculation point in the north direction x and the east direction y.

(2) The terrain compensation effect on gravity disturbance

$$\delta g^c = \delta g^B + \delta g^{cR} = \delta g^B + G \iint_s (\mu' - \mu) \frac{r - r't}{L^3} ds \tag{7.5}$$

where, δg^{cR} is called the local terrain compensation effect on gravity disturbance.

When the calculation point is the same as the move point, the integral of compensation effect on gravity disturbance is singular:

$$\delta g^{cR}|_0 = \frac{R^2}{12\tilde{r}^3} G A_0 \sqrt{A_0/\pi}\left(\mu_{xx} + \mu_{yy}\right) \tag{7.6}$$

(3) Calculation of the terrain Helmert condensation effect

Combining formulas (7.2) and (7.5), the terrain Helmert condensation effect on various anomalous field element outside geoid under the spherical approximation can be expressed as:

$$\alpha^h = \alpha^t - \alpha^c = (\alpha^B + \alpha^R) - (\alpha^B + \alpha^{cR}) = \alpha^R - \alpha^{cR} \tag{7.7}$$

In formula (7.7), the spherical shell Bouguer effect α^B cancels each other, so the terrain Helmert condensation effect is also equal to the difference between the local terrain effect α^R and the local terrain compensation effect α^{cR}.

Substituting formulas (5.5) and (7.4), and formulas (5.7) and (7.5) into formula (7.7), respectively, the terrain Helmert condensation effect on the disturbing potential and gravity disturbance can be obtained, and then terrain Helmert condensation effects on various other types of anomalous field elements can be obtained.

(4) Fast algorithm of local terrain compensation effect on gravity disturbance

Using the local spherical polar coordinate system, let the z-axis be the radial direction (the zenith direction), in this case, $dr = d\tilde{h}$, we have

$$\begin{aligned}\delta g^{cR} &= -G \iint_s (\mu' - \mu) \frac{\partial}{\partial \tilde{h}} \frac{1}{L} ds = G \iint_s (\mu' - \mu) \frac{\tilde{h}}{L^3} - \frac{\mu' - \mu}{L^3}(h' - h) ds \\ &= G \iint_s (\mu' - \mu) \frac{\tilde{h}}{L^3} ds - G \iint_s \frac{\mu' h'}{L^3} \frac{t}{L} ds \\ &\quad + G \iint_s \frac{\mu' h}{L^3} ds + G \iint_s \frac{\mu h'}{L^3} ds - G \iint_s \frac{\mu h}{L^3} ds\end{aligned} \tag{7.8}$$

Each item on the right side of the formula (7.8) can be quickly calculated using the FFT algorithm.

(5) Fast algorithm of local terrain compensation effect on vertical deflection

Considering $\frac{\partial L^{-1}(r,\psi,r')}{\partial \psi} = \frac{r r' sin\psi}{L^3}$, $\frac{\partial \psi}{\partial \varphi} = -cos\alpha$, $\frac{\partial \psi}{\partial \lambda} = -cos\varphi sin\alpha$, we have

$$\xi^{cR} = -\frac{\partial T^{cR}}{\gamma r\partial\varphi} = -\frac{\partial T^{cR}}{\gamma r\partial\psi}\frac{\partial\psi}{\partial\varphi} = \frac{\partial T^{cR}}{\gamma r\partial\psi}cos\alpha = \frac{G}{\gamma r}\iint_s (\mu' - \mu)\frac{\partial L^{-1}(r,\psi,r')}{\partial\psi}cos\alpha ds$$

$$= \frac{G}{\gamma}\int_s (\mu' - \mu)\frac{r' sin\psi}{L^3}cos\alpha ds \tag{7.9}$$

$$\eta^{cR} = -\frac{\partial T^{cR}}{\gamma r cos\varphi\partial\psi}\frac{\partial\psi}{\partial\lambda} = \frac{\partial T^{cR}}{\gamma r\partial\psi}sin\alpha = \frac{G}{\gamma r}\iint_s \frac{\partial L^{-1}(r,\psi,r')}{\partial\psi}(\mu' - \mu)sin\alpha ds$$

$$= \frac{G}{\gamma}\iint_s (\mu' - \mu)\frac{r' sin\psi}{L^3}sin\alpha ds \tag{7.10}$$

(6) Fast algorithm of local terrain compensation effect on disturbing gravity gradient

$$T_{rr}^{cR} = \frac{\partial^2}{\partial r^2}T^{cR} = G\iint_s (\mu' - \mu)\frac{\partial^2}{\partial r^2}\left(\frac{1}{L}\right)ds$$

$$= G\iint_s (\mu' - \mu)\left(3\frac{r - r' cos\psi}{L^5} - \frac{1}{L^3}\right)ds \tag{7.11}$$

7.8 Land-sea unified classic Bouguer and equilibrium effects

7.8.1 The classical reduction method for land Bouguer gravity anomaly

In Stokes theory, the Bouguer gravity anomaly is defined on the geoid, which is equal to the gravity anomaly on the geoid minus the effect of all terrain masses outside the geoid on the gravity at the ground point. The classical algorithm for the Bouguer gravity anomaly on the geoid is:

$$\Delta g_B = \Delta g - g^R - 2\pi G\rho h \tag{8.1}$$

where Δg is the gravity anomaly on the geoid, $-g^R$ is the classic plane terrain correction as well as g^R is equal to the plane approximation of the local terrain effect in PAGravf4.5, and $-2\pi G\rho h$ is called the layer correction as well as $2\pi G\rho h$ is equal to the plane approximation of spherical shell Bouguer effect in PAGravf4.5.

In terrestrial mountainous area, the layer correction $-2\pi G\rho h$ is much less than zero, so the Bouguer gravity anomaly is generally less than zero.

Since the gravity measurement point is generally not on the geoid, it is necessary to make downward continuation of the observed gravity from the measurement point to the geoid to obtain the gravity anomaly Δg on the geoid, and then according to the algorithm formula (8.1) to calculate the classical Bouguer gravity anomaly.

In the classical gravity reduction process, the observed gravity is made downward continuation to the geoid using the space correction $-0.3086h + O(h^2)$ (mGal), which only considers the normal gravity gradient. While the actual situation is that even in hilly area with the terrain altitude of several hundred meters, the contribution of disturbing gravity gradient may reach or exceed the mGal level.

Considering that the height of the measurement point can be easily and accurately measured at present, and the normal gravity at the measurement point can be strictly calculated, PAGravf4.5 will not continue to use the concept of space correction.

PAGravf4.5 firstly calculates the gravity anomaly at the measurement point from the observed gravity and height, in which, the normal gravity calculated using the analytical formula (see Section 2.3.1). Then, the rigorous method is employed to obtain the analytical continuation value of the gravity anomaly from the measurement point to the geoid.

From an ultra-high degree geopotential model, the difference between the model gravity anomaly at the measurement point and on the geoid can be directly the analytical continuation value within the altitude of 1000m. Which is equivalent to removing the model gravity anomaly at measurement point firstly and then restoring model gravity anomaly on the geoid. In mountainous area, the analytical continuation value can be furtherly improved by using the radial gradient continuation of residual gravity anomaly (see Section 2.5).

From the observed gravity and height, PAGravf4.5 calculate the classic Bouguer gravity anomaly on geoid according to the general formula in following:

$$\Delta g_B = \Delta g^s - g^R - 2\pi G\rho h - \Delta g^c \tag{8.2}$$

where Δg^s is the gravity anomaly at the measurement point (see section 2.3 for the calculation method), and Δg^c is the the analytical continuation value.

Since the object affected by terrain is gravity itself, it has nothing to do with normal gravity. Therefore, the algorithm formula for the Bouguer gravity disturbance on the geoid is:

$$\delta g_B = \delta g^s - g^R - 2\pi G\rho h - \delta g^c \tag{8.3}$$

where δg^s is the gravity disturbance at the measurement point (see section 2.3 for the calculation method), and δg^c is the analytical continuation value, which is almost equal to Δg^c.

In formula (8.2) or (8.3), the measurement points can be on the ground, or in near-Earth space outside the ground

It should be emphasized that no matter whether the measurement point is on the ground or in the near-Earth space (such as aviation altitude), the classical Bouguer gravity anomaly and classical Bouguer gravity disturbance can only be defined on the geoid, and the terrain effect can only be the effect of terrain masses on the ground gravity. g^R in the formula (8.2) or (8.3) can only be the local terrain effect on the ground gravity, even if the measurement point is at the altitude of the air.

7.8.2 Calculation of land-sea unified Bouguer gravity anomaly

The existence of terrestrial terrain makes the space outside the geoid exist mass, which need to be removed, resulting in the land complete Bouguer effect. In the marine area, the density of seawater below the sea level (geoid) is less than the terrain density, and the mass loss of seawater layer need to be compensated, resulting in the seawater complete Bouguer effect.

Referring to Section 7.5.4 for the calculation method of the seawater complete Bouguer effect on gravity (gravity anomaly or gravity disturbance), the exact integral formula is:

$$g_b^w = \frac{G\beta}{r} \iint_s \left[\frac{r_0}{\mathcal{L}} - \frac{r_0 + d}{\sqrt{\left(\tilde{h} - d\right)^2 + l^2}} \right] ds \tag{8.4}$$

there $d < 0$ is the sea depth, β is the seawater compensation density (the difference between the terrain density ρ_0 and the seawater density ρ_w), and $= \rho_0 - \rho_w$ =1.64×10³km/m³, $\tilde{h}$ is the height of the calculation point relative to the sea surface, r_0 is the geocentric distance of the sea surface directly below the calculation point, ds is the move areal element on sea surface, $\mathcal{L}$ is the space distance from the move areal element to the calculation point, and l is the spherical distance between the projection point of the calculation point on the sea surface and the move areal element ds.

Since the local terrain effect g^R in (8.1) and the seawater complete Bouguer effect g_b^w in (8.4) are both integral values of a certain range of areas, the offshore ocean gravity is affected by land terrain, and the coastal land gravity is affected by seawater, so both are not zero. It is necessary the land-sea unified Bouguer effect algorithm in the coastal zone.

The height of the sea level is equal to zero, so if the integral range of the local terrain effect includes the sea area, the contribution of the sea area to the local terrain effect is equal to zero. Similarly, the sea depth on land is equal to zero, so if the integral range of the seawater complete Bouguer effect includes land area, the contribution of land area to the seawater Bouguer effect is also zero. It is easy to find that the local terrain effect and seawater complete Bouguer effect are completely separated and seamlessly spliced in the integral domain. Therefore, the two integral formulas are directly added to obtain the calculation formula for the land-sea unified Bouguer gravity anomaly and Bouguer gravity disturbance:

$$\Delta g_B = \Delta g^s - g^R - 2\pi G\rho h - g_b^w - \Delta g^c \tag{8.5}$$

$$\delta g_B = \delta g^s - g^R - 2\pi G\rho h - g_b^w - \delta g^c \tag{8.6}$$

Let $g^B = g^R + 2\pi G\rho h + g_b^w$ (8.7)

In PAGravf 4.5, g^B is called the classical Bouguer effect (see Section 3.5). It is not difficult to see that the classical Bouguer effect g^B for gravity anomaly or gravity disturbance is unified and need not be distinguished.

7.8.3 Airy-Heiskanen terrain equilibrium effect on land

The Bouguer gravity anomaly usually has a large negative value in mountainous areas, and people therefore associate the ‘excess’ material with irregular undulating mountains on the crust, which may be compensated by the corresponding loss material in the magma layer below.

Let the depth from the sea level (geoid) to the magma level be the compensation depth D. The Airy-Heiskanen model believes that the lower crust is a magma layer with a density of ρ_1=3.27×10³km/m³, and a mountain floats above the magma layer with a density of the crustal density ρ_0=2.67×10³km/m³. The part of the mountain body above the sea level is the visible terrain. The higher the mountain body is, the deeper the part that sinks into the magma (called the mountain root), and the mountain body and the mountain root are approximately symmetrical to the magma surface. A density difference $\Delta\rho_1 = \rho_1 - \rho_0$=0.6×10³km/m³ is formed between the mountain body and the mountain root, which is the local density deficit in the magma layer.

Suppose that the surplus material of the terrain is filled into the depleted part below it and compensated. The compensation density is exactly equal to the depletion density $\Delta\rho_1$=0.6×10³km/m³, and the compensation density make the gravity increase. The gravity value change caused by the compensation is the terrain equilibrium effect.

Let the terrain height be h and the mountain root depth be b, it can be known from the floating static equilibrium condition

$$b\Delta\rho_1 = \rho_0 h \quad \Rightarrow \quad b = \frac{\rho_0}{\Delta\rho_1} h = 4.45h \tag{8.8}$$

Let the z-axis be the direction of the plumb line, then the terrain equilibrium effect is equal to

$$g_I = -G\Delta\rho_1 \iint_\sigma \int_D^{D+b} \frac{z-z'}{L^3} dz\, d\sigma \tag{8.9}$$

7.8.4 Calculation of land-sea unified equilibrium gravity anomaly

The ocean has a layer of low-density seawater ρ_w=1.03×10³km/m³ and a layer of oceanic crust with a density equal to ρ_0=2.67×10³km/m³. The self-weight of the two layers of material is less than the buoyancy of the magma, so it need supplement the material to achieve static balance, which leads to the magma material upwelling to the ocean area, the formation of mountain anti-root.

The compensation $\beta = \rho_0 - \rho_w = 1.64$×10³km/m³ for the density deficit of the seawater layer, which produces the seawater complete Bouguer effect, has been expressed by formula (8.4). After the seawater density compensated, the static equilibrium condition of the ocean mountain anti-root becomes:

$$b'\Delta\rho_1 = \beta d \quad \Rightarrow \quad b' = \frac{\beta}{\Delta\rho_1} d = 2.73d \tag{8.10}$$

where d is the sea depth.

The mass loss of the land mountain root requires mass compensation, so the land equilibrium effect and the plane Bouguer effect are roughly inverse. On the contrary, the ocean mountain anti-root is excess mass that need be removed, the ocean equilibrium effect and the seawater Bouguer effect are also roughly inverse. The ocean equilibrium effect is equal to

$$g_I^o = -G\Delta\rho_1 \iint_\sigma \int_{D-b'}^{D} \frac{z-z'}{L^3} dz\, d\sigma \tag{8.11}$$

Since the terrain equilibrium effect in (8.9) and the ocean equilibrium effect in (8.11) are both integral values of a certain range of areas, the terrain equilibrium effect is not zero in the offshore ocean area, and the ocean equilibrium effect is also not zero in the coastal land area. Thence it is necessary the land-sea unified equilibrium effect algorithm in the coastal zone.

The height of the sea level is equal to zero, so if the integral range of the terrain equilibrium effect includes the sea area, the contribution of the sea area to the terrain equilibrium effect is equal to zero. Similarly, the sea depth on land is equal to zero, so if the integral range of the ocean equilibrium effect includes land area, the contribution of land area to the ocean equilibrium effect is also zero. It is easy to find that the terrain equilibrium effect and the ocean equilibrium effect are completely separated and seamlessly spliced in the integral domain. Therefore, the two integral formulas are directly added to obtain the calculation formula for the land-sea unified equilibrium gravity anomaly and equilibrium gravity disturbance:

$$\Delta g_B = \Delta g^s - g^B - g_I - g_I^o - \Delta g^c \tag{8.12}$$

$$\delta g_B = \delta g^s - g^R - g_I - g_I^o - \delta g^c \tag{8.13}$$

Let $g^I = g_I + g_I^o$ (8.14)

In PAGravf 4.5, g^I is called the classical equilibrium effect (see Section 3.5). Similarly, the classical equilibrium effect g^I for gravity anomaly or gravity disturbance is unified and need not be distinguished.

7.8.5 Sign analysis of the land and sea Bouguer / equilibrium effect

The land layer effect is to remove the effect of terrain mass outside geoid on surface gravity, and the seawater Bouguer effect is the effect on surface gravity after seawater density compensated to terrain density. Therefore, the sign of the land layer effect is inverse to that of the seawater Bouguer effect.

The land equilibrium effect is the effect of filling the depleted part of the mountain root with excess terrain material on surface gravity, and the sign of land equilibrium effect is inverse to that of the layer effect. The ocean equilibrium effect is the effect on surface gravity after the process mass of the ocean mountain anti-root removed, and the sign of ocean equilibrium effect is inverse to that of the seawater Bouguer effect.

In PAGravf4.5, the layer effect (equal to the negative layer correction) is greater than zero, so the seawater Bouguer effect and the land equilibrium effect are less than zero, and the ocean equilibrium effect is greater than zero.

If + means greater than zero, - means less than zero, we have: the layer effect (+), sea water Bouguer effect (-), land equilibrium effect (-), and ocean equilibrium effect (+).

If described by the concept of classical terrain correction, we have: the layer correction (-), sea water Bouguer correction (+), land equilibrium correction (+), and ocean equilibrium correction (-).

7.9 Integral algorithm formula of anomalous gravity field

7.9.1 Stokes and Hotine integral formulas outside geoid

(1) It is known that the gravity anomaly Δg on an equipotential surface outside the geoid, the disturbing potential $T(r,\theta,\lambda)$ or the height anomaly $\zeta(r,\theta,\lambda)$ at the calculation

point (r,θ,λ) outside the geoid can be calculated by Stokes integral Formula:

$$T(r,\theta,\lambda)=\gamma\zeta(r,\theta,\lambda)=\frac{1}{4\pi}\iint_s \Delta g' S(r,\psi,r')ds \tag{9.1}$$

where r' is the geocentric distance of the move area element ds (the move point) on the equipotential surface where the gravity anomaly $\Delta g'$ is located, $S(r,\psi,r')$ is called the generalized Stokes kernel function, and:

$$S(r,\psi,r')=\frac{2}{L}+\frac{1}{r}-\frac{3L}{r^2}-\frac{5r'cos\psi}{r^2}-\frac{3r'}{r^2}cos\psi ln\frac{r-r'cos\psi+L}{2r} \tag{9.2}$$

where L is the space distance from the move point to the calculation point.

When the calculation point is the same as the move point, the integral is singular:

$$\zeta|_0=\frac{A_0}{\gamma}\Delta g_0 \tag{9.3}$$

(2) It is known that the gravity disturbance δg on an equipotential surface outside the geoid, the disturbing potential $T(r,\theta,\lambda)$ or the height anomaly $\zeta(r,\theta,\lambda)$ at the calculation point (r,θ,λ) outside the geoid can be calculated by Hotine integral Formula:

$$T(r,\theta,\lambda)=\gamma\zeta(r,\theta,\lambda)=\frac{1}{4\pi}\iint_s \delta g' H(r,\psi,r')ds \tag{9.4}$$

where $H(r,\psi,r')$ is called the generalized Hotine kernel function, and:

$$H(r,\psi,r')=\frac{2}{L}-\frac{1}{r}-\frac{3r'cos\psi}{r^2}-\frac{1}{r'}ln\frac{r-r'cos\psi+L}{r(1-cos\psi)} \tag{9.5}$$

When the calculation point is the same as the move point, the integral is singular:

$$\zeta|_0=\frac{A_0}{\gamma}\delta g_0 \tag{9.6}$$

If r and r' are taken as constants, the generalized Stokes/Hotine integral can be calculated by the fast FFT algorithm.

7.9.2 Vening-Meinesz integral formulas outside geoid

Taking the horizontal derivatives on both sides of the generalized Stokes formula, we have:

$$\xi=\frac{-1}{4\pi r\gamma}\iint_s \Delta g'\frac{\partial S(r,\psi,r')}{\partial\psi}\frac{\partial\psi}{\partial\varphi}ds,\quad \eta=\frac{-1}{4\pi r cos\varphi\gamma}\iint_s \Delta g'\frac{\partial S(r,\psi,r')}{\partial\psi}\frac{\partial\psi}{\partial\lambda}ds \tag{9.7}$$

From $cos\psi=sin\varphi sin\varphi'+cos\varphi cos\varphi' cos(\lambda'-\lambda)$, (9.8)

differentiating both sides, we get:

$$-sin\psi\frac{\partial\psi}{\partial\varphi}=cos\varphi sin\varphi'-sin\varphi cos\varphi' cos(\lambda'-\lambda) \tag{9.9}$$

$$-sin\psi\frac{\partial\psi}{\partial\lambda}=cos\varphi cos\varphi' sin(\lambda'-\lambda) \tag{9.10}$$

From the spherical trigonometry formula, we can get:

$$sin\psi cos\alpha=cos\varphi sin\varphi'-sin\varphi cos\varphi' cos(\lambda'-\lambda) \tag{9.11}$$

$$sin\psi sin\alpha=cos\varphi' sin(\lambda'-\lambda) \tag{9.12}$$

Combining formulas (9.9) ~ (9.12), we can get:

$$\frac{\partial\psi}{\partial\varphi}=-cos\alpha,\quad \frac{\partial\psi}{\partial\lambda}=-cos\varphi sin\alpha \tag{9.13}$$

Substitute formula (9.13) into (9.7), we have:

$$\xi = \frac{1}{4\pi r\gamma}\iint_s g'\frac{\partial S(r,\psi,r')}{\partial\psi}cos\alpha ds,\quad \eta = \frac{1}{4\pi r\gamma}\iint_s \Delta g'\frac{\partial S(r,\psi,r')}{\partial\psi}sin\alpha ds \tag{9.14}$$

Considering $L = \sqrt{r^2 + r'^2 - 2rr'cos\psi}$, we have:

$$\frac{\partial}{\partial\psi}L = \frac{rr'}{L}sin\psi,\quad \frac{\partial}{\partial\psi}\frac{1}{L} = -\frac{1}{L^2}\frac{\partial}{\partial\psi}L = -\frac{rr'}{L^3}sin\psi \tag{9.15}$$

$$\frac{\partial}{\partial\psi}ln\frac{r-r'cos\psi+L}{2r} = \frac{1}{r-r'cos\psi+L}\left(\frac{rr'}{L}sin\psi + r'sin\psi\right) = \frac{r'sin\psi}{r+L-r'cos\psi}\frac{L+r}{L} \tag{9.16}$$

$$\frac{\partial}{\partial\psi}S(r,\psi,r') = \frac{\partial}{\partial\psi}\left(\frac{2}{L}+\frac{1}{r}-\frac{3L}{r^2}-\frac{5r'cos\psi}{r^2}-\frac{3r'cos\psi}{r^2}ln\frac{r-r'cos\psi+L}{2r}\right)$$

$$= \frac{\partial}{\partial\psi}\frac{2}{L}-\frac{3}{r^2}\frac{\partial}{\partial\psi}L+\frac{5r'sin\psi}{r^2}+\frac{3r'sin\psi}{r^2}ln\frac{r+L-r'cos\psi}{2r}-\frac{3r'cos\psi}{r^2}\frac{\partial}{\partial\psi}ln\frac{r+L-r'cos\psi}{2r}$$

$$= \left(-\frac{2rr'}{L^3}-\frac{3r'}{rL}+\frac{5r'}{r^2}+\frac{3r'}{r^2}ln\frac{r-r'cos\psi+L}{2r}-\frac{3r'cos\psi}{r^2}\frac{r'}{r-r'cos\psi+L}\frac{L+r}{L}\right)sin\psi$$

$$= \left[-\frac{2r}{L^3}-\frac{3}{rL}+\frac{5}{r^2}+\frac{3}{r^2}ln\frac{r-r'cos\psi+L}{2r}-\frac{3r'(L+r)cos\psi}{r^2L(r-r'cos\psi+L)}\right]r'sin\psi \tag{9.17}$$

In the same way, by calculating the horizontal derivatives on both sides of the generalized Hotine formula, we can get:

$$\xi = \frac{1}{4\pi r\gamma}\iint_s \delta g'\frac{\partial H(r,\psi,r')}{\partial\psi}cos\alpha ds,\quad \eta = \frac{1}{4\pi r\gamma}\iint_s \delta g'\frac{\partial H(r,\psi,r')}{\partial\psi}sin\alpha ds \tag{9.18}$$

Because of

$$\frac{\partial}{\partial\psi}ln\frac{r-r'cos\psi+L}{r(1-cos\psi)} = \frac{r(1-cos\psi)}{r-r'cos\psi+L}\frac{\left(\frac{rr'}{L}sin\psi+r'sin\psi\right)r(1-cos\psi)+(r-r'cos\psi+L)rsin\psi}{r^2(1-cos\psi)^2}$$

$$= \frac{sin\psi}{r-r'cos\psi+L}\frac{\frac{L+r}{L}r'(1-cos\psi)+(r-r'cos\psi+L)}{1-cos\psi} = \left[\frac{r'(L+r)}{(r-r'cos\psi+L)L}+\frac{1}{1-cos\psi}\right]sin\psi, \tag{9.19}$$

we have:

$$\frac{\partial}{\partial\psi}H(r,\psi,r') = \frac{\partial}{\partial\psi}\left(\frac{2}{L}-\frac{1}{r}-\frac{3r'cos\psi}{r^2}-\frac{1}{r'}ln\frac{r-r'cos\psi+L}{r(1-cos\psi)}\right)$$

$$= \frac{\partial}{\partial\psi}\frac{2}{L}+\frac{3r'sin\psi}{r^2}-\frac{1}{r'}\frac{\partial}{\partial\psi}ln\frac{r-r'cos\psi+L}{r(1-cos\psi)}$$

$$= \left[-\frac{2rr'}{L^3}+\frac{3r'}{r^2}-\frac{L-r}{(r-r'cos\psi+L)L}+\frac{1}{r'(1-cos\psi)}\right]sin\psi \tag{9.20}$$

Formulas (9.14) and (9.18) are also called generalized Vening-Meinesz integral formulas, and formulas (9.17) and (9.20) are generalized Vening-Meinesz kernel functions.

Using the formula (9.14), the vertical deflection at any point outside the geoid be calculated from the gravity anomaly on a certain equipotential surface. And using the formula (9.18), the vertical deflection at any point outside the geoid be calculated from the gravity disturbance on a certain equipotential surface.

If r and r' are taken as constants, the generalized Vening-Meinesz integral formulas (9.14) and (9.18) can be calculated by the fast FFT algorithm.

7.9.3 Integral formula of inverse operation of anomalous gravity field element

(1) Calculation of the gravity disturbance by integral on the height anomaly

According to the definition of gravity disturbance, take the vertical derivative of the Poisson integral formula of disturbing potential T

$$\delta g = \frac{\partial T}{\partial n} \approx -\frac{\gamma \partial \zeta}{\partial r} = -\frac{\gamma}{2\pi} \iint_s \frac{\zeta - \zeta_p}{l^3} ds \qquad (9.21)$$

where n is the vertical line direction (reverse to the radial direction r), and l is the distance between the calculation point and the move point on the sphere:

$$l = 2r \sin\frac{\psi}{2} \qquad (9.22)$$

Formula (9.21) is also known as the inverse Hotine integral formula under spherical approximation.

When the calculation point is the same as the move point, the integral is singular:

$$\delta g|_0 = \frac{\gamma\sqrt{A_0/\pi}}{4}\left(\zeta_{xx} + \zeta_{yy}\right) \qquad (9.23)$$

where ζ_{xx} and ζ_{yy} are the second-order horizontal partial derivatives of the height anomaly at the calculation point, and $\gamma\zeta_{xx}$ and $\gamma\zeta_{yy}$ are the northward direction of the horizontal gravity gradient and the eastward direction of the horizontal gravity gradient, respectively.

Using formula (9.21), the gravity disturbance on the equipotential surface can be calculated from the height anomaly on the surface.

Since the gravity disturbance δg is the derivative of the disturbing potential T along the vertical direction n, formular (9.21) requires that the boundary surface where the height anomaly is located should be an equipotential surface.

(2) Calculation of the gravity anomaly by integral on the height anomaly

Substitute the basic gravimetric equation into formular (9.21) to get:

$$\Delta g = -\frac{\gamma}{2\pi} \iint_s \frac{\zeta - \zeta_p}{l^3} ds - \frac{\zeta\gamma}{2r} \qquad (9.24)$$

Formula (9.24) is also known as the inverse Stokes integral formula under spherical approximation.

Using formula (9.24), the gravity anomaly on the equipotential surface can be calculated from the height anomaly on the surface.

(3) Calculation of the height anomaly by integral on the vertical deflection

$$\zeta = \frac{r}{4\pi} \iint_\sigma ctg\frac{\psi}{2} (\xi cos\alpha + \eta sin\alpha)\, d\sigma \qquad (9.25)$$

When the calculation point is the same as the move point, the integral is singular:

$$\zeta|_0 = \frac{A_0}{4\pi}\left(\xi_y + \eta_x\right) \qquad (9.26)$$

where ξ_y and η_x are the partial derivatives of ξ and η in the east and north directions, respectively.

Using formula (9.26), the height anomaly on the equipotential surface can be calculated from the vertical deflection on the surface.

(4) Calculation of the gravity anomaly by integral on the vertical deflection

$$\Delta g = -\frac{\gamma}{4\pi}\iint_{\sigma}\left(3csc\psi - csc\psi csc\frac{\psi}{2} - tg\frac{\psi}{2}\right)(\xi cos\alpha + \eta sin\alpha)\,d\sigma \tag{9.27}$$

When the calculation point is the same as the move point, the integral is singular:

$$\Delta g|_0 = -\frac{\gamma\sqrt{A_0/\pi}}{4}\left(\xi_y + \eta_x\right) \tag{9.28}$$

Using formula (9.27), the gravity anomaly on the equipotential surface can be calculated from the vertical deflection on the surface.

(5) Calculation of the gravity disturbance by integrating on the vertical deflection

From the basic gravimetric equation, and the formulas (9.25) and (9.27), the integral formula for the gravity disturbance from the vertical deflection can be obtained:

$$\delta g = \frac{-\gamma}{4\pi}\iint_{\sigma}\left(3csc\psi - csc\psi csc\frac{\psi}{2} - tg\frac{\psi}{2} - 2ctg\frac{\psi}{2}\right)(\xi cos\alpha + \eta sin\alpha)\,d\sigma \tag{9.29}$$

When the calculation point is the same as the move point, the integral is singular:

$$\delta g|_0 = -\frac{\gamma}{2\pi}\left(\sqrt{\pi A_0} + \frac{A_0}{r}\right)\left(\xi_y + \eta_x\right) \tag{9.30}$$

Using formula (9.29), the gravity disturbance on the equipotential surface can be calculated from the vertical deflection on the surface.

Formulas (9.25), (9.27) and (9.29) are also known as the inverse Vening-Meinesz integral formula under spherical approximation.

If r is taken as constant, formulas (9.21), (9.24), (9.25), (9.27) and (9.29) can all be calculated by the fast FFT algorithm.

7.9.4 Positive and negative operation formula of anomalous field element integral

(1) Poisson integral formula of anomalous field element

Any type of anomalous gravity field element μ can be expressed by the linear combination of the disturbing potential or its partial derivatives. Therefore, the radial gradient and Poisson integral formula of field element are similar.

Given the anomalous gravity field element on a certain boundary surface, the Poisson integral relation satisfied by the same type of field element at any point (r,θ,λ) outside the geoid:

$$\mu(r,\theta,\lambda) = \frac{1}{4\pi r}\iint_{S}\mu'\frac{r^2 - r'^2}{L^3}ds \tag{9.31}$$

When the calculation point is the same as the move point, $\psi \to 0$, $r' \to rt$, $L \to r\psi$ and $r - r't \to r\psi^2$, the integral is singular. Considering

$$ds = r'^2 sin\psi d\psi d\alpha = \pi r^2\psi_0^2 \tag{9.32}$$

we have $\frac{1}{4\pi r}\iint_{S}\frac{r^2 - r'^2}{L^3}ds = \frac{1}{2r}\int_0^{\psi_0} r^2\frac{\psi^2}{r^3\psi^3}r^2\psi d\psi = \frac{1}{2}\psi_0 = \frac{1}{2r}\sqrt{ds/\pi},$ (9.33)

hence $\mu|_0 = \frac{\mu'}{2r}\sqrt{ds/\pi}.$ (9.34)

(2) Radial gradient integral formula for anomalous field elements

Given the anomalous gravity field element on a certain equipotential surface, the radial gradient of the field element in the Stokes boundary value theory can be calculated by the following integral formula:

$$\frac{\partial\mu}{\partial r}=\frac{1}{2\pi}\iint_S \frac{\mu-\mu'}{l^3}ds \tag{9.35}$$

If r and r' are taken as constants, the integral formulas (9.31) and (9.35) can be calculated by the fast FFT algorithm.

(3) Integral positive and negative operation formula for disturbing gravity gradient

Given the disturbing gravity gradient T_{rr} on some an equipotential surface outside the geoid, the gravity disturbance $\delta g=-T_r$ at any calculation point (r,θ,λ) outside the geoid satisfies the following integral formula:

$$\delta g(r,\theta,\lambda)=\frac{1}{4\pi}\iint_S T_{rr}H(r,\psi,r')ds \tag{9.36}$$

where $H(r,\psi,r')$ is the generalized Hotine kernel function.

Given the gravity disturbance δg on a certain equipotential surface, the disturbing gravity gradient at any point on the equipotential surface can be calculated by the following integral formula:

$$T_{rr}=\frac{1}{2\pi}\iint\frac{\delta g-\delta g'}{l^3}ds \tag{9.37}$$

(4) Calculation of the disturbing gravity gradient by integral on the gravity disturbance

Given the gravity disturbance δg on a certain boundary surface, the disturbing gravity gradient T_{rr} at the any point (r,θ,λ) outside the geoid can also be calculated.

Using the Poisson integral formular (9.31) for the gravity disturbance δg, we have:

$$\delta g(r,\theta,\lambda)=\frac{1}{4\pi r}\iint_S \delta g'\frac{r^2-r'^2}{L^3}ds \tag{9.38}$$

Considering $T_{rr}=\frac{\partial}{\partial r}\left(\frac{\partial}{\partial r}T\right)=-\frac{\partial}{\partial r}(\delta g)$, taking the partial derivatives of both sides of (9.38) with respect to r, we get:

$$T_{rr}=-\frac{1}{4\pi r}\iint_S \delta g'\frac{\partial}{\partial r}\frac{r^2-r'^2}{L^3}ds=\frac{1}{4\pi r}\iint_S \delta g'\frac{r^3-5rr'^2+(r^2+3r'^2)r'^2cos\psi}{L^5}ds \tag{9.39}$$

The formula (9.39) for calculating the disturbing gravity gradient outside the geoid from the gravity disturbance on the boundary surface is derived from the Poisson integral formula for solving the first boundary value problem. Therefore, it is not required that the boundary surface should be a gravity equipotential surface.

7.10 Theory and algorithm of gravity field approach using spherical radial basis functions

The disturbing potential $T(x)$ at the point x outside the Earth can be expressed as a linear combination of normalized surface harmonic basis functions:

$$T(x)=\frac{GM}{r}\sum_{n=2}^{N}\left(\frac{a}{r}\right)^n\sum_{m=-n}^{n}\bar{F}_{nm}\bar{Y}_{nm}(e) \tag{10.1}$$

where $x=r\cdot e=r(sin\theta cos\lambda, sin\theta sin\lambda, cos\theta)$, r,λ,θ are the geocentric distance, longitude and colatitude of the point x outside the Earth, respectively, $\bar{F}_{nm}$ are the fully normalized Stokes coefficients, also known as the geopotential coefficients, GM, a are the geocentric gravitational constant and equatorial radius of the Earth, respectively, called as the scale parameters, and $\bar{Y}_{nm}$ is the normalized surface harmonic function:

$$\bar{Y}_{nm}(e) = \bar{P}_{nm}(cos\theta)cosm\lambda, \quad \bar{F}_{nm} = \delta\bar{C}_{nm}, \quad m \geq 0$$
$$\bar{Y}_{nm}(e) = \bar{P}_{n|m|}(cos\theta)sin|m|\lambda, \quad \bar{F}_{nm} = \bar{S}_{n|m|}, \quad m < 0 \tag{10.2}$$

where $\bar{P}_{nm}(cos\theta)$ is the fully normalized associative Legendre function, n is called the degree of the geopotential coefficient, and m is called order of geopotential coefficients.

The equatorial radius a of the Earth in formula (10.1) represents the boundary surface of the geopotential coefficients as the geoid. If it is replaced by the radius $\mathcal{R}$ of the Bjerhammar sphere, the boundary surface of the geopotential coefficients becomes the Bjerhammar spherical surface. In this case, the geopotential coefficient $\bar{F}_{nm}$ is also converted into $\bar{E}_{nm}$ due to the change of the scale parameter, and $a^n\bar{F}_{nm} = \mathcal{R}^n\bar{E}_{nm}$, the formula (10.1) becomes:

$$T(x) = \frac{GM}{r}\sum_{n=2}^{N}\left(\frac{\mathcal{R}}{r}\right)^n\sum_{m=-n}^{n}\bar{E}_{nm}\bar{Y}_{nm}(e) \tag{10.3}$$

7.10.1 Spherical radial basis function representation of external disturbing potential

The disturbing potential $T(x)$ at any point x outside the Earth can also be expressed as a linear combination of K spherical radial basis functions (SRBF):

$$T(x) = \frac{GM}{r}\sum_{k=1}^{K}d_k\Phi_k(x, x_k) \tag{10.4}$$

where $x_k = \mathcal{R}\cdot e_k$ is the SRBF node defined on the Bjerhammar sphere, also known as the SRBF center or SRBF pole, d_k is the SRBF coefficient, K is the number of the SRBF nodes, equal to the number of SRBF coefficients, $\Phi_k(x, x_k)$ is the spherical radial basis function of the disturbing potential can be abbreviated as $\Phi_k(x) = \Phi_k(x, x_k)$.

The spherical radial basis function $\Phi_k(x, x_k)$ can be furtherly expanded into the Legendre series:

$$\Phi_k(x, x_k) = \sum_{n=2}^{N}\phi_n P_n(\psi_k) = \sum_{n=2}^{N}\frac{2n+1}{4\pi}B_n\left(\frac{\mathcal{R}}{r}\right)^n P_n(\psi_k) \tag{10.5}$$

where ϕ_n is the degree n Legendre coefficient of SRBF, which characterizes the shape of SRBF and basically determines the spatial and spectral figures of SRBF, also known as shape factor. When the spectral domain degree n need be not emphasized, B_n is also called the Legendre coefficient of SRBF. $\mu = \mathcal{R}/r$ is also called the bandwidth parameter because it is related to the spectral domain bandwidth of the radial basis function $\Phi_k(x)$.

Substitute the formula (10.5) into (10.4) to get:

$$T(x) = \frac{GM}{4\pi r}\sum_{n=2}^{N}(2n+1)B_n\left(\frac{\mathcal{R}}{r}\right)^n\sum_{k=1}^{K}d_k P_n(\psi_k)$$
$$= \frac{GM}{4\pi r}\sum_{k=1}^{K}d_k\sum_{n=2}^{N}(2n+1)B_n\left(\frac{\mathcal{R}}{r}\right)^n P_n(\psi_k) \tag{10.6}$$

Considering the addition theorem of spherical harmonics:

$$P_n(\psi_k) = P_n(e, e_k) = \frac{4\pi}{2n+1}\sum_{m=-n}^{n} \bar{Y}_{nm}(e)\bar{Y}_{nm}(e_k), \tag{10.7}$$

then the formula (10.5) can be written as

$$T(x) = \frac{GM}{r}\sum_{n=2}^{N} B_n \left(\frac{\mathcal{R}}{r}\right)^n \sum_{m=-n}^{n}\sum_{k=1}^{K} d_k \bar{Y}_{nm}(e)\bar{Y}_{nm}(e_k) \tag{10.8}$$

Comparing formulas (10.1), (10.3) and (10.8), we have:

$$\bar{F}_{nm} = \left(\frac{\mathcal{R}}{a}\right)^n \bar{E}_{nm} = B_n \left(\frac{\mathcal{R}}{a}\right)^n \sum_{k=1}^{K} d_k \bar{Y}_{nm}(e_k) \tag{10.9}$$

Using formula (10.9), the geopotential coefficient $\bar{F}_{nm}$ can be calculated from the spherical radial basis function coefficient d_k. After that, the geopotential coefficient can be employed to calculate various anomalous gravity field elements outside the Earth.

The position, distribution, and amount of the SRBF nodes (centers) $\{x_k\}$ on the Bjerhammar sphere are the key indicators for gravity field approach using spherical radial basis function, which determine the spatial degrees of freedom (spatial resolution) and spatial feature of regional gravity field, like the degree of the global geopotential coefficient model.

7.10.2 Spherical radial basis functions suitable for gravity field approach

The gravity field approach is according to the geodetic boundary value theory, and the radial basis function employed for the gravity field approach must satisfy the Laplace equation. Some spherical radial basis kernel function such as the point mass kernel function, Poisson kernel function, radial multipole kernel function and Poisson wavelet kernel function are all harmonic.

Let x be the calculation point outside the Earth and x_k be the SRBF center on the Bjerhammar sphere $\Omega_{\mathcal{R}}$.

(1) The point mass kernel function

The point mass kernel function is an inverse multiquadric function (IMQ) proposed by Hardy (1971), which is the kernel function of the gravitational potential integral formula $V = G\iiint \frac{dm}{L}$, and its analytical expression is:

$$\Phi_{IMQ}(x, x_k) = \frac{1}{L} = \frac{1}{|x - x_k|} \tag{10.10}$$

where L is the space distance between x and x_k. Since $\Delta(1/L) = 0$, the point mass kernel function $\Phi_{IMQ}(x, x_k)$ satisfies the Laplace equation.

(2) The Poisson kernel function

The Poisson kernel function is derived from the Poisson integral formula of the anomalous gravity field element, and its analytical expression is:

$$\Phi_P(x, x_k) = -2r\frac{\partial}{\partial r}\left(\frac{1}{L}\right) - \frac{1}{L} = \frac{r^2 - r_k^2}{L^3} \tag{10.11}$$

(3) The radial multipole kernel function

The analytical expression of the radial multipole kernel function is:

$$\Phi_{RM}^m(x, x_k) = \frac{1}{m!}\left(\frac{\partial}{\partial r_k}\right)^m \frac{1}{L} \tag{10.12}$$

where m can be called the order of the radial multipole kernel function, and the zero-order radial multipole kernel function is the point mass kernel function $\Phi_{IMQ}(x, x_k) =$

$\Phi^{0}_{RM}(x,x_k)$.

(4) The Poisson wavelet kernel function

The analytical expression of the Poisson wavelet kernel function is:

$$\Phi^{m}_{PW}(x,x_k)=2(\chi_{m+1}-\chi_m),\quad \chi_m=\left(r_k\frac{\partial}{\partial r_k}\right)^m\frac{1}{L} \qquad (10.13)$$

The zero-order Poisson wavelet kernel function is the Poisson kernel function $\Phi_P(x,x_k)=\Phi^{0}_{PW}(x,x_k)$.

(5) Calculation of spherical radial basis functions

The spherical radial basis function analytical expressions (10.10) ~ (10.13) are usually expressed in the Legendre series (10.5), and then calculated according to the Legendre expansion to highlight the spectral domain figures of the anomalous gravity field element.

PAGravf4.5 normalizes the Legendre expansion of the spherical radial basis function $\Phi_k(x,x_k)$, and then calculates the spherical radial basis function (SRBF) using the normalized Legendre expansion. When dealing with different types of observed field elements, the SRBF of various field elements are uniformly divided by the normalization coefficient of disturbing potential SRBF to maintain the analytical relationship between different types of field elements.

Let the spherical angular distance $\psi_k=0$ from x_k to x, then $cos\psi_k=1$, $P_n(cos\psi_k)=P_n(1)=1$, substitute it into formula (10.5), we have the general expression of the normalization coefficient of disturbing potential SRBF:

$$\Phi^0=\sum_{n=2}^{N}\frac{2n+1}{4\pi}B_n\mu^n \qquad (10.14)$$

The Legendre expansion of the normalized disturbing potential (height anomaly) spherical radial basis function is:

$$\Phi_k(x,x_k)=\frac{1}{\Phi^0}\sum_{n=2}^{N}\phi_n P_n(\psi_k)=\frac{1}{\Phi^0}\sum_{n=2}^{N}\frac{2n+1}{4\pi}B_n\mu^n P_n(\psi_k) \qquad (10.15)$$

The above four forms of disturbing potential SRBF and their corresponding Legendre coefficients are shown in Table 2, where the constant factor 4π in the Legendre coefficient B_n has been removed in advance.

SRBF	$\Phi_k(x,x_k)$	ϕ_n	B_n
Point mass kernel	$\frac{1}{L}=\frac{1}{\lvert x-x_k\rvert}$	μ^n	$\frac{1}{2n+1}$
Poisson kernel function	$\frac{r^2-r_k^2}{L^3}$	$(2n+1)\mu^n$	1
radial multipole kernel	$\frac{1}{m!}\left(\frac{\partial}{\partial r_k}\right)^m\frac{1}{L}$	$C_n^m\mu^{n-m}\ (n\geq m)$	$\frac{C_n^m}{2n+1}\mu^{-m}$
Poisson wavelet kernel	$2(\chi_{m+1}-\chi_m)$ $\chi_m=\left(r_k\frac{\partial}{\partial r_k}\right)^m\frac{1}{L}$	$(-nln\mu)^m(2n+1)\mu^n$	$(-nln\mu)^m$

7.10.3 Spherical radial basis function representation for various gravity field elements

According to the definition of the anomalous gravity field element, the spherical radial basis function parameterized form for various anomalous gravity field elements can be derived from the disturbing potential SRBF series (the rightmost expression) of (10.6).

$$\zeta(x)=\frac{T}{\gamma}=\frac{GM}{4\pi r\gamma}\sum_{k=1}^{K}d_k\sum_n(2n+1)B_n\left(\frac{\mathcal{R}}{r}\right)^n P_n(\psi_k) \tag{10.16}$$

$$\delta g(x)=-\frac{\partial T}{\partial r}=\frac{GM}{4\pi r^2}\sum_{k=1}^{K}d_k\sum_n(2n+1)(n+1)B_n\left(\frac{\mathcal{R}}{r}\right)^{n-1} P_n(\psi_k) \tag{10.17}$$

$$\Delta g(x)=-\frac{\partial T}{\partial r}-\frac{2T}{r}=\frac{GM}{4\pi r^2}\sum_{k=1}^{K}d_k\sum_n(2n+1)(n-1)B_n\left(\frac{\mathcal{R}}{r}\right)^{n-1} P_n(\psi_k) \tag{10.18}$$

$$\xi(x)=\frac{GM}{4\pi r^2\gamma}\sum_{k=1}^{K}d_k\, cos\alpha_k\sum_n(2n+1)B_n\left(\frac{\mathcal{R}}{r}\right)^n \frac{\partial P_n(\psi_k)}{\partial\psi_k} \tag{10.19}$$

$$\eta(x)=\frac{GM}{4\pi r^2\gamma}\sum_{k=1}^{K}d_k\, sin\alpha_k\sum_n(2n+1)B_n\left(\frac{\mathcal{R}}{r}\right)^n \frac{\partial P_n(\psi_k)}{\partial\psi_k} \tag{10.20}$$

$$T_{rr}(x)=\frac{GM}{4\pi r^3}\sum_{k=1}^{K}d_k\sum_n(2n+1)(n+1)(n+2)B_n\left(\frac{\mathcal{R}}{r}\right)^{n-1} P_n(\psi_k) \tag{10.21}$$

where α_k is the geodetic azimuth of ψ_k.

For the regional gravity field approach, the reference geopotential model (such as the EGM2008 model) is usually employed to remove the reference model value from the observed anomalous field element, and the residual gravity field is refined by the observed residual field element.

In this case, the minimum and maximum degree range in formulas (10.16) ~ (10.21) (the spectral bandwidth of the gravity field) is closely related to the selected reference geopotential model and the feature of regional gravity field in the target area, which can only be determined after verifying and analysis from actual observation data.

7.10.4 Spherical Reuter grid construction and SRBF nodes design

PAGravf4.5 adopts the global and regional consistent spherical Reuter grid, constructs the spherical radial basis function SRBF centers according to the given Reuter grid level, and then using the adaptive algorithm, make the space distribution of SRBF nodes be consistent with the space distribution of observations everywhere.

(1) Unit spherical Reuter grid construction algorithm

Given the Reuter grid level Q (even number), the geocentric latitude interval $d\varphi$ of the unit spherical Reuter grid in the spherical coordinate system and the geocentric latitude φ_i of the center of the cell grid i are respectively

$$d\varphi=\frac{\pi}{Q},\quad \varphi_i=-\frac{\pi}{2}+\left(i-\frac{1}{2}\right)d\varphi,\quad 1\le i<Q \tag{10.22}$$

The grid number J_i in the prime vertical circle direction at latitude φ_i, the longitude interval $d\lambda_i$ and the side length dl_i are respectively

$$J_i=\left[\frac{2\pi cos\varphi_i}{d\varphi}\right],\quad d\lambda_i=\frac{2\pi}{J_i},\quad dl_i=d\lambda_i cos\varphi_i \tag{10.23}$$

It is not difficult to find that $dl_i\approx d\varphi$. Let

$$\varepsilon_i=\frac{ds_i-ds}{ds}=\frac{dl_i-d\varphi}{d\varphi}=\frac{d\lambda_i}{d\varphi}cos\varphi_i-1 \tag{10.24}$$

where ds is the cell grid area near the equator, ds_i is the cell grid area at the prime vertical circle φ_i, and ε_i is the relative deviation of the parallel circle cell grid area relative to the cell grid area near the equator.

ε_i is generally small, about a few ten-thousandth, and the value is related to the Reuter grid level Q. Near the equator, we have $ds = d\varphi \cdot d\varphi$, $\varepsilon_{Q/2} = 0$.

Given the range of longitude and latitude of the local area, you can directly determine the minimum and maximum value of i according to the formula (10.22), and then calculate the maximum J_i at each prime vertical circle according to the formula (10.23), to determine the regional Reuter grid whose level is Q without calculating the global grid.

(2) Regional SRBF nodes design with adaptive observation space distribution

PAGravf4.5 presents a simple Reuter grid fitting algorithm to design the SRBF centers that adapts to the space distribution of observations. Firstly, construct a regional Reuter grid from the given level Q, and then count the number J of effective observations in each cell Reuter grid. When J is less than a given number (as the input parameter), eliminate the SRBF center. After traversing all cell Reuter grids, generate the SRBF centers set that adapts to the space distribution of observations.

Obviously, when the observation data is a regular grid, the SRBF centers are also regularly distributed, and when the observations are irregularly distributed, the SRBF centers are also irregularly distributed. The denser the distribution of observations, the denser the distribution of SRBF centers. That is, the space distribution of SRBF centers is consistent with the space distribution of observations everywhere.

7.10.5 SRBF coefficients estimation and gravity field approach

After the constant $GM/(4\pi)$ removed, it does not change the analytical relationship between the anomalous gravity field elements. Therefore, formulas (10.16) ~ (10.21) are rewritten as:

$$\zeta(x) = \frac{1}{r\gamma}\sum_{k=1}^{K} d_k \sum_n (2n+1)B_n\mu^n P_n(\psi_k) \tag{10.25}$$

$$\delta g(x) = \frac{1}{r^2}\sum_{k=1}^{K} d_k \sum_n (2n+1)(n+1)B_n\mu^{n-1} P_n(\psi_k) \tag{10.26}$$

$$\Delta g(x) = \frac{1}{r^2}\sum_{k=1}^{K} d_k \sum_n (2n+1)(n-1)B_n\mu^{n-1} P_n(\psi_k) \tag{10.27}$$

$$\xi(x) = \frac{1}{r^2\gamma}\sum_{k=1}^{K} d_k \cos\alpha_k \sum_n (2n+1)B_n\mu^n \frac{\partial P_n(\psi_k)}{\partial\psi_k} \tag{10.28}$$

$$\eta(x) = \frac{1}{r^2\gamma}\sum_{k=1}^{K} d_k \sin\alpha_k \sum_n (2n+1)B_n\mu^n \frac{\partial P_n(\psi_k)}{\partial\psi_k} \tag{10.29}$$

$$T_{rr}(x) = \frac{1}{r^3}\sum_{k=1}^{K} d_k \sum_n (2n+1)(n+1)(n+2)B_n\mu^{n-1} P_n(\psi_k) \tag{10.30}$$

Substituting the Legendre coefficient B_n in Table 2 into the above equations, we can obtain the basic observation equations for regional gravity field approach with the (residual) anomalous gravity field element $F(x_i)$ as the observations and the SRBF coefficients $\{d_k\}$ as the unknowns.

$$L = \{F(x_i)\}^T = A\{d_k\}^T + e \quad (i = 1,\cdots,M, k = 1,\cdots,K) \tag{10.31}$$

where A is the $M \times K$ design matrix, e is the $M \times 1$ observation error vector, M is the number of observations, K is the number of RBF centers, that is, the number of unknowns $\{d_k\}$, and x_i is the position of the observations.

PAGravf4.5 proposes an algorithm that can improve the performance of parameter estimation by suppressing edge effect. When the RBF center v is located at the margin of the calculation area, its corresponding SRBF coefficient is set to zero, that is, $d_v = 0$ as the observation equation to suppress the edge effect. The normal equation with the additional suppression of edge effect constructed by PAGravf4.5 is:

$$[A^T PA + \epsilon \Xi]\{d_k\}^T = A^T PL \tag{10.32}$$

where Ξ is a diagonal matrix, whose element is equal to 1 only when the SRBF center corresponding to its subscript is in the margin of the area, and the others are zero. ϵ is equal to the diagonal standard deviation of the cofactor matrix $A^T PA$.

In PAGravf4.5, The action distance dr of all SRBF centers is required to be equal to maintain the spatial consistency of the approach performance of gravity field. Where dr corresponds to the domain of the SRBF argument, so any observation is a linear combination of the spherical radial basis functions of the SRBF centers only within the radius dr.

PAGravf4.5 improves the ill-conditioned or singularity of $A^T PA$ by adding some observation equations that can suppresses edge effect to improve the stability and reliability of parameter estimation, to instead of the regularization of the normal equation without geophysical meaning.

You can choose the LU triangular decomposition (square root method), Cholesky decomposition or unknowns smallest norm method to solve the normal equation (10.32).

7.10.6 Regional gravity field modelling from various heterogeneous observations by SRBF

It has always been a hot and difficult issue in physical geodesy to combine various types of observations to model the regional gravity field. Like the surface harmonic coefficient expansion of anomalous gravity field elements, various types of observations can be represented by spherical radial basis function expansion, such as equations (10.25) ~ (10.30). Estimating the spherical radial basis function coefficients with equations (10.25) ~ (10.30) as observation equations, we can model gravity field from various types of observations.

(1) The crucial issues of gravity field modelling using spherical radial basis function from various types of observations

The regional gravity field modelling from various heterogeneous observations by SRBF need deal with three crucial issues, namely ① The SRBF representations from various types of observations should strictly keep the analytical relationship between different types of observations. ② How to determine the contribution of different types of observations to the SRBF coefficients $\{d_k\}$. ③ Investigate the spectral center & bandwidth of target field element, observations and SRBF, and then study the relationship between the three.

For the first issue, only the SRBF Legendre expansion of height anomaly is normalized, and the SRBF Legendre expansion of other types of observations are divided all by the SRBF normalization coefficient of height anomaly. In this way, the analytical relationships can be strictly maintained between different types of observations.

The way to deal with the second issue is to construct observation equations and normal equations from different types of observations firstly, introduce some parameters related to the error or space distribution of observations, and then combine these normal equations to form a new normal equation.

The third issue is related to the observation situations, the nature of gravity field and the modelling algorithm. The spectral center and bandwidth of the observation, target field element and SRBF need be comprehensively analyzed in different parameter combinations case, and then according to the principle of fully resolving the spectrum of the target field element, optimize the relevant scheme and parameters.

(2) Observation contribution adjustment and edge effect suppression

PAGravf4.5 proposes a SRBF gravity field approach method that can suppress edge effect and adjust the contribution of the given type observations at the same time. The normal equation is:

$$\sum_i^{i\neq j}\left(\frac{1}{\varepsilon_i}A_i^T P_i A_i\right)+\frac{\kappa^2}{\varepsilon_j}A_j^T P_j A_j+\epsilon\Xi\{d_k\}^T=\sum_i^{i\neq j}\left(\frac{1}{\varepsilon_i}A_i^T P_i L_i\right)+\frac{\kappa^2}{\varepsilon_j}A_j^T P_j L_j \qquad (10.33)$$

where ε_j is the cofactor matrix combination parameter of the given type of adjustable observation, ε_i is the cofactor matrix combination parameter of the i $(i \neq j)$ observation, and κ is the contribution rate of the adjustable observation j.

PAGravf4.5 multiplies the normal equation coefficient matrix $A_j^T P_j A_j$ and constant matrix $A_j^T P_j L_j$ of the adjustable observation j by κ^2 respectively, to increase ($\kappa > 1$) or decrease ($\kappa < 1$) the contribution of the adjustable observation.

For example, the GNSS-levelling residual height anomaly in the observations can be set as the adjustable observation with the contribution rate $\kappa > 1$ to improve the analytical fusion of GNSS-levelling and other observations. For another example, let the nearshore altimetric elements adjustable with the contribution rate $\kappa < 1$, we can suppress the influence of the shallow water altimetric errors and improve the separation of sea surface topography.

(3) Parameter estimation method with different types of heterogeneous observations

The parameter estimation with several types of observations usually employs the variance component estimation method. In this case, the normal equation need be solved by an iterative solution. The initial variance is the variance of the source observations, and the residual observation variance of the previous iteration result is employed in the iterative process.

Variance component estimation method affected by observation errors, not only has the risk of no stable solution, but also interferes with the analytical nature of the gravity field approach algorithm. PAGravf4.5 proposes a cofactor matrix diagonal standard deviation method to combinate different types of heterogeneous observations for estimation of the SRBF coefficients, to instead of the common variance component estimation method. Which takes ε_i and ε_j as the cofactor matrix diagonal standard deviation of the observations i and cofactor matrix diagonal standard deviation of the adjustable observation j, respectively.

Using the cofactor matrix diagonal standard deviation method, the properties of the parameter estimation solution are only related to the space distribution of the observations without influence of observation errors, so the normal equation does not need be iteratively calculated. Which is conducive to combination of various types of observations with extreme differences in space distribution and improve the analytical nature of the SRBF approach algorithm.

(4) The cumulative SRBF approach method to achieve the best approach of the gravity field

The target field elements are equal to the convolution of the observations and the filter SRBF. When the target field elements and the observations are of different types, it is difficult for one SRBF to effectively match the spectral center and bandwidth of the observations and the target field element at the same time, which would make the spectral leakage of the target field element. In addition, the SRBF type, the minimum and maximum degree of Legendre expansion and the SRBF centers distribution also affect the approach performance of gravity field. Therefore, only the optimal estimation of SRBF coefficient with the burial depth as parameter is not enough to ensure the best approach of gravity field.

To solve this key problem, according to the linear additivity of the gravity field, PAGravf4.5 proposes a cumulative SRBF approach scheme to replace the optimal estimation scheme of SRBF coefficients with the burial depth as parameter. Using the multiple cumulative SRBF approach scheme, it is not necessary to determine the optimal burial depth.

When each SRBF approach adopts SRBF with different spectral figure, the cumulative SRBF approach can fully resolve the spectral domain signal of target field element from multiple SRBF spectral centers and bandwidths, and then optimally restore the target field element in space domain.

The validity principle of once SRBF approach: (1) The residual target field element grid is continuous and differentiable (view the drawing), and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

The character of cumulative SRBF approach scheme of gravity field: the essence of each SRBF approach is to employ the previous approach results as the reference gravity field, and then refine the residual target field element by remove -restore scheme.

Selecting the adjustable observation and its contribution rate κ, we can effectively deal with the problem of high-precision gravity field approach from heterogeneous observations with extreme differences in space distribution, quality and accuracy. With $\kappa = 0$ for some a type of observations, we can effectively detect the gross error and evaluate the quality and external accuracy. With $\kappa > 1$ for several high-precision observations, we can effectively improve the contribution of the several observations such as the astronomical vertical deflection or GNSS-levelling observations.

● **The typical technical features of SRBF approach program in PAGrav4.5:**

① The analytical function relationships between gravity field elements are strict, and the SRBF approach performance has nothing to do with the observation errors.

② Various heterogeneous observations in the different altitudes, cross-distribution, and land-sea coexisting cases can be directly employed to estimate the full element models of gravity field without reduction, continuation, and griding..

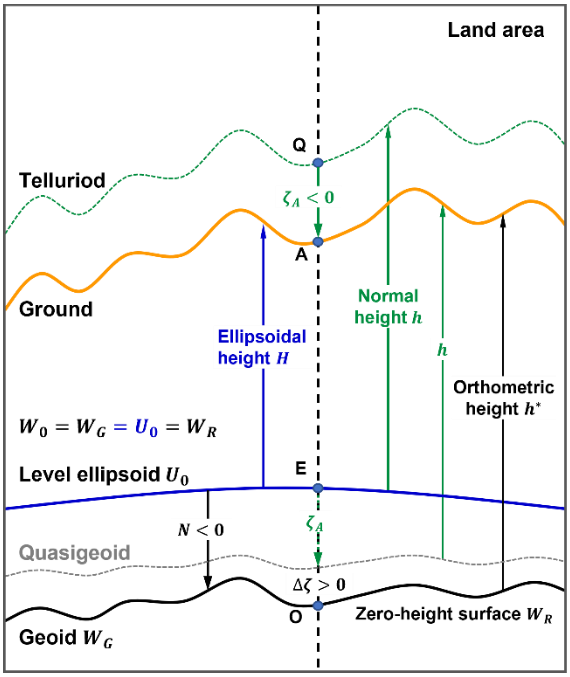
③ Can integrate very little astronomical vertical deflection or GNSS-levelling data, and effectively absorb the edge effect.

④ Has the strong capacity in detection of observation gross errors, measurement of external accuracy indexes, and control of computational performance.

7.11 Height system and its analytic relationship with gravity field

7.11.1 Relationship between the height systems and gravity field

Let the geopotential at the ground point A be W_A, the latitude and longitude of point Q is the same as that of point A, and its normal geopotential U_Q is equal to W_A, then Q is located on the telluroid at A, and QA is equal to the height anomaly ζ_A at point A where the arrow downward means $\zeta_A < 0$, as shown in Figure. In the high-altitude areas, the ground height anomaly $\zeta < 0$, and the telluroid is above the ground.



Without loss of generality, let the geopotential W_R of the zero-height surface in regional height datum, the geoidal geopotential W_G $(= U_0)$, and the global geopotential W_0 (which can be understood as the geopotential of the global height datum) are exactly equal, namely:

$$W_0 = W_G = W_R \qquad (11.1)$$

In this case, the geoid is a closed surface whose geopotential is equal to the normal potential of the ellipsoidal surface, and the potential difference of the height datum is equal to zero, that is, $\delta W_R = W_0 - W_R = 0$. The difference between the geopotential W_A of the zero-height surface and the geopotential W_A of the ground point A is called the geopotential number of the point A, namely $c_A = W_R - W_A = W_0 - W_A$.

The orthometric height is defined in the gravity field space, and it is the ratio of the geopotential number c_A of the point A to the mean gravity $\bar{g}_A$ between the point A and the point O, and the point O is on the zero-height surface namely on the height datum surface.

$$h_A^* = \frac{c_A}{\bar{g}_A} = \frac{W_R - W_A}{\bar{g}_A} = \frac{W_0 - W_A}{\bar{g}_A} = \frac{1}{\bar{g}_A}\int_O^A g dn \qquad (11.2)$$

where dn is the line element between the ground point A and the O point on the height datum surface, and g is the gravity at the line element.

The normal height is defined in the normal gravity field space, which is the ratio of the normal potential number $(= U_0 - U_Q)$ of the telluroid Q at the ground point A to the mean normal gravity $\bar{\gamma}_Q$ between the Q point on the telluroid and the Q point on the normal ellipsoid surface:

$$h_A = \frac{U_0 - U_Q}{\bar{\gamma}_Q} = \frac{1}{\bar{\gamma}_Q}\int_E^Q \gamma dN \qquad (11.3)$$

where dN is the line element between the point E and the Q, and γ is the normal gravity at the line element.

The Molodensky condition assumes that the normal geopotential of the point Q located on the telluroid at point A is equal to the geopotential number of point A, that is, $U_0 - U_Q = c_A = W_0 - W_A$, and substituting it into the formula (11.3), then the normal height of Molodensky is:

$$h_A = \frac{U_0 - U_Q}{\bar{\gamma}_Q} = \frac{W_0 - W_A}{\bar{\gamma}_Q} = \frac{1}{\bar{\gamma}_Q}\int_O^A g dn \tag{11.4}$$

Equation (11.4) is the normal height definition formula adopted by the Chinese current height system.

7.11.2 The rigorous analytical relationship between orthometric and normal height systems

The solution of the Stokes boundary value problem is the disturbing potential of the geoid and in entire Earth space outside the geoid, that is, the Stokes boundary value problem simultaneously determines the geoidal height and the height anomaly outside the geoid, see the formular (9.1).

It is easy to find that the ground height anomaly is also the solution to the Stokes boundary value problem. In particular, the Stoke boundary value problem solution constrains the rigorous analytical relationship between the ground height anomaly ζ and the geoidal height N:

$$\zeta = N + \Delta\zeta = N + \int_0^{h^*} \frac{\partial \zeta}{\partial h} dh = N - \int_0^{h^*} \frac{\delta g}{\gamma} dh \tag{11.5}$$

where dh is the line element between the ground and the geoid, and δg and γ are the gravity disturbance (analytic gravity disturbance) and normal gravity at the line element dh, respectively.

$\Delta\zeta$ in formula (11.5) is the difference between the height anomaly and the geoidal height, that is, the difference between the orthometric height and the normal height. PAGravf4.5 programs 5.5 and 5.6 can be used for this calculation.

According to the basic conditions for the solution of the Stokes boundary value problem, the gravity between the ground and the geoid should be equivalent to the gravity analytically continued to this point from the gravity outside the ground, which is also called as analytical gravity g^*. Rather than the actual gravity g being affected by the terrain and surrounded by the mass, the analytical gravity g^* has a strict analytical relationship with the solution of the Stokes boundary value problem. The solution condition of the Stokes boundary value problem also requires that the actual gravity and the analytical gravity are equal everywhere outside the ground.

The solution to the Stokes boundary value problem points out that the height anomaly ζ_o on the geoid is the geoidal height N. From this, it is easy to find that the zero normal height surface and the zero orthometric height surface coincide everywhere, that is the geoid, whose geopotential are all equal.

In high-altitude areas, the determination accuracy of $\Delta\zeta = \zeta - N$ in formula (11.5) can be effectively improved from the observed gravity or by using regional gravity field data to refine the analytical gravity disturbance on the integral line element.

The mean gravity and integral line element gravity in the definition of orthometric height formula (11.2) replaced with analytical gravity, the strict orthometric height definition that satisfies the solution requirements of the Stokes boundary value problem is obtained:

$$h_A^* = \frac{1}{\bar{g}_A^*}\int_O^A g^* dn \quad (11.6)$$

The zero orthometric height surface coincides with the zero normal height surface, both of which are the geoid. Obviously, only using the analytical gravity g^*, we can ensure that the orthometric height, normal height, height anomaly, geoid and their interrelationships are analytically compatible in Stokes boundary value theory. PAGravf4.5 calls the rigorous orthometric height as the analytical orthometric height.

7.11.3 The problem of quasi-geoid as height datum surface

The geoid can be uniquely determined or continuously refined according to its geopotential value, and it can be employed as the orthometric height starting surface, which meets the requirements of the uniqueness of the geodetic datum. However, it is not theoretically rigorous to regard the quasi-geoid as the normal height starting surface.

(1) The zero normal height surface is the equipotential surface whose geopotential is equal to the geopotential at the height datum zero-point. It is the geoid, not the so-called quasi-geoid.

(2) Two points with the same latitude and longitude but different heights have different height anomalies. Therefore, if the normal height is considered to start from the quasi-geoid, there must be two different starting points in the vertical direction.

(3) In most cases, the measurement points will not be just on the specific ground elevation digital model surface employed in the quasi-geoid modelling. It is necessary to add a gradient (or gravity disturbance) correction for the height anomaly at the measurement point from the quasi-geoid model. see the section 5.1 for the calculation procedure.

PAGravf4.5 downplays the concept of quasi-geoid and does not regard so-called quasi-geoid as the starting surface of normal height. The height anomalies in PAGravf4.5 are strictly in one-to-one correspondence with their spatial locations.

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3	Calculation of global geopotential model and its spectral character analysis	PrModelgravfdcalc
4	Calculation of observed anomalous field elements and error analysis of geoid	ProbsAnomousgrav
5	Correction of boundary value problem for field element on non-equipotential surface	PrBoundaryvalueAdj
6	Analytical continuation of anomalous field elements using multi-order radial gradient	PrGradicontinuation
7	Gross error detection and basis function gridding of discrete gravity field elements	PrGerrweighgridate
8	Computation of local terrain effect on various field elements outside the geoid	TerLocalterraininfl
9	Computation of land, ocean, and lake complete Bouguer effect on gravity outside geoid	TerCompleteBougure
10	Computation of terrain Helmert condensation effect on various field elements outside geoid	TerHelmertcondensat
11	Computation of residual terrain effect on various	Renterraineffect

	field elements outside geoid	
12	Computation of land-sea unified classical gravity Bouguer / equilibrium effect	TerSurfacegravinfl
13	Ultrahigh degree spherical harmonic analysis on land-sea terrain and construction of model	TerGloharmanalysis
14	Spherical harmonic synthesis of complete Bouguer or residual terrain effects	TerHarmrntinfluence
15	Computation process demo of various terrain effects outside geoid	Terraininflexercise
16	External height anomaly computation using Stokes/Hotine integral	IntgenStokesHotine
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19	Gradient and Poisson integral of external gravity field element	Intgendistgradient
20	Feature and performance analysis of spherical radial basis functions	SRBFperformance
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22	Full element modelling on gravity field using SRBFs from heterogeneous observations	SRBFheterogeneous
23	Modelling process exercise of regional gravity field and geoid	Gravfmdlexercise
24	Height deference correction of height anomaly and calculation of height system difference	AppHgtsysdifferent
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29	GNSS replaces leveling to calculate the orthometric or normal height	AppGNSSrepleveling
30	Converting of general ASCII records into PAGravf4.5 format	EdPntrecordstandard
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